BEHAVIOR OF A LUMPY ARTIFICIAL TRANSMISSION LINE AS THE FREQUENCY IS INDEFINITELY INCREASED.

A Thesis Submitted to the Faculty of the
Massachusetts Institute of Technology, in
Part Fulfillment of the Requirements for
the Degree of Master of Science.

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In presenting this thesis, the writer desires to express her indebtedness to Dr. A. E. Kennelly for his valuable criticism and advice, and to Mr. Edy Velander for his helpful suggestions in regard to the laboratory work.

TABLE OF CONTENTS

An artificial a.c. line is designed to represent some real line at a particular frequency. As the frequency is changed the conjugate smooth line represented by the artificial line ceases to be the counterpart of the actual smooth line. The problem under consideration is to determine the nature of the conjugate smooth line as the frequency is indefinitely increased and also the highest frequency at which the artificial line approximately represents the same real line which it represented at low frequencies up to 60 cycles.

The artificial line used in this work is a π - line consisting of thirty sections, each section representing, at frequencies up to 60 cycles per second, approximately 50 miles of No. 000 A.W.G. aluminum stranded conductor, diameter 0.47 in., interaxial spacing for three phase system 90.5 inches. The sections are so arranged that it may be operated as a three phase line with length of 500 miles, or as a single phase line with length of 1500 miles and ground return.

* Nominal Linear Constants of Artificial 85 sq. mm. (168,000 circ. mils) Aluminum Line, taking each Section as representing 80 km. (49.7 miles) of conductor.

	Per wire km.	Per wire mile
Linear resistance r ohms at 0° C.......	0.278	0.445
Linear resistance 5 ohms at 20° C......	0.301	0.485
Linear inductance l henrys............	1.13×10^{-3}	1.82×10^{-3}
Linear capacitance c farads.............	9.38×10^{-9}	15.1×10^{-9}
Linear leakance g mhos	0.12×10^{-6}	0.19×10^{-6}
Linear Hyperbolic angle hyps. at 60~..........	$0.00135\angle 71^\circ.5$	$0.00218\angle 71^\circ.5$

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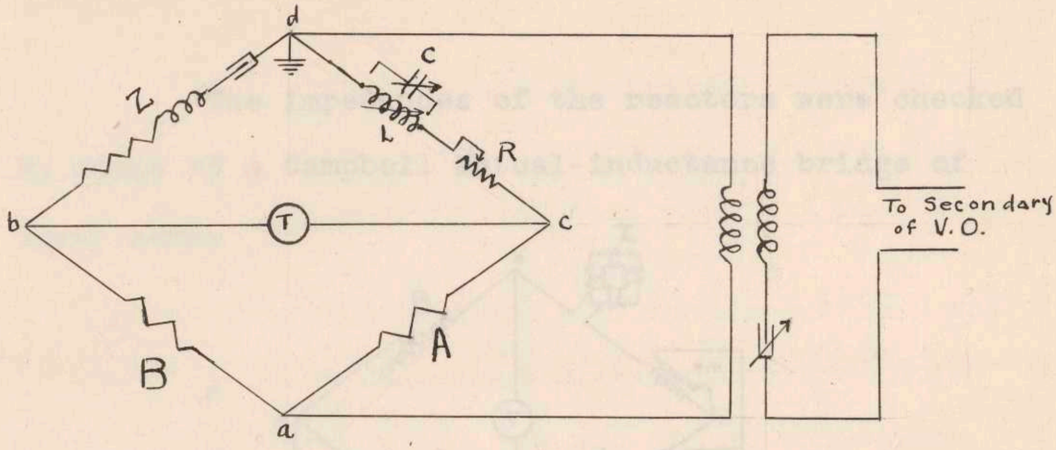
*"Artificial Electric Lines", page 201, A.E. Kennelly.

