A STUDY OF THE AERODYNAMIC DAMPING OF AIRFOILS IN PITCHING OSCILLATIONS

by

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Submitted in Partial Fulfillment of the Requirements for
the Bachelor of Science Degree in Aeronautical Engineering

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May 16, 1940.

May 9, 1940

Professor George W. Swett,
Secretary of the Faculty,
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Dear Sir:

In partial fulfillment of the requirements for the degree of Bachelor of Science, we hereby submit a thesis entitled A STUDY OF THE AERODYNAMIC DAMPING OF AIRFOILS IN PITCHING OSCILLATIONS.

Respectfully,
Signature redacted
Signature redacted

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A STUDY OF THE AERODYNAMIC DAMPING OF AIRFOILS IN PITCHING OSCILLATIONS

INTRODUCTION:

A complete investigation into the many varied aspects of wing flutter should include an investigation of the effects of as many variables as possible upon the stability of the motion once it has been started. Knowledge would be very desirable, for instance, of the effect on the aerodynamic damping of an oscillating wing of such variables as angle of attack, airfoil shape, rate of oscillation of the wing, amplitude of the oscillation, forward speed, surface smoothness, and aspect ratio. The character and magnitude of the damping effect contributed by each variable is very important in determining the stability of the motion.

The existence of negative damping on an oscillating wing, for instance, is very significant, since in this case the airstream feeds energy to the oscillation, and the amplitude may build up to very large values and a probable structural failure and fatal accident occur. Thus negative damping is obviously very undesirable, while the reverse is true of positive damping. The character and magnitude of the aerodynamic damping present may determine the outcome when, by some means, a wing has begun to flutter.

Previously, flutter has been thought most dangerous and likely at high speeds and low angles of attack. No published investigations have been made into the possibility of serious negative damping at low speeds and high angles of attack. This thesis has for its purpose an investigation into this possibility of large negative damping at high angles of attack, as well as a study of the separate effect of as many as possible of the previously-mentioned variables.

Although actual wing flutter is a highly complicated motion of several possible types and does not necessarily take the form of an oscillation such as was used in this thesis, the complication, cost, and time required for a more accurate duplication of the motion prohibited its study in this thesis, and it was decided to adhere to the form of a pure oscillation.

SUMMARY OF CONCLUSIONS:

1) Both the character and magnitude of the aerodynamic damping present vary decidedly with Reynolds' Number. At the same $V/\omega c$ (= 10), for instance, it can be seen from Run #3, as an example, that it is possible to get positive damping for a Reynolds' Number corresponding to 20 M.P.H. airspeed while negative damping is obtained at a Reynolds' Number corresponding to 40 M.P.H. airspeed.

2) In general, there is a tendency for the damping to become more negative as the angle of attack increases, especially with rearward axis (46% chord). This is apparent in the majority of curves of C_p vs. angle of attack, even for cases of different axis position and amplitude. This seems to indicate that the possibility of flutter at high angles of attack should not be neglected.

3) At higher frequencies ($V/\omega c$ of 10), the damping seems to be more negative with forward axis (26.2% chord), but the reverse is true at lower frequencies ($V/\omega c$ of 20). In each case the damping is more negative with increasing angle of attack and rearward axis position.

4) At a high frequency ($V/\omega c$ of 10), the damping becomes more negative with smaller amplitude and exhibits the same trend of more negative damping at higher angles of attack. At a lower frequency ($V/\omega c$ of 20), the trend is just the opposite–that is, the damping becomes less

negative with smaller amplitude,but still exhibits the general trend of increasing negative damping with increasing angle of attack.

DESCRIPTION OF APPARATUS:

Because of the many quantities that must be continuously varied in a test such as this, the apparatus must be designed to allow quick change of adjustment of the various factors as well as quick assembly and disassembly. Also, the strength required is very high because of the fairly high weight that must be oscillated at quite high speeds.

All tests were run in the M.I.T. 5 foot wind tunnel, which is of the closed-throat, open-return type. The airfoil tested was a laminated mahogany wing, symmetrical section N.A.C.A. 0010, of 5" X 28½" dimensions. This airfoil was held by brass plates screwed onto its surfaces at both ends. Half-inch steel rod, with steel plates butted against the ends adjacent to the airfoil, was used as the axis. These steel plates were provided with holes so that three different axes positions were possible by merely putting the bolts projecting from the ends of the airfoil into different holes.

Because of the necessity for keeping friction down, especially starting and stopping friction, ball bearings were used - one near each end of the airfoil, and one at the point where the axis passed through the tunnel wall.

Power to oscillate the wing was supplied by a D.C. shunt-wound series motor. The speed of this motor could be varied by rheostats connected in series with the line. A diagram of the electrical hook-up is given in Figure 1. A flywheel was attached to the motor shaft, and to this flywheel was attached an eccentric arm running to an arm on the airfoil shaft. The eccentric can be seen in Figure 3. Setscrews were used to hold the arm onto the shaft, so that the angle of attack could be quickly changed by adjusting these setscrews. The amplitude was adjusted by moving the point of attachment of the eccentric arm to the flywheel inward or outward along the radius.

The revolutions per minute of the motor, or the oscillations per minute of the airfoil,were measured by means of a hand-type direct-reading tachometer. Power input to the oscillating airfoil was measured by means of an ammeter and a voltmeter, connected as shown in Figure 1.

The amplitude of the oscillation was measured with the help of a stroboscope.

Several photographs, both inside and outside the tunnel, are shown in Figures 2, 3, and 4.

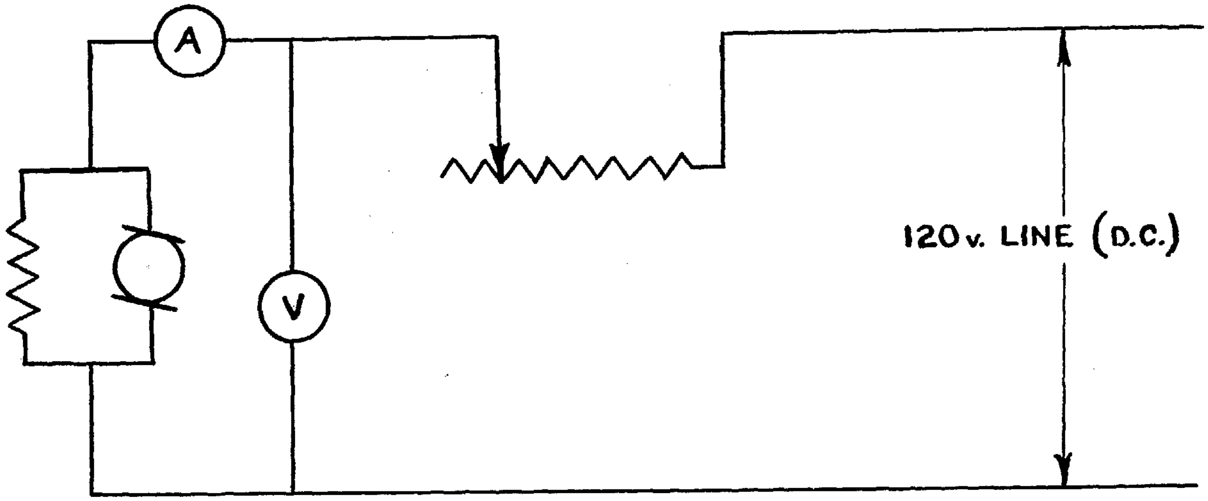

Figure 1

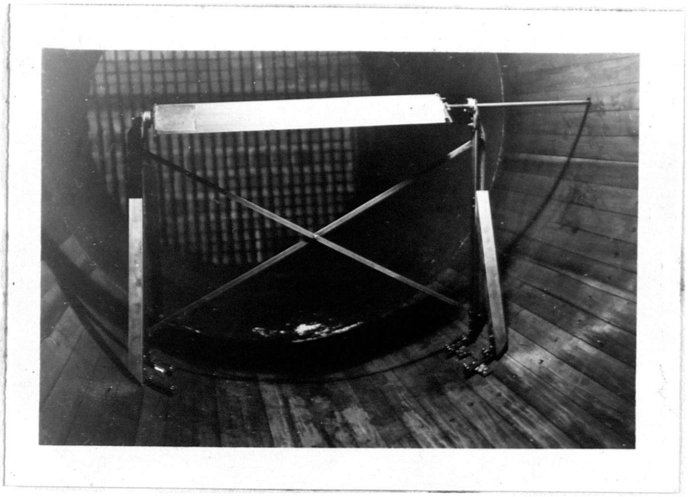

Figure 2

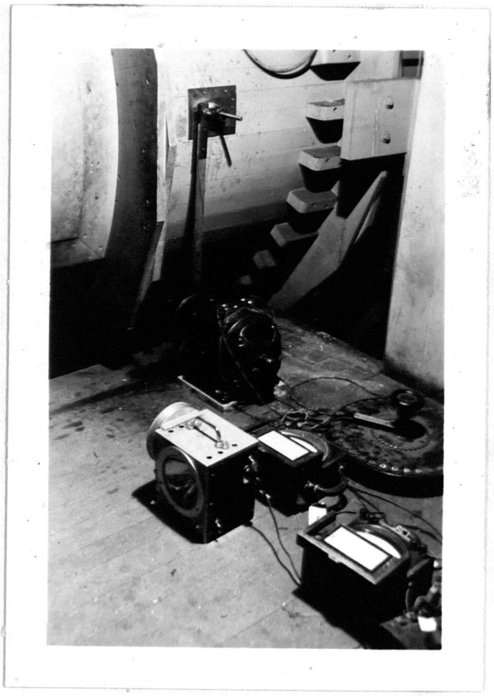

Figure 3

Figure 4

PROCEDURE OF TEST:

The first step in each run was to obtain a set of data for the no-wind condition. The motor speed was varied from as low a speed as possible up to a speed giving about 20 oscillations per second. Original plans called for speeds up to 30 oscillations per second, but the rapid increase in power required with speed and the increasing effect on the vibration of a slight unbalance in the apparatus caused the upper limit to be lowered. Further, it developed that trends were brought out sufficiently well in the range used.

At each motor speed readings were taken of the motor revolutions per minute and the ammeter and voltmeter, so that the power input to the motor was known. A curve could then be drawn of power input in watts against motor speed for the no-wind condition. This curve then served as the basic reference line. Runs were then made, using the same procedure, at several different wind speeds.

A series of curves were then available on one sheet of the motor input in watts against the motor speed for several different wind speeds. Every time a variable such as axis position or amplitude was changed, another series of tests would be run. The same three angles of attack were covered for every series, so that the isolated effect of the variables could be traced through and determined.

PLOTTING CURVES :

When the different series of curves of motor input in watts against motor speed (oscillations per minute) were available, the next step was to transfer them to general or non-dimensional form. This was done by transferring the damping power in watts into a non-dimensional coefficient and plotting this against the quantity $V/\omega c$, which is a non-dimensional measure of the wave set up in the airfoil wake by the oscillation. These quantities are then dimensionless, and for a given Reynold's Number these points for different speeds should fall on the same curve.

Finally, cross-plots were made of the effect of the separate variables. In each case the damping coefficient C_p was plotted against the angle of attack. The effect of each variable was then brought out by holding the other variables fixed and varying the desired quantity. To bring out the effect of Reynold's Number, for instance, a plot of C_p against the angle of attack was made for a fixed $V/\omega c$ and various Reynolds' Numbers. In the case of amplitude and axis position, for which only enough time was available to make two changes, a tabulation was made of the damping factor for various values of $V/\omega c$ and Reynold's Number. From this comparison estimates were made of the effect of these two variables.

DERIVATION OF PARAMETERS :

The abscissa for all the general curves is the quantity $V/\omega c$,where

V is the forward velocity in feet per second

ω is the angular velocity in radians per second

c is the wing chord in feet

The ordinate for all the general curves is the quantity C_p ,which is a non-dimensional damping coefficient. At any wind speed, this quantity can be obtained by taking the damping power in watts at the desired oscillation speed and substituting in the following relation:

$$C_p = \frac{(\text{Damping power in watts})\ 550}{(\omega)\ (q)\ (c)^3\ 746}$$

where

c is the wing chord in feet

ω is the angular velocity in radians per second

q is the dynamic pressure in pounds per sq. ft.

SUMMARY OF RUNS

Run No.	Axis (% chord)	* Amplitude (degrees)	Wind Speeds (M.P.H.)	Mean Angle of Attack
1	26.2	60	15,30,45,60	15.5°
2	26.2	60	15,30,45,60	6.4°
3	26.2	60	20,40,60	-2.0°
4	46.0	60	20,40,60	-2.0°
5	46.0	60	20,40,60	6.4°
6	46.0	60	20,40,60	15.5°
7	46.0	51	20,40,60	15.5°
8	46.0	51	20,40,60	6.4°
9	46.0	51	20,40,60	-2.0°

* From full up to full down.

DISCUSSION OF RESULTS :

In general, the curves seem to be consistent. Further, the curves in each series exhibit the same trends.

The very pronounced effect of Reynolds' Number on both the character and magnitude of the aerodynamic damping present is probably due to the variation in the flow pattern around the airfoil with Reynolds' Number (wind speed, since the chord is fixed).

The pronounced effect of oscillating speed ($V/\omega c$) is probably due to flow variations. At low values of $V/\omega c$, or high oscillating speeds, the flow would tend to follow the airfoil form more closely , while at high values of $V/\omega c$, or low oscillating speeds, the flow would have more tendency to separate.

No detailed attempt will be made to explain the variation of damping with angle of attack or amplitude because of the large number of unknown variables. It would probably be sufficient to say that lift lag of an oscillating airfoiland the change in slope of the lift and moment curves in the region beyond the burble point are major factors.

Also, the theory for oscillating airfoils has been built up especially for small amplitudes, while the amplitudes in this test were larger than those covered by theory.

Nothing will be said on how the conslusions for this test, based on a symmetrical section, can be applied to cambered airfoils.

LIMITATIONS OF TEST :

Not much difficulty was experienced with the apparatus over the test range except that at a motor speed of about 700 R.P.M. resonance of the apparatus occurred in each case, and it was impossible to get accurate readings at this speed. This speed was passed over in each test. Sufficient points were nevertheless available to get smooth curves over the entire test range.

The second series of curves-that is, runs 4 through 6, were of unexplainable shape in certain ranges. This is probably due to experimental error, since the most vibration trouble was experienced with the apparatus in these runs. The other 6 runs, however, gave reasonable curves.

Probably the most chance for experimental error came from the motor speed measurement, since there was some tendency for motor speed fluctuation, especially at low speeds.

Although the support for the airfoil in the tunnel was necessarily rather strong in construction and thus tended to introduce turbulence, this effect was probably negligible because of the distance from the airfoil. The airfoil was polished except at its ends, where the necessity of screws in the fastening device to the oscillation axis introduced some roughness.

The aspect ratio attained, 5.7, was rather low because of limitations of available equipment. Limitations of time prevented the testing of two cambered airfoils available.

SUGGESTIONS FOR FUTURE INVESTIGATORS

The results of this investigation seem to indicate that tests of this nature should be run at largest Reynolds' Numbers to be of more value for full scale airplanes, because of the decided variation in damping characteristics with Reynolds' Number. In order to obtain low values of $V/\omega c$ but large values of Reynolds' Number, it would be desirable to increase the physical size of the wing as well as running at as high an oscillating speed as possible. This would increase Reynolds' Number but would tend to decrease $V/\omega c$.

Also, it would be advisable to carefully counterbalance the motor so that high oscillating speeds could be obtained, and also to provide a minimum of play in the joints. A heavier motor flywheel would perhaps be desirable, since it might hold the motor speed more constant at low speed. Considerable care to eliminate vibration is also desirable.

Further, more detailed investigation than was possible in the short time available in this thesis should be made into the isolated effect of the many possible variables already mentioned.

It would be advisable in future tests to have a smaller amplitude and to reduce the weight of the oscillating system as much as possible.

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Run #1

Axis of rotation- 26.2% chord
Angle of attack- 15.5 degrees
Amplitude of oscillations- 60 degrees

-No Wind-

Motor R.P.M.	Amperes	Volts	Watts
220	1.30	12.9	16.8
365	1.45	16.9	24.5
500	1.56	21.5	33.6
550	1.64	24.0	39.4
600	1.74	26.0	45.2
770	1.97	40.6	80.0
850	2.05	50.0	102.5
910	2.07	70.0	145.0

-Wind On = 15 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
315	1.20	14.5	17.5
450	1.28	19.0	24.3
585	1.50	25.7	38.5
870	2.01	54.0	108.3
1035	2.16	86.0	185.7

-Wind On - 30 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
200	1.30	15.0	19.5
385	1.38	180	24.9
500	1.40	21.1	29.6
760	1.75	38.9	68.1
860	1.90	50.0	95.0
910	1.91	61.0	116.4
1090	2.10	87.0	182.5

-Wind On- 45 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
200	1.34	160	21.5
365	1.44	18.9	27.2
530	1.64	24.0	39.4
700	1.71	35.5	61.8
835	1.95	47.0	91.6
1040	2.05	84.5	173.0

-Wind On- 60 M.PH.-

Motor R.P.M.	Amperes	Volts	Watts
200	1.53	14.7	22.5
395	1.55	20.0	31.0
550	1.69	26.0	43.9
780	1.80	45.0	81.0
810	1.85	47.0	86.9
1060	2.15	88.0	189.0

Run #2

Axis of rotation- 26.2% chord
Angle of attack- 6.4 degrees
Amplitude of oscillation- 60 degrees

-See Run #1 for No Wind readings-

-Wind On- 15 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
220	1.44	14.0	20.2
400	1.48	19.0	28.1
540	1.60	23.1	36.9
700	1.78	32.2	57.3
820	1.99	45.0	89.5
1000	2.00	85.0	170.0

- Wind On- 30 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
220	1.24	13.9	17.2
420	1.30	20.0	25.9
530	1.42	22.0	31.2
620	1.50	26.0	39.0
715	1.59	32.0	50.8
810	1.69	40.9	69.2
1060	1.86	86.0	160.0

-Wind On - 45 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
220	1.45	14.5	21.0
410	1.45	19.3	28.0
560	1.51	25.2	37.9
620	1.62	25.2	40.8
650	1.62	29.0	46.9
780	1.67	37.5	62.7
840	1.87	47.0	88.0
930	1.82	76.0	138.0
1080	1.89	91.0	172.0

-Wind On- 60 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
200	1.46	14.0	20.4
410	1.50	20.0	30.0
480	1.56	21.0	32.8
710	1.72	32.0	55.0
790	1.78	40.0	71.2
900	1.80	65.0	117.0
1050	1.84	82.0	149.0

Run # 3

Axis of rotation- 26.2% chord
Angle of attack- -2 degrees
Amplitude of oscillations- 60 degrees

-See Run #1 for No Wind readings-

-Wind On- 20 M.p.h.-

Motor R.P.M.	Amperes	Volts	Watts
230	1.40	14.0	19.6
400	1.50	18.0	27.0
560	1.76	24.5	43.1
700	1.85	35.0	64.7
850	2.11	48.6	102.5
980	2.35	72.0	169.0
1000	2.35	74.0	174.0

-Wind On- 40 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
200	1.60	16.0	25.6
420	1.40	20.5	28.8
550	1.69	26.5	44.7
670	1.70	34.0	57.8
810	1.83	44.0	80.5
1020	1.91	73.5	140.0

- Wind On- 60 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
250	1.56	17.0	26.5
340	1.60	18.0	28.8
470	1.55	23.0	35.7
630	1.73	29.8	51.6
825	1.80	48.2	86.8
1060	1.90	80.5	153.0
1090	2.00	85.0	170.0

Run #4

Axis of rotation- 46.0% chord
Angle of attack- -2 degrees
Amplitude of oscillations- 60 degrees

-No Wind-

Motor R.P.M.	Amperes	Volts	Watts
250	1.18	13.0	15.3
350	1.18	15.0	17.7
380	1.20	16.0	19.2
500	1.28	20.1	25.8
610	1.39	26.0	36.1
700	1.46	30.0	43.8
780	1.35	43.0	58.0
960	1.41	67.5	95.2
1010	1.50	73.0	109.5
1050	1.52	78.0	118.5

-Wind On- 20 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
360	1.10	16.5	18.2
390	1.14	17.2	19.6
490	1.20	21.4	25.7
585	1.31	27.5	36.0
670	1.35	30.2	40.7
700	1.36	32.0	43.5
775	1.52	39.0	59.3
1010	1.45	69.0	100.0
1045	1.50	74.9	112.2

-Wind On- 40 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
440	1.05	19.0	20.0
500	1.10	20.0	22.0
600	1.24	24.9	30.9
685	1.16	32.1	37.3
1000	1.39	68.2	94.6
1020	1.43	70.6	100.0
1050	1.43	74.0	105.6

-Wind On- 60 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
535	1.06	19.0	20.1
615	1.26	23.8	30.0
660	1.15	28.2	32.4
800	1.31	42.0	55.0
980	1.35	71.0	95.9
1035	1.42	74.3	105.0
1100	1.55	85.3	132.0

Run #5

Axis of rotation- 46.0% chord
Angle of attack- 6.4 degrees
Amplitude of oscillations- 60 degreees

- See Run #4 for No Wind readings-

-Wind On- 20 M.PH.-

Motor R.P.M.	Amperes	Volts	Watts
405	1.10	16.4	18.1
455	1.16	19.0	22.0
540	1.25	22.1	27.6
650	1.39	27.9	38.7
765	1.56	34.0	52.7
980	1.35	71.0	95.8
1040	1.40	75.0	105.0
1090	1.49	81.0	120.5

-Wind On- 40 M.PH.-

Motor R.P.M.	Amperes	Volts	Watts
510	1.05	20.8	21.8
600	1.15	25.0	28.8
660	1.25	28.1	35.1
980	1.41	66.5	93.7
1010	1.41	69.0	97.3
1040	1.47	76.0	112.0
1080	1.50	84.0	126.0

-Wind On- 60 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
400	1.00	10.5	10.5
600	1.05	19.0	20.0
650	1.10	23.0	25.3
1000	1.54	66.5	102.5
1030	1.45	72.0	104.5
1085	1.50	82.0	123.0

Run # 6

Axis of rotation- 46.0% chord
Angle of attack- 15.5 degrees
Amplitude of oscillations- 60 degrees

-See Run #6 for No Wind readings-

-Wind on- 20 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
370	1.16	14.3	16.6
430	1.20	16.2	19.4
500	1.29	20.1	25.9
610	1.35	24.0	32.4
690	1.39	28.0	38.9
790	1.34	41.0	54.9
1020	1.40	67.1	93.9
1060	1.46	74.0	108.0

- Wind On- 40 M.PH. -

Motor R.P.M.	Amperes	Volts	Watts
570	1.02	22.5	23.0
620	1.05	25.0	26.0
650	1.08	26.1	28.2
690	1.10	27.9	30.7
785	1.29	36.0	46.4
1020	1.30	70.0	91.0
1040	1.35	70.0	94.5
1095	1.35	80.1	108.0

-Wind On- 60 MP.H.-

Motor R.P.M.	Amperes	Volts	Watts
700	0.95	30.0	28.5
715	0.94	31.0	29.1
730	1.09	32.9	35.8
735	1.10	33.0	36.3
810	1.25	40.0	50.0
950	1.30	;60.0	78.0
960	1.40	62.0	86.8
1000	1.34	69.0	92.4
1080	1.35	79.0	105.0
1110	1.38	85.0	117.0
1110	1.40	86.0	120.0

Run#7

Axis of rotation- 46.0% chord
Angle of attack- 15.5 degrees
Amplitude of oscillation- 51 degrees

- No Wind -

Motor R.P.M.	Amperes	Volts	Watts
200	1.35	15.2	20.5
530	1.01	25.2	25.5
690	0.93	33.1	30.8
860	0.96	45.0	43.2
900	1.06	47.1	50.0
980	1.02	60.0	61.2
1050	1.01	70.0	70.7
1100	1.00	77.5	77.5
1130	0.99	83.0	82.2