The measurements made upon this line were all impedance measurements. A bridge balance was obtained with both resistance and reactance. The bridge arms A and B were two approximately equal non-inductive resistances of 250 ohms each. The source of power was the Vreeland Oscillator for all frequencies above 60 cycles. A standard variable condenser was used when the impedance was condensively reactive, and a variometer when the impedance was inductively reactive. A telephone was used as a detector. One point of the bridge, d, was grounded to stabilize the voltage distribution. Since the bridge was symmetrical the balance was not affected by capacity currents. To get rid of higher harmonics a tuned circuit was used consisting of the secondary coils of the Vreeland Oscillator, a variable condenser and the primary of a transformer, the secondary of the transformer supplied the power to the bridge.



$$Z_L = R + j\,\omega L \qquad Z_C = R - j\,\frac{1}{\omega C}$$

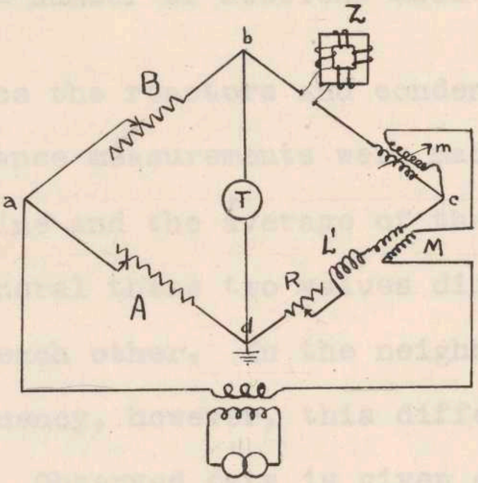
$$Y_C = \frac{1}{Z_C} = \frac{R}{R^2+(\frac{1}{\omega C})^2} + j\,\frac{\frac{1}{\omega C}}{R^2+(\frac{1}{\omega C})^2}$$

$$= R\,\omega^2C^2 + j\,\omega C \quad \text{(when R is small relative to } (\tfrac{1}{\omega C})^2\text{)}$$

$$L_Z = L \qquad C_Z = C \text{ approximately}$$

$$R_Z = R \qquad G_Z = R\,\omega^2C^2 \text{ approximately.}$$

The impedances of the reactors were checked by means of a Campbell Mutual-inductance bridge of equal arms.



(M + m) is read directly.

$$L_Z = L' + 2\ (M + m), \qquad r_Z = R.$$

To obtain experimentally Z'_0, and θ', the home end impedances of the line with the distant end both grounded and free were measured.

$$\text{Then } Z'_0 = \sqrt{Z_g Z_f}$$

$$\theta' = \frac{1}{n} \tanh^{-1} \sqrt{\frac{Z_g}{Z_f}}$$

When Z_g = impedance of the line, distant end grounded.

Z_f = impedance of the line, distant end free.

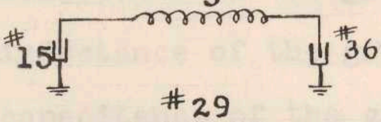
n = number of sections used.

Since the reactors and condensers were not uniform, impedance measurements were made from both ends of the line and the average of the two values taken. In general these two values did not differ greatly from each other. In the neighborhood of the critical frequency, however, this difference may be considerable. Observed data is given on pages 21-25, calculations from data on pages 36-40.

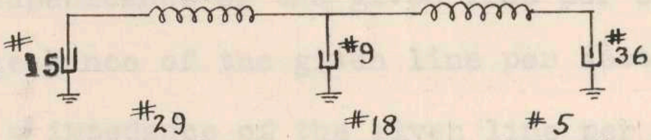
Z_g and Z_f were measured for one, two and three sections. The condensers and reactors were selected for the three arrangements so as to make the average constants per section in each arrangement as nearly uniform as possible.