-Wind On- 20 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
250	1.14	12.5	14.3
410	1.11	14.6	16.2
640	1.00	23.5	23.5
790	0.90	35.0	31.5
970	0.95	55.0	52.2
1070	0.91	71.5	65.0

-Wind On- 40 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
150	1.19	10.5	12.5
270	1.12	12.0	13.5
450	1.15	13.0	16.0
630	1.05	21.0	22.0
710	1.04	25.1	26.1
760	1.10	27.5	30.2
1000	0.87	62.0	54.0
1050	0.90	67.0	60.3
1070	0.86	74.0	63.5

- Wind On- 60 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
200	1.15	11.2	12.9
760	0.85	29.4	25.0
930	0.90	44.0	39.6
980	0.85	54.0	45.8
810	0.95	29.0	27.5
1050	0.85	64.0	53.3
1090	0.85	69.0	58.6

Run #8

Axis of rotation- 46.0% chord
Angle of attack- 6.4 degrees
Amplitude of oscillation- 51 degrees

-See Run #7 for No Wind readings-

-Wind On-20 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
300	1.35	14.5	19.50
380	1.30	15.7	20.4
535	1.20	18.7	22.5
780	1.12	18.9	35.0
890	1.00	45.0	45.0
840	1.15	34.5	39.7
1020	1.01	57.5	58.0
1085	0.96	72.0	69.1

-Wind On- 40 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
300	1.30	13.5	17.5
460	1.15	16.9	19.4
610	1.14	20.0	22.8
750	1.06	27.0	28.6
840	1.09	32.5	35.4
960	1.02	47.0	48.0
1060	0.89	68.5	60.9

-Wind On- 60 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
370	0.89	11.0	9.8
470	0.94	16.0	15.0
600	0.94	20.2	19.0
660	0.95	23.5	22.3
840	1.00	37.5	37.5
950	1.00	47.5	47.5
980	0.90	63.0	56.5
1010	0.90	65.0	58.4
1100	0.91	76.0	69.2

Run #9

Axis of rotation- 46.0% chord
Angle of attack- -2 degrees
Amplitude of oscillation - 51 degrees

-See Run #7 for No Wind readings-

-Wind On- 20 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
340	1.25	15.0	18.7
450	1.20	17.5	210
650	1.00	26.5	26.5
740	1.18	26.5	31.3
800	1.21	28.0	33.9
970	0.90	57.0	51.1
1010	0.95	58.5	55.5
1080	0.95	67.0	63.6
1120	0.95	74.0	70.3

-Wind On- 40 M.P.H.-

Motor R.P.M.	Amperes	Volts	Watts
250	1.41	11.0	15.6
575	0.85	25.0	21.2
690	0.89	27.5	24.4
710	1.05	25.0	27.5
740	0.95	29.0	27.5
925	1.02	41.0	41.8
1000	0.95	53.0	50.3
1110	0.95	68.5	65.0

-Wind On- 60 M.P.H. -

Motor R.P.M.	Amperes	Volts	Watts
350	0.85	13.0	11.1
530	0.80	21.5	17.2
610	0.87	22.5	19.6
710	0.95	27.0	25.6
820	1.02	345	35.2
1060	0.95	66.0	62.7
1090	0.96	68.5	65.7
1110	0.96	75.5	72.5

Results Of Calculations

Run #1

V- 15 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p	V/ωc
-2.6	200	-.0955	15.9
-3.8	400	-.0698	7.95
-5.0	600	-.0613	5.30
-9.0	800	-.0826	3.98
-30.4	1000	-.24200	3.18

V- 30 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p	V/ωc
3.8	200	.0352	31.8
1.5	400	.00695	15.9
-5.2	600	-.0148	10.6
-12.0	800	-.0278	7.96
-50.0	1000	-.0925	6.36

V- 45 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p	V/ωc
5.1	200	.0209	47.70
4.1	400	.0084	23.85
2.1	600	.0029	15.90
-4.1	800	-.0042	11.94
-43.3	1000	-.0350	9.54

V- 60 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p	V/ωc
6.10	200	.0140	63.6
6.20	400	.0070	31.8
4.30	600	.0033	21.2
-3.50	800	-.0020	15.92
-33.40	1000	-.0155	12.72

Run #2

V- 15 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
3.5	200	.1290
2.1	400	.0386
-2.1	600	-.0258
-9.5	800	-.0874
-29.5	1000	-.2170

V- 30 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
2.5	200	.0231
-.3	400	-.0014
-7.8	600	-.0240
-23.8	800	-.0550
-65.6	1000	-.1220

V- 45 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
4.5	200	.0185
2.2	400	.00451
-4.2	600	-.00574
-22.3	800	-.02290
-70.3	1000	-.05760

V- 60 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
5.8	200	.0133
3.6	400	.0041
-3.0	600	-.0023
-18.0	800	-.0103
-64.2	1000	-.0296

Run #3

V- 20 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p	$V/\omega c$
3.3	200	.0672	21.1
3.0	400	.0310	10.6
3.0	600	.0207	7.06
-3.3	800	-.0171	5.30
-20.9	1000	-.0865	4.24

V- 40 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p	$V/\omega c$
7.7	200	.0400	42.2
7.5	400	.0195	21.2
4.1	600	.0070	14.12
-11.2	800	-.0145	10.60
-65.5	1000	-.0678	8.48

V- 60 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p	$V/\omega c$
8.5	200	.0196	63.3
6.2	400	.0071	31.8
2.8	600	.0021	21.2
-10.0	800	-.0046	15.9
-64.7	1000	-.0290	12.72

Run #4

V- 20 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-.50	200	-.01035
-.40	400	-.00415
-1.00	600	-.00690
-2.80	800	-.01450
-5.10	1000	-.02110
-7.00	1200	-.02410

V- 40 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-1.8	200	-.0093
-2.2	400	-.0057
-4.1	600	-.0071
-7.2	800	-.0093
-9.9	1000	-.0103
-10.5	1200	-.0091

V- 60 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-8.9	200	-.0204
-6.6	400	-.0076
-6.1	600	-.0047
-6.3	800	-.0036
-6.3	1000	-.0029
-4.8	1200	-.0018

Run #5

V- 20 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-1.2	200	-.0248
-1.2	400	-.0124
-1.6	600	-.0110
-3.0	800	-.0155
-5.3	1000	-.0220
-7.0	1100	-.0264

V- 40 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-4.6	200	-.0238
-4.1	400	-.0106
-4.8	600	-.0083
-5.0	800	-.0065
-4.1	1000	-.0043
-4.6	1100	-.00435

V- 60 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-12.1	200	-.0230
-10.1	400	-.0116
-11.5	600	-.0088
-11.9	800	-.0068
-7.4	1000	-.0034
-3.4	1100	-.00142

Run #6

V- 20 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-1.9	200	-.0394
-2.0	400	-.0207
-2.8	600	-.0193
-5.2	800	-.0269
-10.9	1000	-.0453
-15.1	1200	-.0520

V- 40 M.P.H.

Damping Power ø (Watts)	R.P.M. (Motor)	c_p
-5.5	200	-.0285
-5.5	400	-.0142
-8.2	600	-.0141
-13.8	800	-.0179
-18.2	1000	-.0189
-21.8	1200	-.0188

V- 60 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-11.7	500	-.01075
-13.0	600	-.01000
-15.8	800	-.00910
-14.0	1000	-.00645
-15.5	1200	-.00594

Run #7

V- 20 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-6.3	200	-.1300
-6.3	400	-.0650
-5.9	600	-.0410
-6.0	800	-.0310
-7.1	1000	-.0290
-8.2	1100	-.0310
-12.0	1200	-.0410

V- 40 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-7.6	200	-.0394
-7.3	400	-.0191
-6.8	600	-.0118
-6.7	800	-.0087
-7.0	1000	-.0073
-9.2	1200	-.0080

V- 60 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-7.7	200	-.0177
-8.6	400	-.0099
-9.8	600	-.0075
-11.1	800	-.0064
-13.2	1000	-.0061
-15.7	1100	-.0066
-20.1	1200	-.0077

Run #8

V- 20 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-1.7	200	-.0352
-2.2	400	-.0228
-2.4	600	-.0166
-2.5	800	-.0129
-4.4	1000	-.0182
-6.2	1100	-.0234
-8.6	1200	-.0297

V- 40 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-3.3	200	-.0171
-4.0	400	-.0104
-4.7	600	-.0081
-5.5	800	-.0071
-8.3	1000	-.0086
-10.7	1100	-.0101
-13.8	1200	-.0119

V- 60M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-10.8	200	-.0246
-9.8	400	-.0112
-7.5	600	-.0058
-5.1	800	-.0029
-4.5	1000	-.0021
-5.6	1100	-.0021
-8.3	1200	-.0032

Run #9

V- 20 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-2.2	200	-.0455
-2.5	400	-.0258
-2.7	600	-.0186
-3.9	800	-.0202
-6.7	1000	-.0278
-9.9	1100	-.0372

V- 40 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-4.7	200	-.0244
-4.9	400	-.0127
-5.8	600	-.0100
-6.2	800	-.0081
-6.8	1000	-.0071
-9.9	1100	-.00935

V- 60 M.P.H.

Damping Power (Watts)	R.P.M. (Motor)	C_p
-10.4	200	-.0239
-9.0	400	-.0103
-7.4	600	-.0057
-8.2	800	-.0047
-6.8	1000	-.0031
-7.9	1100	-.0034

EFFECT OF AXIS POSITION AND AMPLITUDE ON DAMPING :

A. Effect of Axis Position

Reynold's Number- 235,000
Amplitude- 60 degrees
$V/\omega c$ - 10

$\alpha = 15.5°$

Axis Position (% chord)	C_p
26.2%	-.034
46.0%	-.006

$\alpha = 6.4°$

Axis Position (% chord)	C_p
26.2%	-.0550
46.0%	-.00275

$\alpha = -2.0°$

Axis Position (% Chord)	C_p
26.2%	-.1200
46.0%	.0005

Reynold's Number- 235,000
Amplitude- 60 degrees
$V/\omega c$ - 20

$\alpha = 15.5°$

Axis Position (% chord)	C_p
26.2%	.0025
46.0%	-.0100

$\alpha = 6.4°$

Axis Position (% chord)	C_p
26.2%	-.0040
46.0%	-.0080

$\alpha = -2°$

Axis Position (% chord)	C_p
26.2%	.0050
46.0%	-.0045

B. Effect of Amplitude

Reynold's Number- 235,000
Axis position- 46% chord
$V/\omega c$- 10

$\alpha = 15.5^\circ$

Amplitude (degrees)	C_p
60	-.0060
51	-.0085

$\alpha = 6.4^\circ$

Amplitude (degrees)	C_p
60	-.00275
51	-.0040

$\alpha = -2.0^\circ$

Amplitude (degrees)	C_p
60	.0005
51	-.0050

Reynold's Number- 235,000
Axis position- 46% chord
$V/\omega c$- 20

$\alpha = 15.5^\circ$

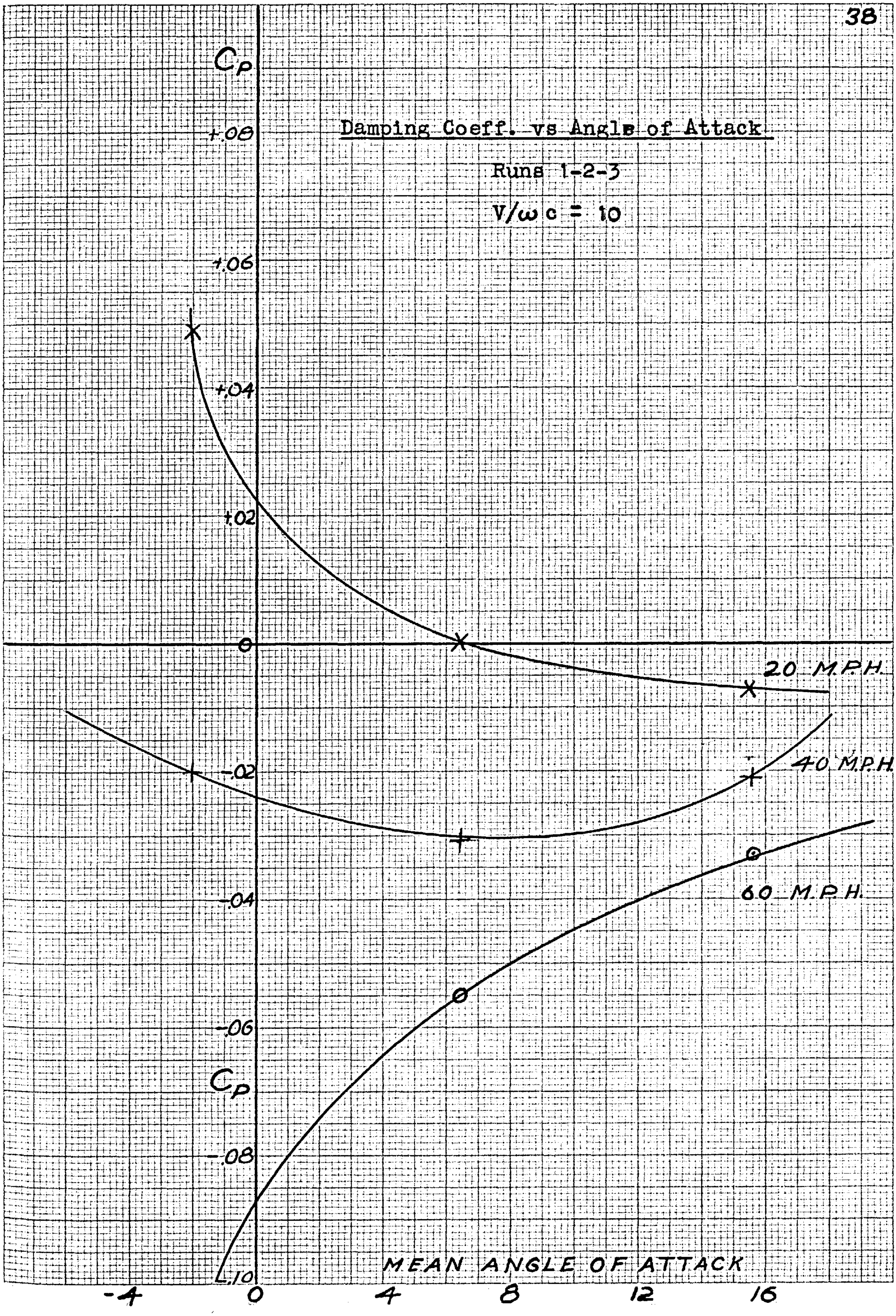
Amplitude (degrees)	C_p
60	-.0100
51	-.0070

$\alpha = 6.4^\circ$

Amplitude (degrees)	C_p
60	-.0080
51	-.0050

$\alpha = -2.0^\circ$

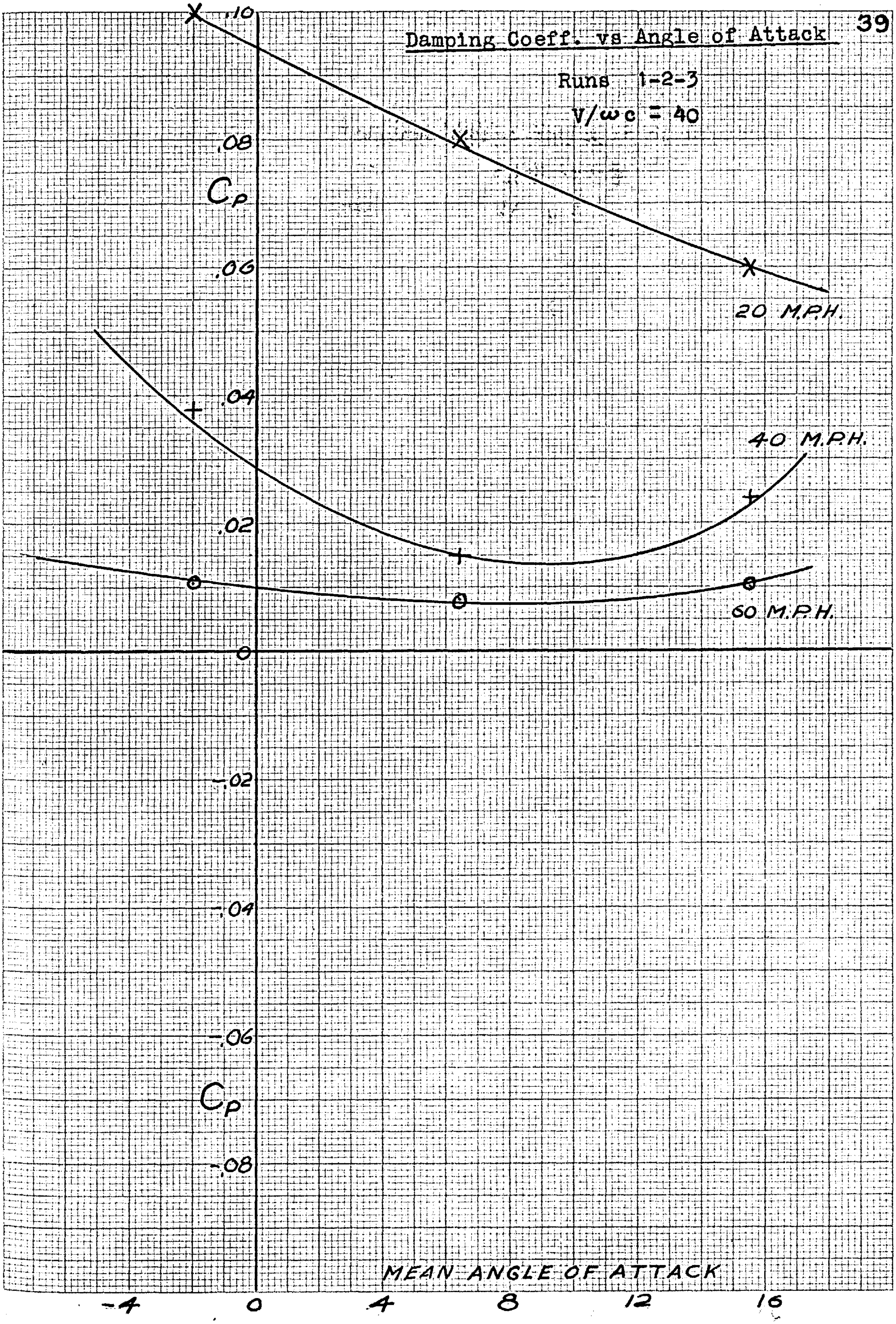
Amplitude (degrees)	C_p
60	-.0045
51	-.0055