Arrangement of reactors and condensers:

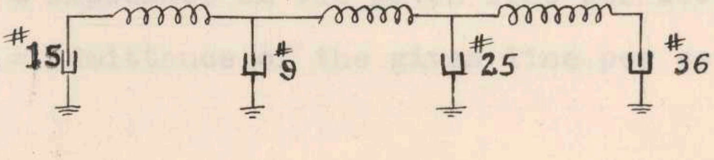
1 section



2 sections



3 sections



The resistance and inductance of the 3 reactors versus ω are plotted on curve sheets No. 1 and No. 2. Observed data and calculations are given on pages 18, 19 & 28. The leakance and capacitance of the two full condensers and the two half condensers versus ω are plotted on curve sheets No. 3 and No. 4. Observed data and calculations are given on pages 20 and 29.

To simplify calculations the assumption is made that the linear constants of the real line, which is represented by the artificial at low frequencies, vary with frequency in the same manner as the constants of the artificial line, i.e., that the real line has the resistance, inductance, leakance and capacitance of the artificial line as given on curve sheets 1, 2, 3 and 4, smoothly distributed.

*Theory of Artificial Lines:

Let R = resistance of the given line per section

L = inductance of the given line per section

C = capacitance of the given line per section

G = leakance of the given line per section

$Z = R + j\,\omega\,L$ = impedance of the given line per section

$Y = G + j\,\omega\,C$ = admittance of the given line per section.

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*"Artificial Electric Lines",- A.E. Kennelly.

When these constants are smoothly distributed as in a real line

$\theta = \sqrt{XY}$ = propogation constant per section of real line.

$Z_o = \sqrt{Z/Y}$ = surge impedance of real line.

When these constants are lumped in an artificial line, a real line, called the conjugate smooth line, is represented by the artificial line.

$\theta' = 2 \sinh^{-1} \frac{\theta}{2}$ = propogation constant per section of conjugate smooth line.

$Z'_o = Z_o / \cosh \frac{\theta'}{2}$ = surge impedance of conjugate smooth line.

Let R' = resistance per section of the conjugate smooth line

L' = inductance per section of the conjugate smooth line

C' = capacitance per section of the conjugate smooth line

G' = leakance per section of the conjugate smooth line

Then $R' + j\omega L' = Z' = Z'_o \, \theta'$

$G' + j\omega C' = Y' = \theta' / Z'_o$

Knowing the resistance, inductance, capacitance and leakance per section of the artificial line at various frequencies it is possible by means of the above formulae to calculate the propogation constant, surge impedance, linear resistance, inductance, capacitance and leakance of the conjugate smooth line. Calculations are given on pages 30 to 36.

In making these calculations the work was greatly reduced by the use of the "Chart Atlas" by Dr. Kennelly, and the complex quantity slide rule.

Curve sheet No. 5 gives Z_0' versus ω and shows the variation in size and slope of the surge impedance of the conjugate smooth line as the frequency is raised. The dotted curves give corresponding values of Z_0, the surge impedance of the real line having the same constants per section as the artificial line.

Curve sheet No. 6 gives a polar plot of Z_0' and Z_0. This curve crosses itself, showing that the line offers the same surge impedance in the neighborhood of $\omega = 7500$ as at $\omega = 20$. A maximum value of Z_0' is obtained in the neighborhood of $\omega = 8100$. As the frequency is indefinitely raised Z_0 decreases in size and tends to become wholly reactive. Curve sheet No. 7 gives θ' and θ for corresponding values of ω. Up to $|\theta| = 1.0$, ($\omega = 4000$), $|\theta'|$ differs but slightly from $|\theta|$, this difference gradually increases with ω until, when $|\theta| = 2.0$ ($\omega = 8100$) α' the real part of θ' begins to increase very rapidly. As the frequency is further increased β', the imaginary part of θ', does not exceed π.

Curve sheet No. 8 gives α and α' to a larger scale for the lower frequencies.

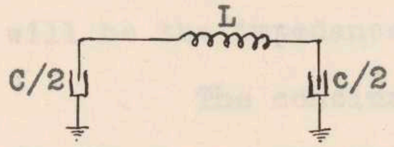
Curve sheet No. 9 gives a polar plot of θ' and θ.