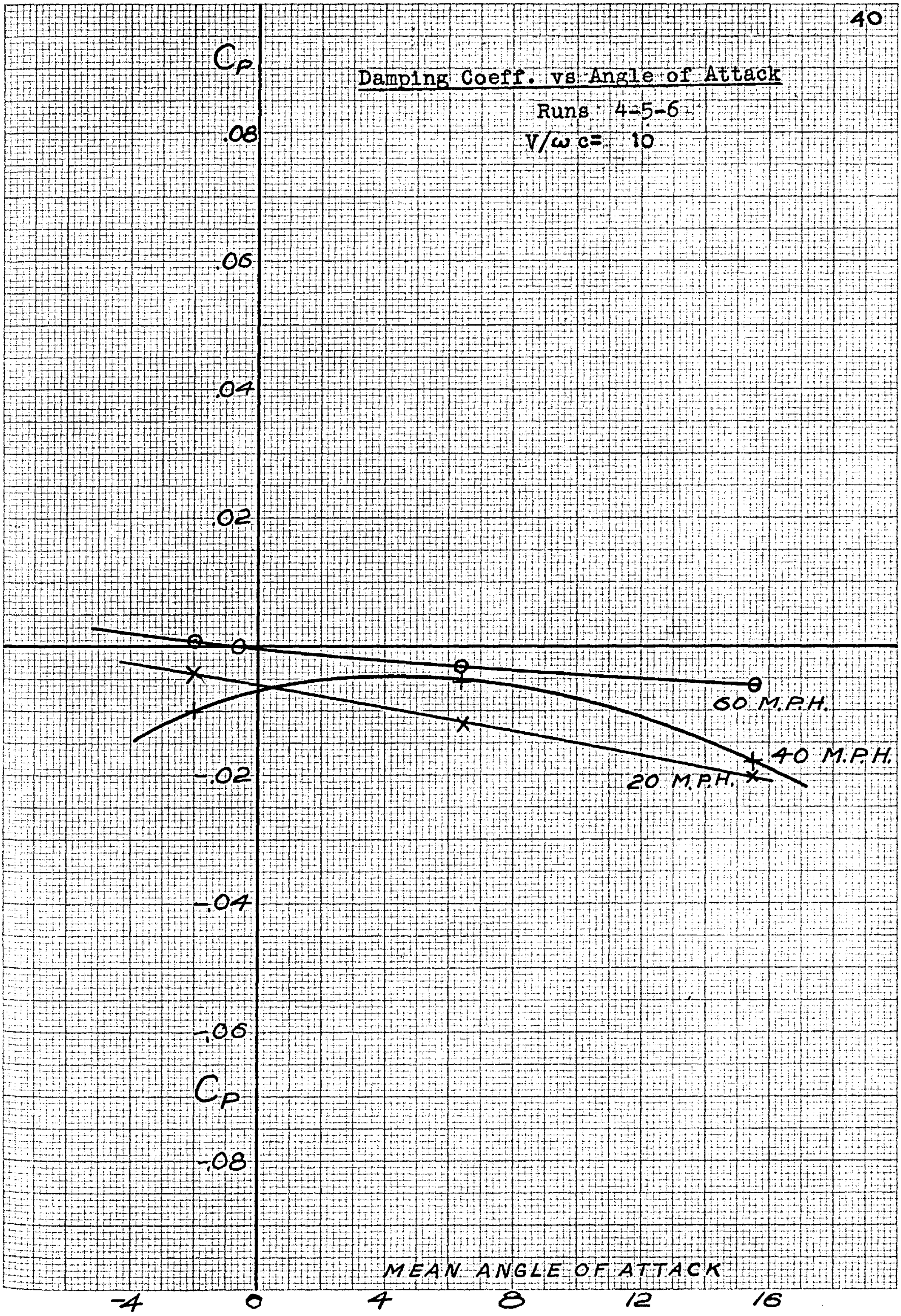


Damping Coeff. vs Angle of Attack

Runs 1-2-3

V/ω c = 10

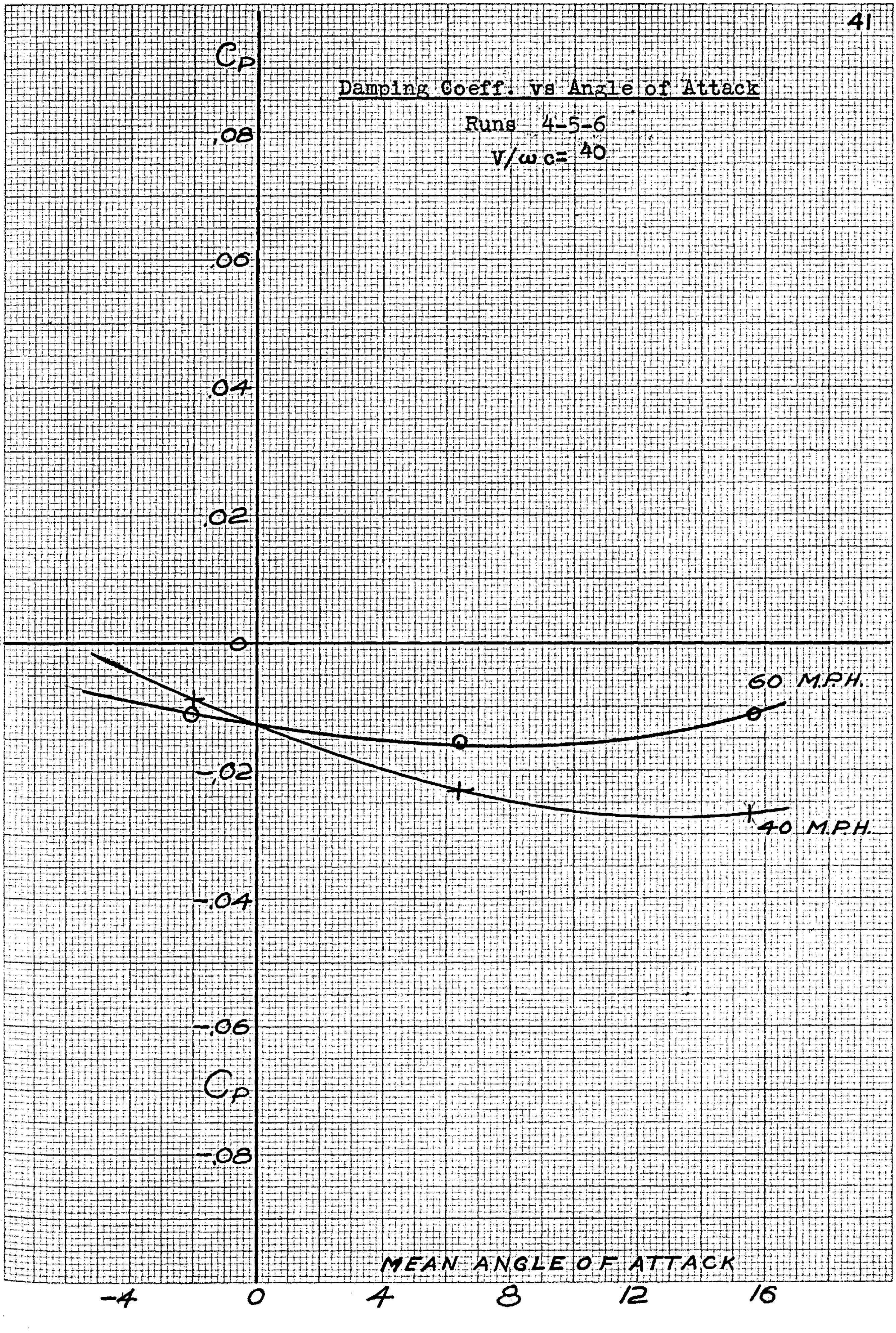

Damping Coeff. vs Angle of Attack
Runs 1-2-3
V/ωc = 40
C_P
.10
.08
.06
.04
.02
0
-.02
-.04
-.06
-.08
C_P
20 M.P.H.
40 M.P.H.
60 M.P.H.
MEAN ANGLE OF ATTACK
-4
0
4
8
12
16



Damping Coeff. vs Angle of Attack

Runs 4-5-6

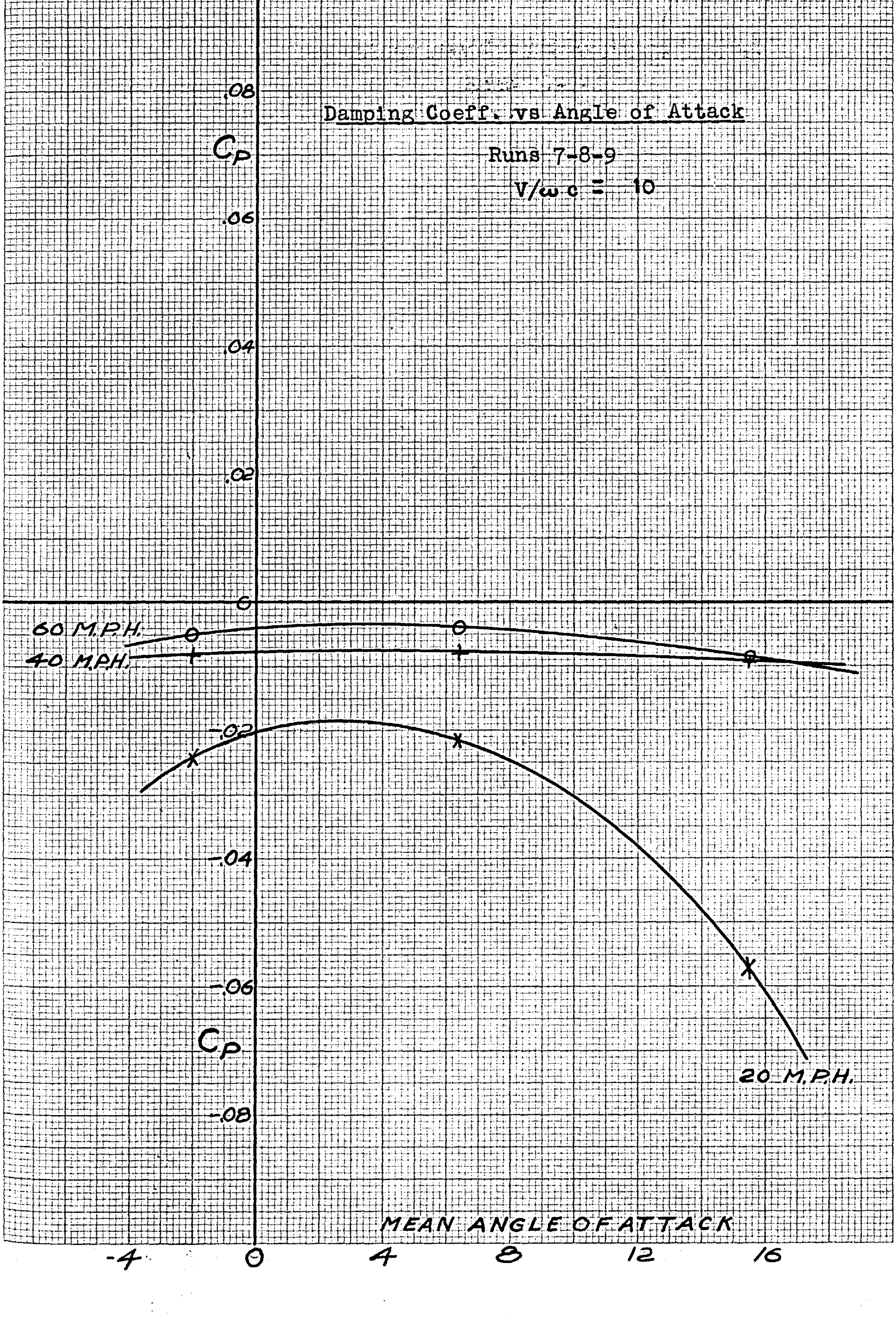
V/ω c= 10



Damping Coeff. vs Angle of Attack

Runs 4-5-6

$V/\omega c = 40$



Damping Coeff. vs Angle of Attack

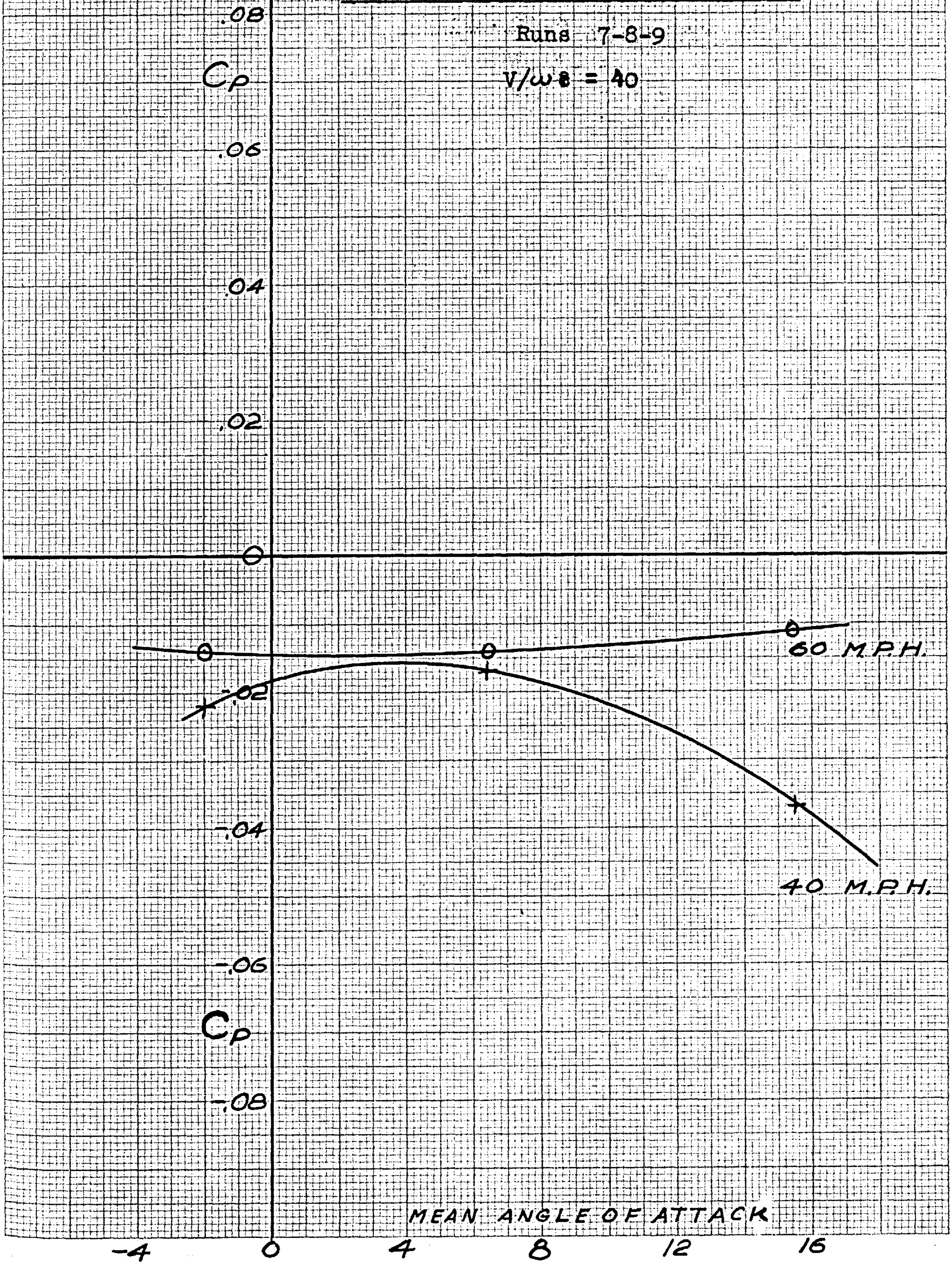
Runs 7-8-9

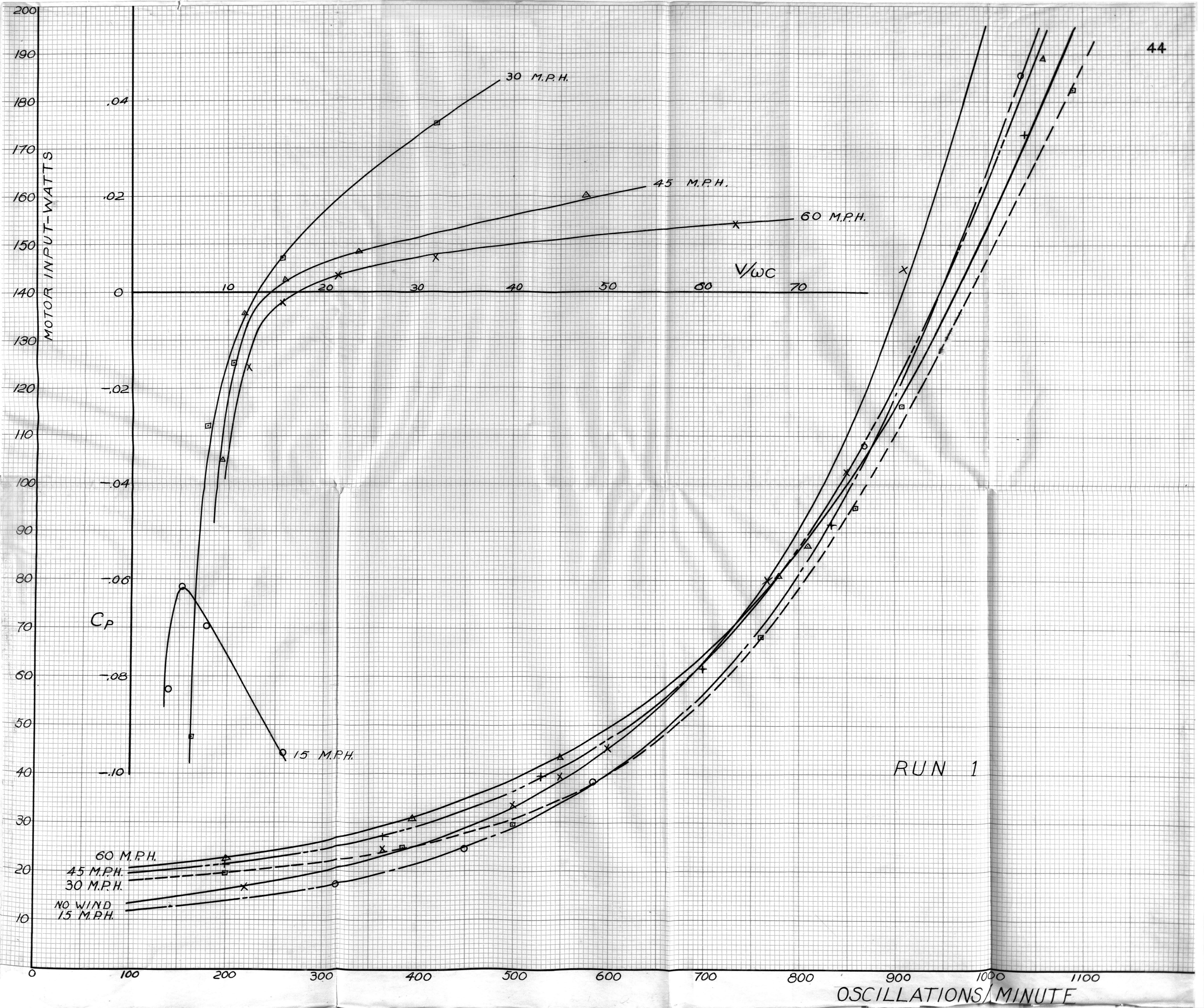
V/ω c = 10

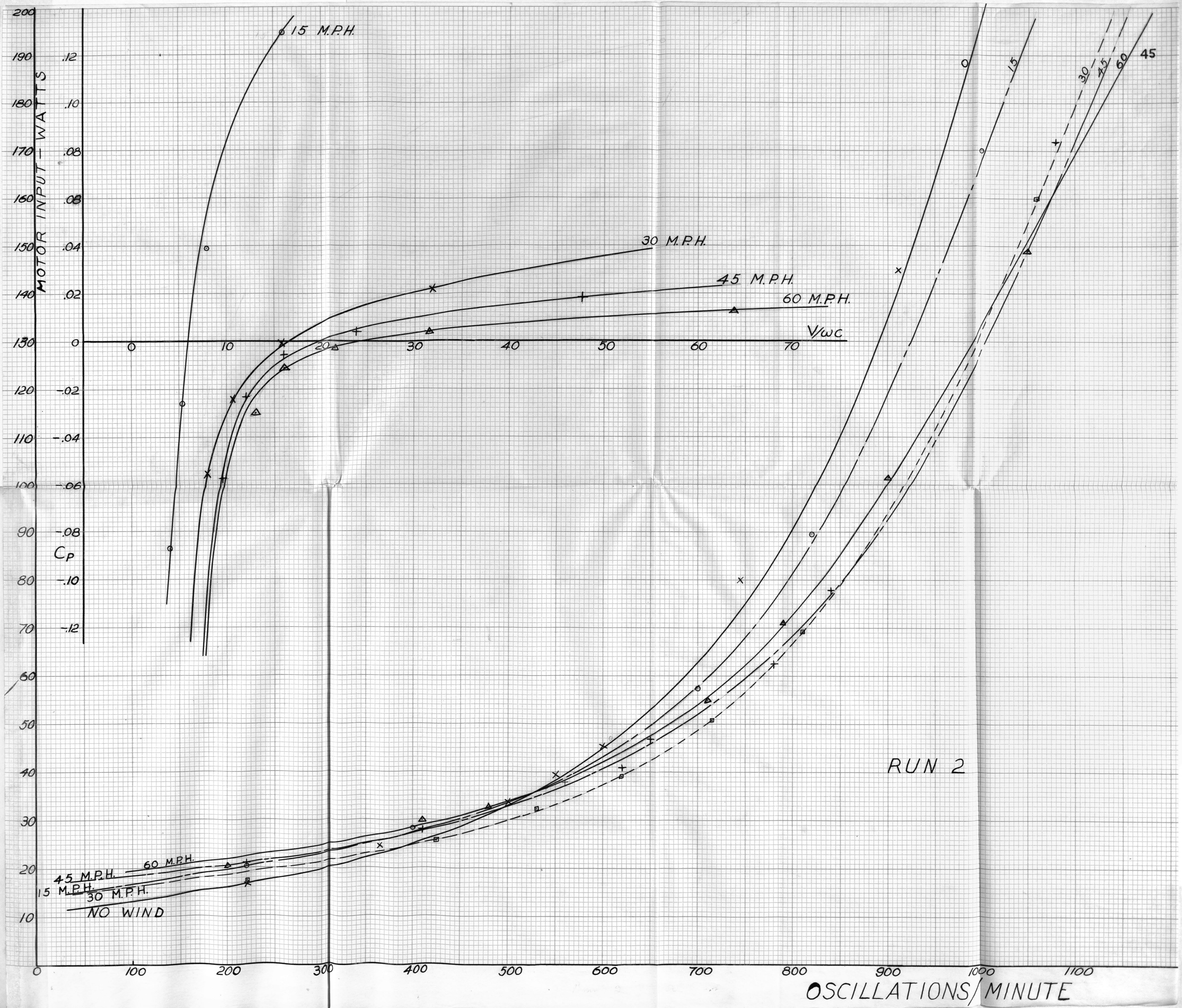
Damping Coeff. vs Angle of Attack

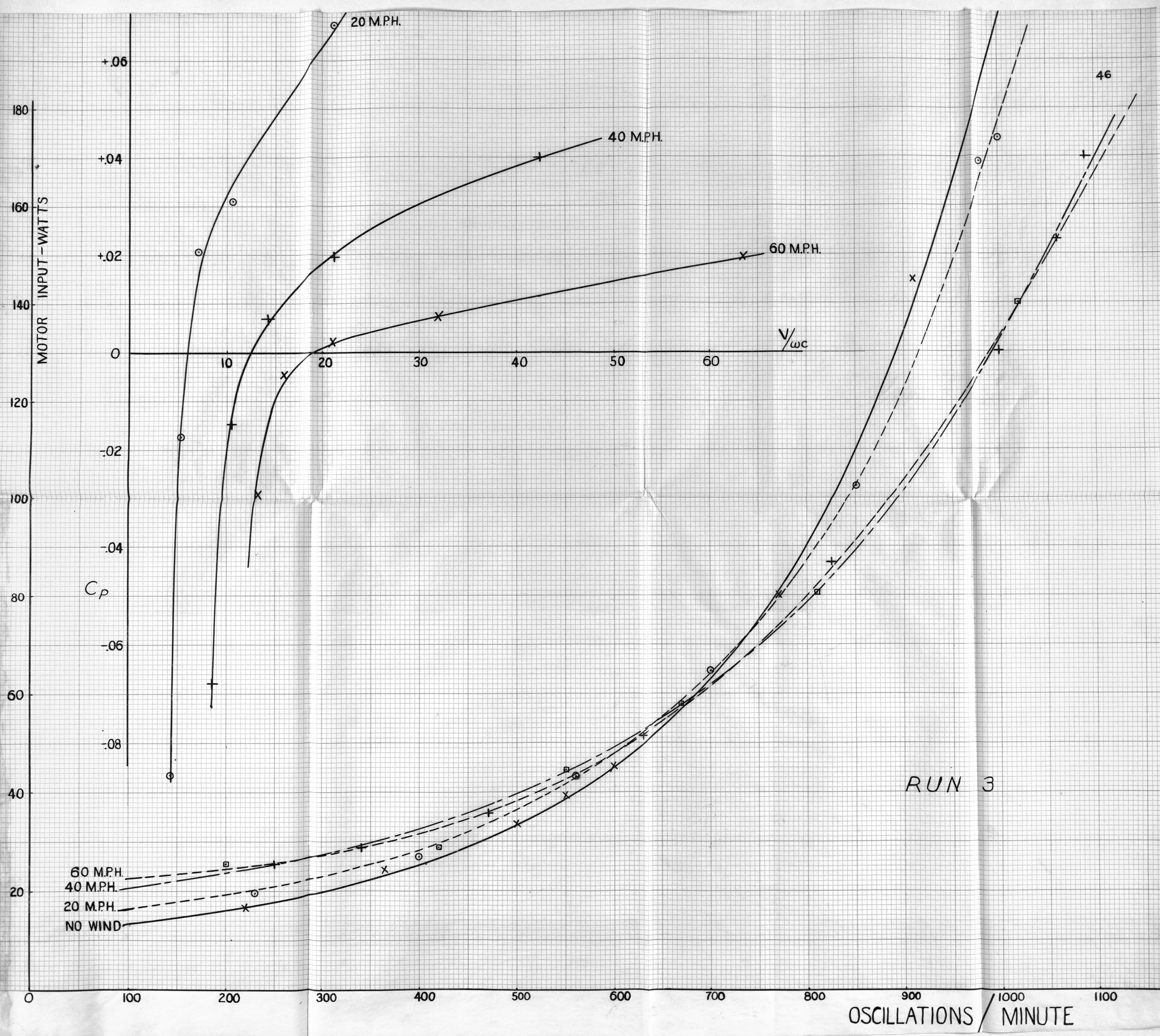
Runs 7-8-9

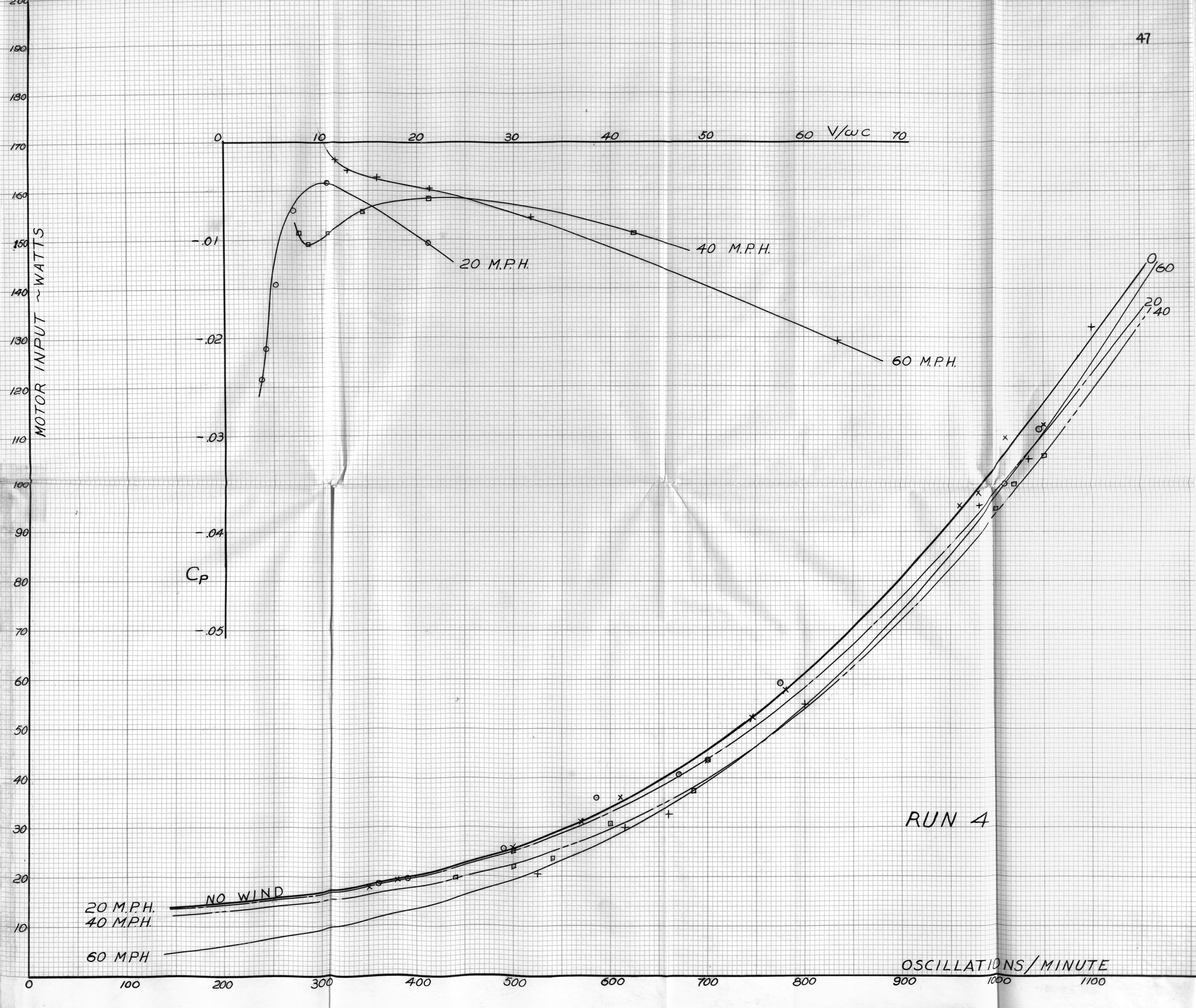
$V/\omega s = 40$

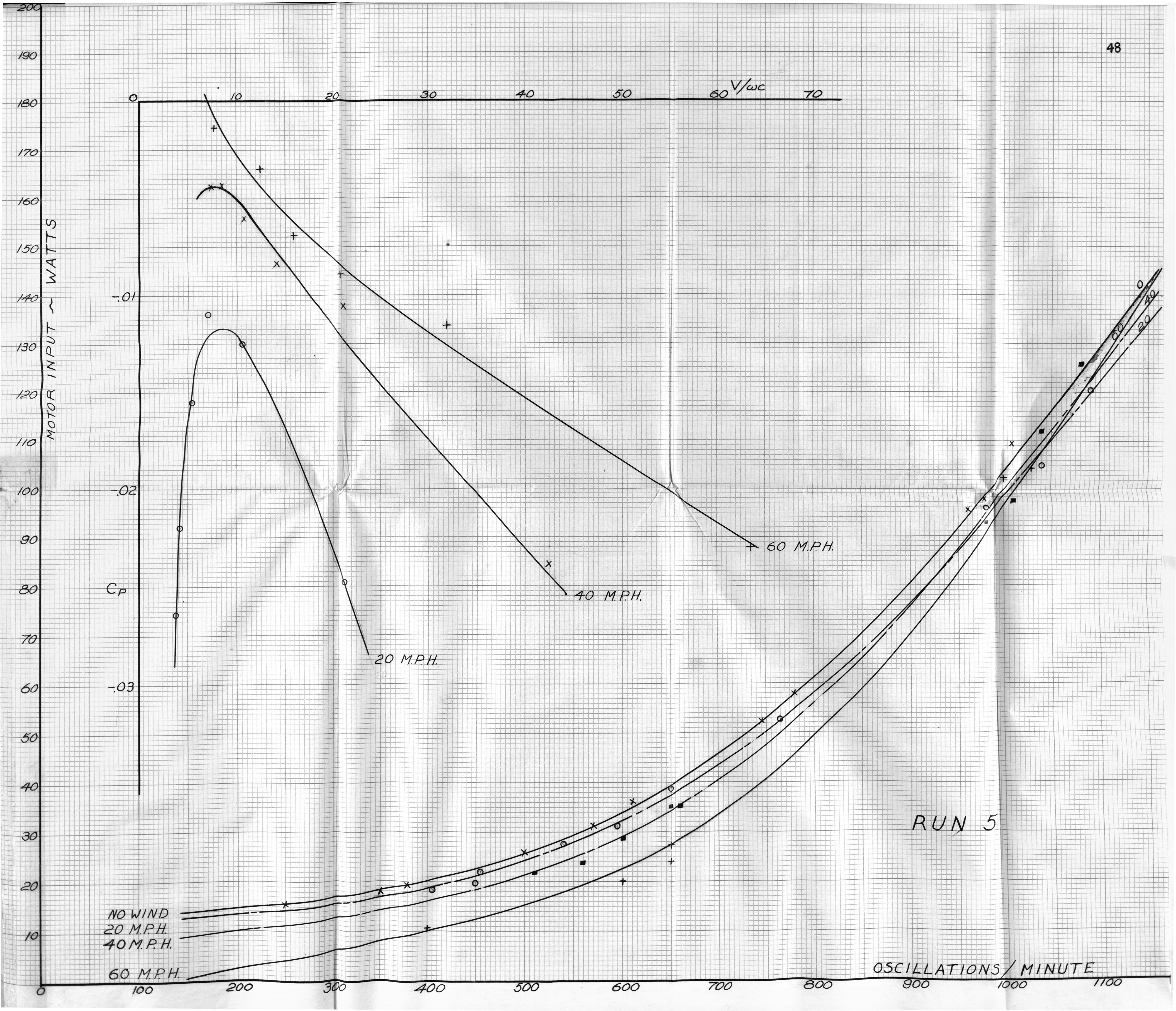


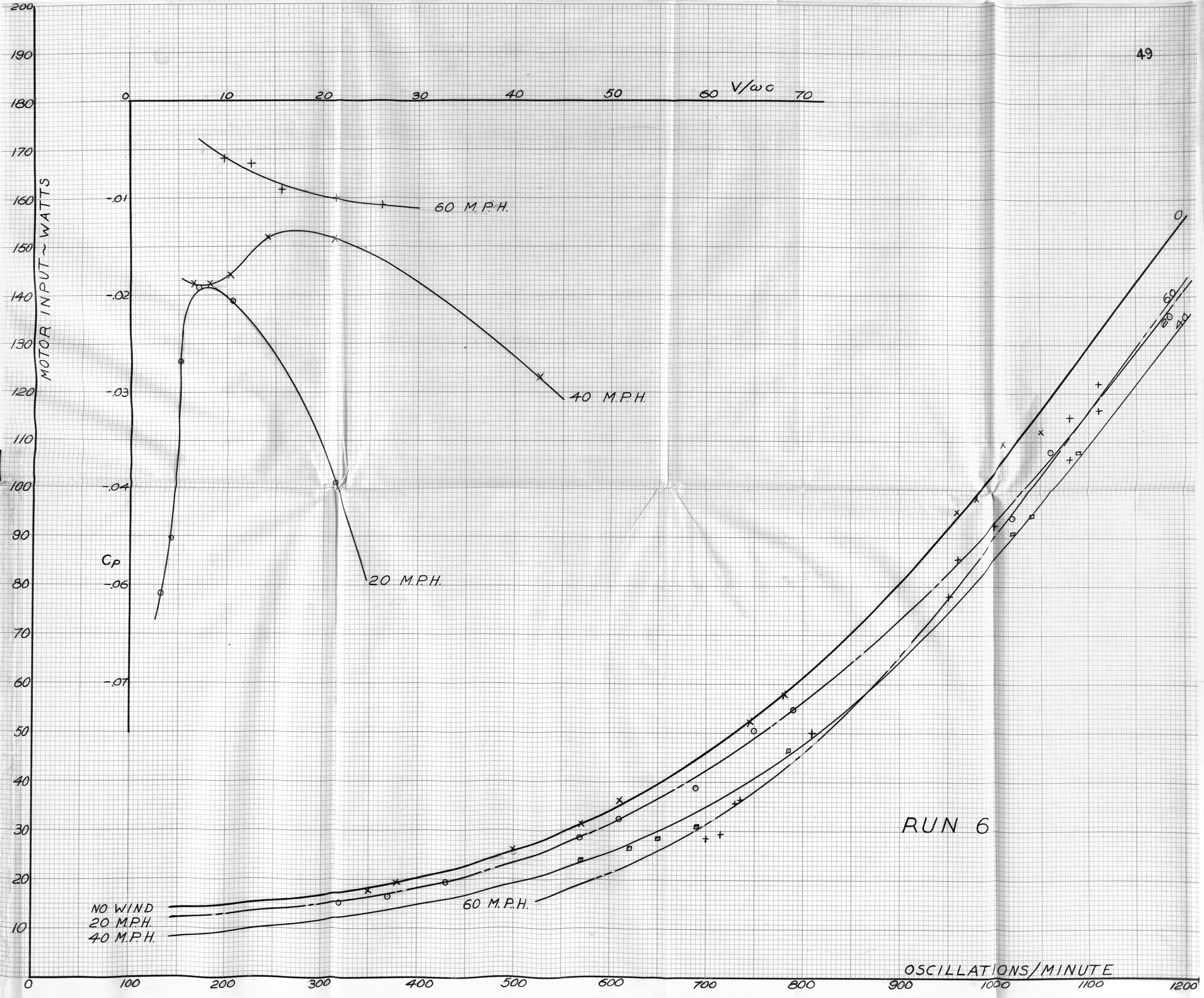

MOTOR INPUT-WATTS
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