Curve sheets Nos. 10, 11, 12, 13 give the resistance, inductance, leakance and capacitance of the conjugate smooth line and the real line. All four curves for the conjugate smooth line have critical values in the neighborhood of $\omega = 8100$, which is the value of ω at which $|\theta| = 2.0$. At this value of ω R' is more than 100 times as great as R, L' is more than 3 times as great as L, while C' is only one quarter as great as C and G' has become negative. As the frequency is further increased R' decreases, L' decreases abruptly and becomes negative, C' increases, G' becomes greater in magnitude but remains negative.

The conclusion, from a study of the curves, is that there is a critical frequency beyond which the conjugate smooth line breaks down and does not even approximately represent the real line. This frequency is that near which $|\theta| = 2.0$. At half this frequency (at $\omega = 4000$) $|\theta'|$, $|Z'|$ and α' differ from $|\theta|$, $|Z_0|$ and α by 4%, 15% and 14% respectively. Above $\omega = 4000$ this percentage difference increases rapidly with ω for Z_0' and α'.

As the frequency is increased beyond the critical frequency the conjugate smooth line has negative inductance and large negative leakance and is not physical realizable. The surge impedance decreases in magnitude, and approaches $0\overline{/90^\circ}$. α' the real part of θ' continues to increase. Since β' the imaginary part of θ' does not exceed π., the wave length is always more than two sections. ($\lambda = \frac{2\pi}{\beta}$) Therefore there are always at least two sections per wave length.

Since $Z_0 = \sqrt{Z_g Z_f}$, the frequency at which Z_0 is maximum may be readily obtained from a consideration of one section of the artificial line if resistance and leakance are neglected.



$$Z_g = \frac{1}{\frac{j\omega C}{2} + \frac{1}{j\omega L}} = \frac{j\,2\omega L}{2 - \omega^2 CL}$$

$$Z_f = \frac{1}{\frac{j\omega C}{2} + \frac{1}{j\omega L - \frac{j2}{\omega C}}} = \frac{j\,2(\omega^2 CL - 2)}{\omega\, C(4 - \omega^2 CL)}$$

$$Z_o = \sqrt{Z_g Z_f} = \frac{2\sqrt{L/C}}{\sqrt{4 - \omega^2 CL}}$$

For maximum Z_o, $4 - \omega^2 CL = 0$

$$\omega = \frac{2}{\sqrt{LC}} = \frac{2}{\sqrt{.0882 \times .695}} = 8060$$

It is apparent that at very high frequencies the surge impedance of the line will be the impedance of the half condenser. Above the critical frequency, therefore, the surge impedance decreases with an increase in frequency and approaches zero as the frequency is indefinitely increased.

It is interesting to note that with a T line, the surge impedance at very high frequencies will be the impedance of the half reactor.

The conclusions reached from an inspection of the curves may be obtained mathematically.

Assume that R and G are small enough to be neglected.

Then $\theta = j\,\omega\sqrt{LC} = j\,2\,K$, $\quad Z_o = \sqrt{L/C}$

$\frac{\theta'}{2} = \sinh^{-1} \theta/2 = \sinh^{-1} j\,K = x + jY$

$\sinh \theta'/2 = \sinh (x + jY) = \sinh X \cos Y + j\cosh X \sin Y$

$= j\,K$

$\therefore \sinh X \cos Y = 0$, and either

$X = 0$ or $Y = \pi/2$.

If $X = 0$, $\cosh X \sin Y = \sin Y = K$

$K = \sin Y \leqq 1.$, $\quad \theta \leqq j2$

If $Y = \pi/2$ $\quad \cosh X \sin Y = \cosh X = K$

$K = \cosh X \geqq 1$) $\quad \theta \geqq j2.$

$\underline{\theta < j2.}$

$Y = \sin^{-1} K$, $\quad \theta' = 0 + j2 \sin^{-1} K$

wave length $\geqq$ 2 sections,

$$\cosh \theta'/2 = \sqrt{1 + \sinh^2 \theta'/2} = \sqrt{1+(\theta/2)^2} = \sqrt{1-K^2}$$