100
200
300
400
500
600
700
800
900
1000
1100
OSCILLATIONS/MINUTE
C_P
.04
.02
0
-.02
-.04
-.06
-.08
-.10
V/ωc
10
20
30
40
50
60
70
30 M.P.H.
45 M.P.H.
60 M.P.H.
15 M.P.H.
60 M.P.H.
45 M.P.H.
30 M.P.H.
NO WIND
15 M.P.H.
RUN 1

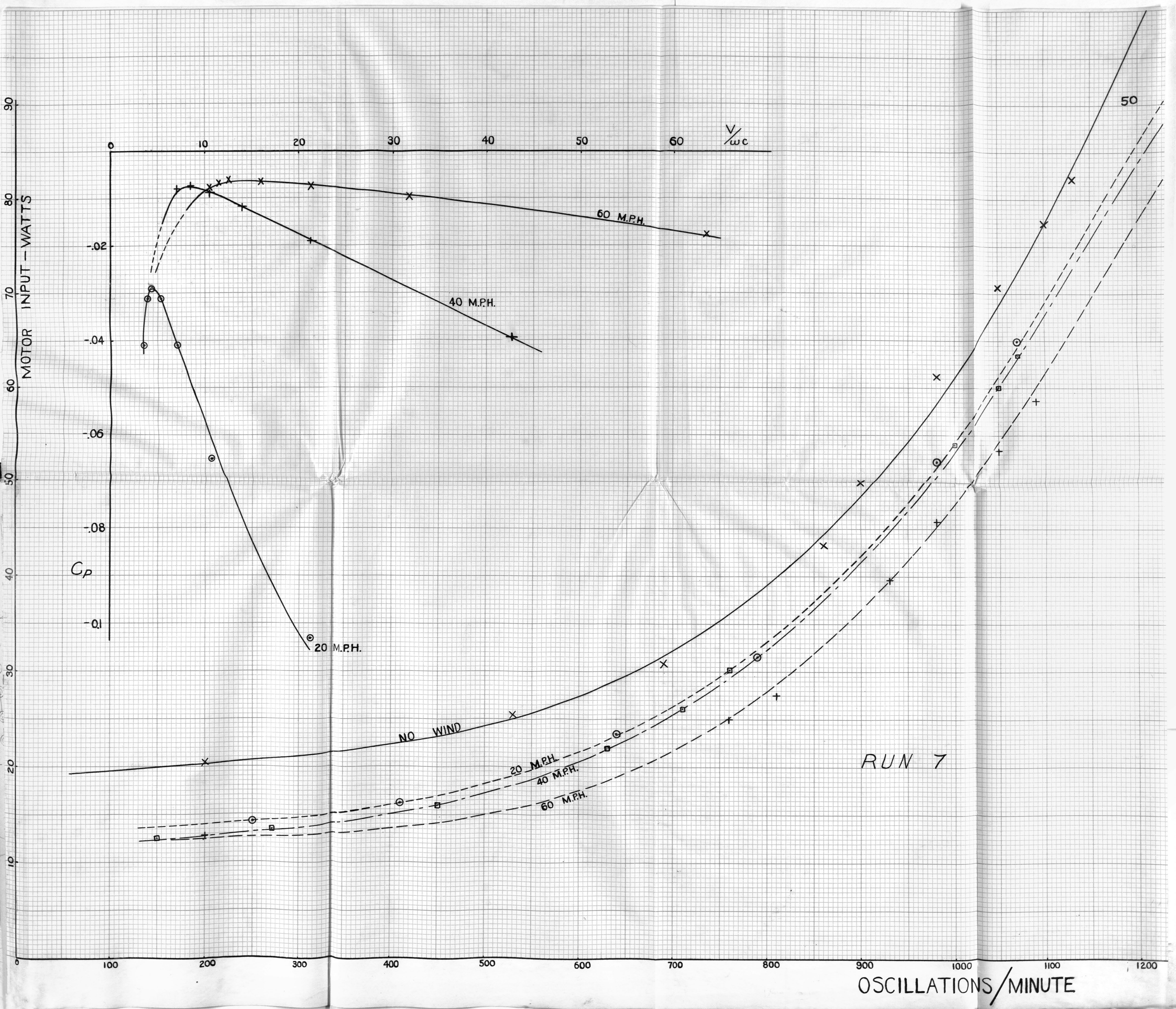

MOTOR INPUT – WATTS
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
.12
.10
.08
.06
.04
.02
0
-.02
-.04
-.06
-.08
-.10
-.12
C_P
0
10
20
30
40
50
60
70
V/ωc
15 M.P.H.
30 M.P.H.
45 M.P.H.
60 M.P.H.
15
30
45
60
45
RUN 2
60 M.P.H.
45 M.P.H.
15 M.P.H.
30 M.P.H.
NO WIND
0
100
200
300
400
500
600
700
800
900
1000
1100
OSCILLATIONS/MINUTE