$$Z'_o = Z_o/\sqrt{1-K^2}$$

$$Z' = R' + j\omega L' = \theta' Z'_o = \frac{j\,2 \sin^{-1} K\, Z_o}{\sqrt{1-K^2}} \times \frac{\theta}{j2K}$$

$$= Z \frac{\sin^{-1} K}{K\sqrt{1-K^2}} = \frac{Z(K + K^3/6 + \frac{1.3}{2.4} K^5/5)}{K(1-K^2/2 - K^4/8)}$$

$$= Z\ (\ 1 + 2/3\ K^2 + \quad)$$

Z' > Z, L' and R' are positive and greater than L and R.

$$Y' = G' + j\omega C' = \frac{\theta'}{Z'_o} = \frac{j\,2 \sin^{-1} K \sqrt{1-K^2}}{Z_o} \ \frac{\theta}{j2K}$$

$$= Y \frac{\sin^{-1} K}{K} \sqrt{1-K^2} = \frac{(K + K^3/6 + \frac{1.3}{2.4}\frac{K^5}{5}\ )}{K}(1-K^2/2 - K^4/8)$$

$$= Y\ (\ 1 - 1/3\ K^2 - \quad)$$

Y' < Y , G' and C' are less than G and C.

$\underline{\theta > j2}$

$X = \cosh^{-1}K \qquad \theta' = 2\cosh^{-1}K + j\pi$

α' increases as $2\cosh^{-1}\left|\theta/2\right|$, β' remains constant.

wave length = 2 sections.

$\cosh\theta'/2 = \sqrt{1+(\theta/2)^2} = j\sqrt{K^2-1}$

$$Z' = R' + j\omega L' = \theta' Z'_o = \frac{(2\cosh^{-1}K + j\pi)\,Z_o}{j\sqrt{K^2-1}} \times \frac{\theta}{j2K}$$

$$= Z\,\frac{\cosh^{-1}K + j\pi/2}{-K\sqrt{K^2-1}}$$

$$\left|\frac{\cosh^{-1}K + j\pi/2}{-K\sqrt{K^2-1}}\right| > 1 \qquad \text{when } 1 < K < 1.56$$

$$\left|\frac{\cosh^{-1}K + j\pi/2}{-K\sqrt{K^2-1}}\right| < 1 \qquad \text{when } K > 1.56$$

$|Z'| > |Z| \qquad$ when $|\theta| < 3.12$

$|Z'| < |Z| \qquad$ when $|\theta| > 3.12$

$$\overline{Z'} = \overline{Z} - \left(180° - \tan^{-1}\frac{\pi/2}{\cosh^{-1}X}\right)$$

The phase difference between Z' and Z varies from $-90°$ to $-180°$. When $\theta = j2$, $\underline{\overline{Z'}} = \underline{\overline{Z}} - 90°$. L' becomes negative, therefore, beyond the critical frequency, and remains negative as the frequency is indefinitely raised, while R' decreases gradually toward zero.

$$Y' = G' + j\omega C' + \theta' / Z'_0 = \frac{(2 \cosh^{-1} K + j\pi)\, j\sqrt{K^2-1}}{Z_0} \; \frac{\theta}{j2K}$$

$$= Y \, \frac{(\cosh^{-1} K + j\pi/2) \sqrt{K^2 - 1}}{K}$$

$$\left| \frac{(\cosh^{-1} K + j\pi/2) \sqrt{K^2 - 1}}{K} \right| < 1 \qquad \text{when } 1 < K < 1.23$$

$$\left| \frac{(\cosh^{-1} K + j\pi/2) \sqrt{K^2 - 1}}{K} \right| > 1 \qquad \text{when } K > 1.23$$