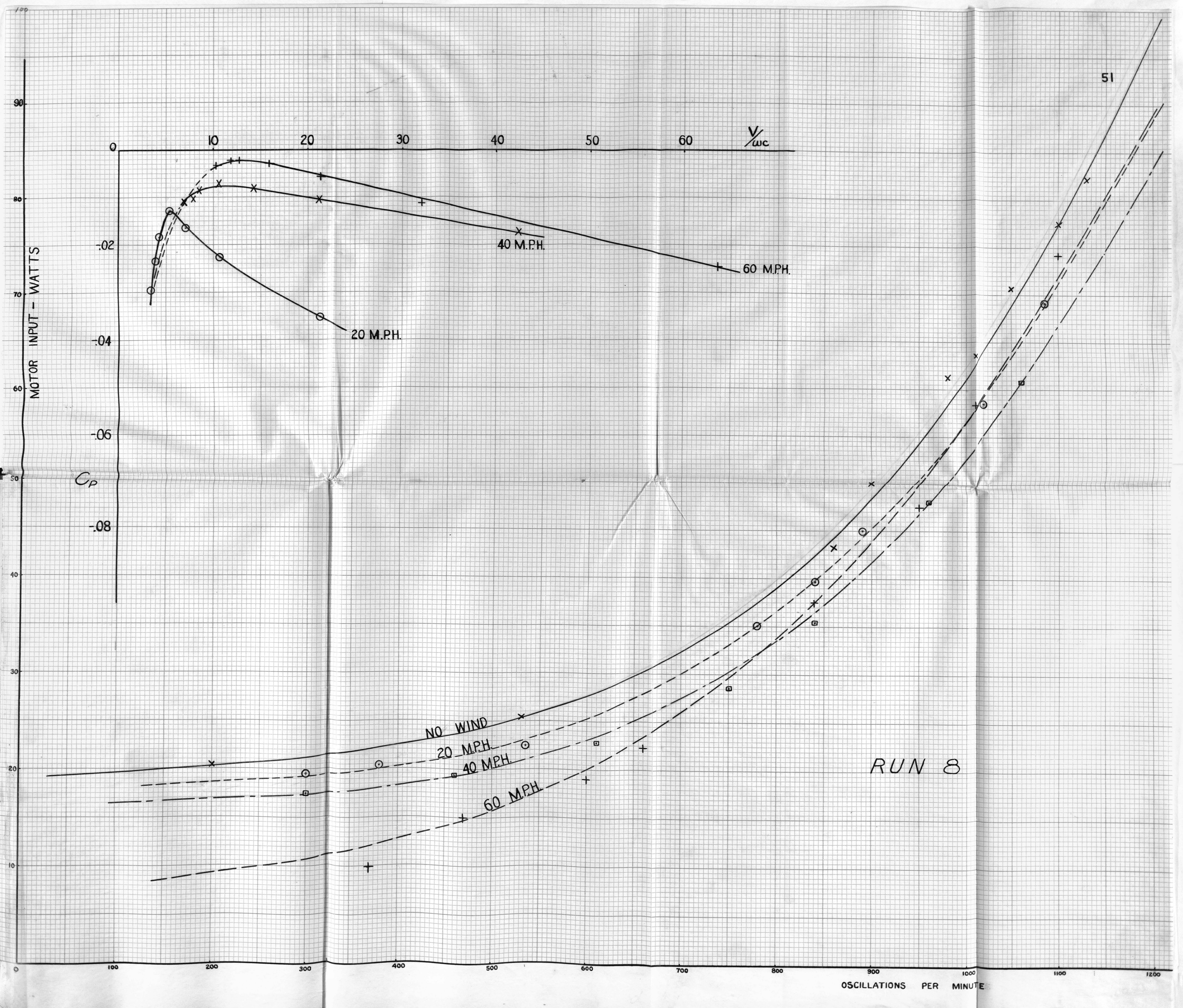

20 M.P.H.
40 M.P.H.
60 M.P.H.
+.06
+.04
+.02
0
-.02
-.04
-.06
-.08
V/ωc
10
20
30
40
50
60
C_P
MOTOR INPUT-WATTS
180
160
140
120
100
80
60
40
20
0
60 M.P.H.
40 M.P.H.
20 M.P.H.
NO WIND
RUN 3
100
200
300
400
500
600
700
800
900
1000
1100
OSCILLATIONS / MINUTE

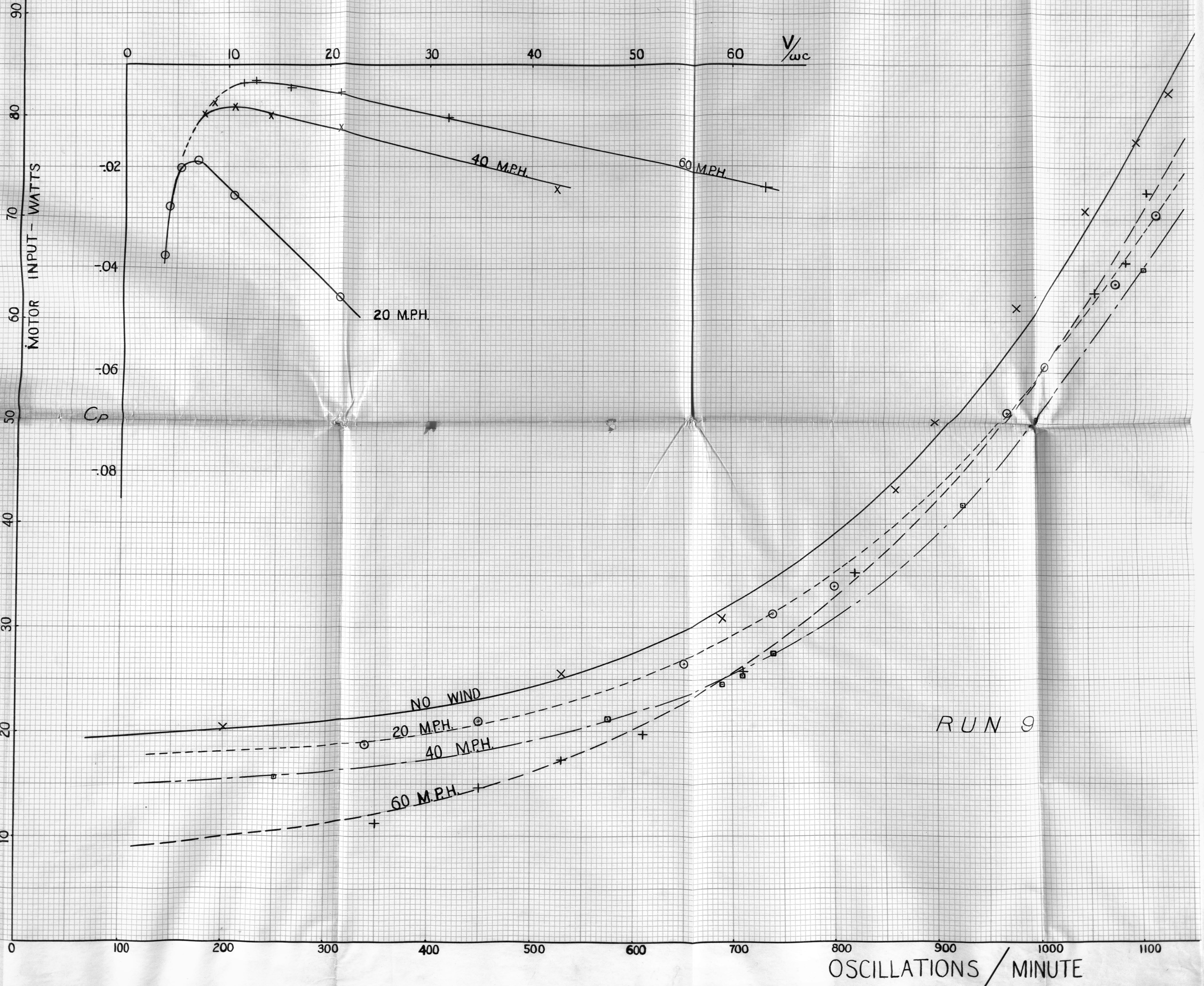

MOTOR INPUT ~ WATTS
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
100
200
300
400
500
600
700
800
900
1000
1100
OSCILLATIONS / MINUTE
NO WIND
20 M.P.H.
40 M.P.H.
60 MPH
0
60
20
40
RUN 4
V/ωc
0
10
20
30
40
50
60
70
C_P
-.01
-.02
-.03
-.04
-.05
20 M.P.H.
40 M.P.H.
60 M.P.H.


MOTOR INPUT ~ WATTS
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
V/ωc
0
10
20
30
40
50
60
70
Cp
-.01
-.02
-.03
60 M.P.H.
40 M.P.H.
20 M.P.H.
RUN 5
NO WIND
20 M.P.H.
40 M.P.H.
60 M.P.H.
0
60
40
20
OSCILLATIONS / MINUTE
100
200
300
400
500
600
700
800
900
1000
1100


MOTOR INPUT ~ WATTS
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
OSCILLATIONS/MINUTE
V/ωc
0
10
20
30
40
50
60
70
C_P
-.01
-.02
-.03
-.04
-.06
-.07
60 M.P.H.
40 M.P.H.
20 M.P.H.
NO WIND
20 M.P.H.
40 M.P.H.
60 M.P.H.
0
60
20
40
RUN 6


RUN 7
MOTOR INPUT – WATTS
10
20
30
40
50
60
70
80
90
OSCILLATIONS/MINUTE
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
NO WIND
20 M.P.H.
40 M.P.H.
60 M.P.H.
50
C_P
-.02
-.04
-.06
-.08
-.1
V/ωc
0
10
20
30
40
50
60
20 M.P.H.
40 M.P.H.
60 M.P.H.


MOTOR INPUT - WATTS
10
20
30
40
50
60
70
80
90
100
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
OSCILLATIONS PER MINUTE
NO WIND
20 M.P.H.
40 M.P.H.
60 M.P.H.
RUN 8
C_P
-.02
-.04
-.06
-.08
0
10
20
30
40
50
60
$V/\omega c$
20 M.P.H.
40 M.P.H.
60 M.P.H.


MOTOR INPUT - WATTS
90
80
70
60
50
40
30
20
10
0
100
200
300
400
500
600
700
800
900
1000
1100
OSCILLATIONS / MINUTE
NO WIND
20 M.P.H.
40 M.P.H.
60 M.P.H.
RUN 9
C_P
.02
.04
.06
.08
V/ωc
0
10
20
30
40
50
60
20 M.P.H.
40 M.P.H.
60 M.P.H.