$$|Y'| < |Y| \qquad \text{when } |\theta| < 2.46$$

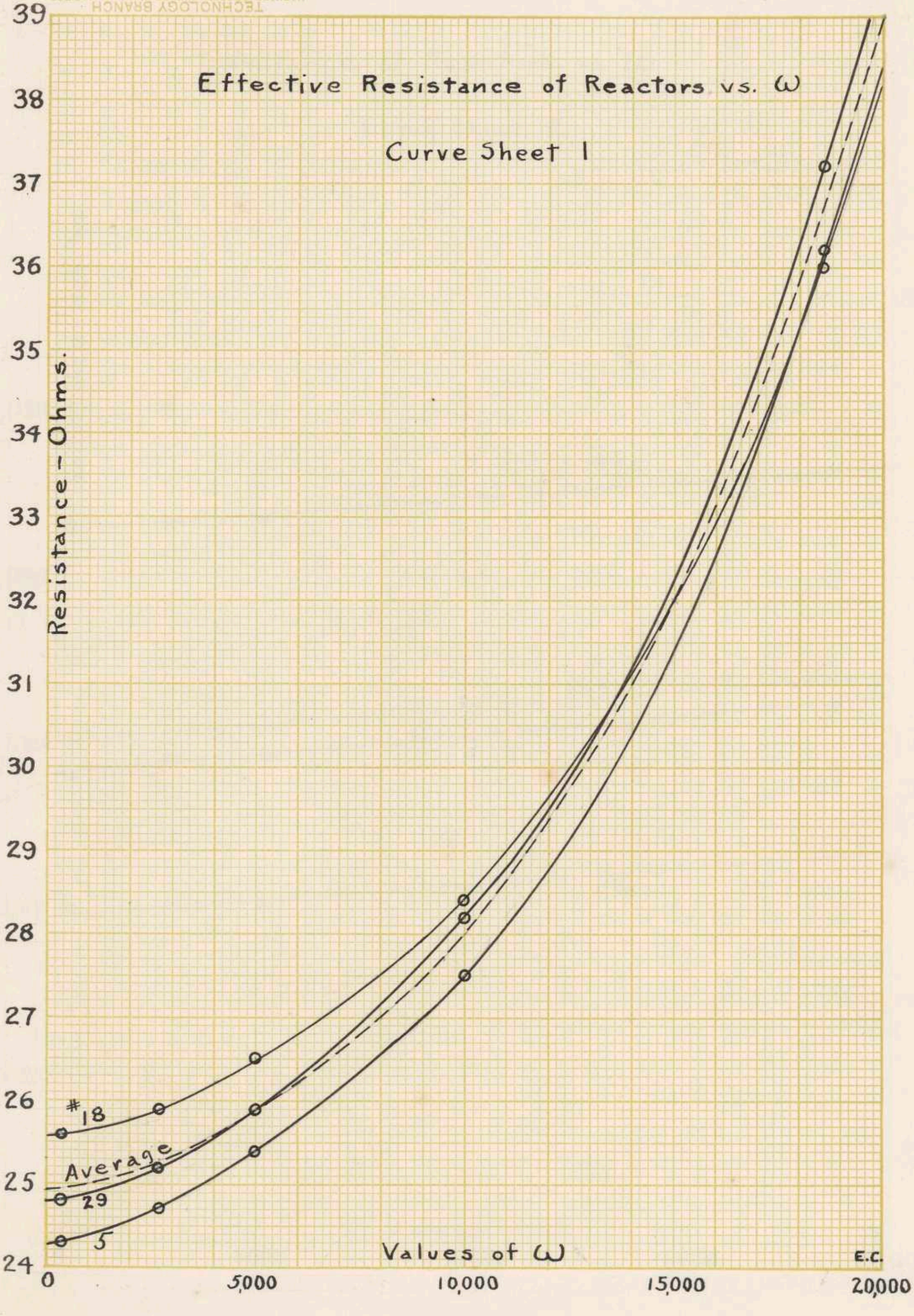
$$|Y'| > |Y| \qquad \text{when } |\theta| > 2.46$$

$$\underline{\overline{Y'}} = \underline{\overline{Y}} + \tan^{-1} \frac{\pi/2}{\cosh^{-1} K}$$

The phase difference between Y' and Y varies from

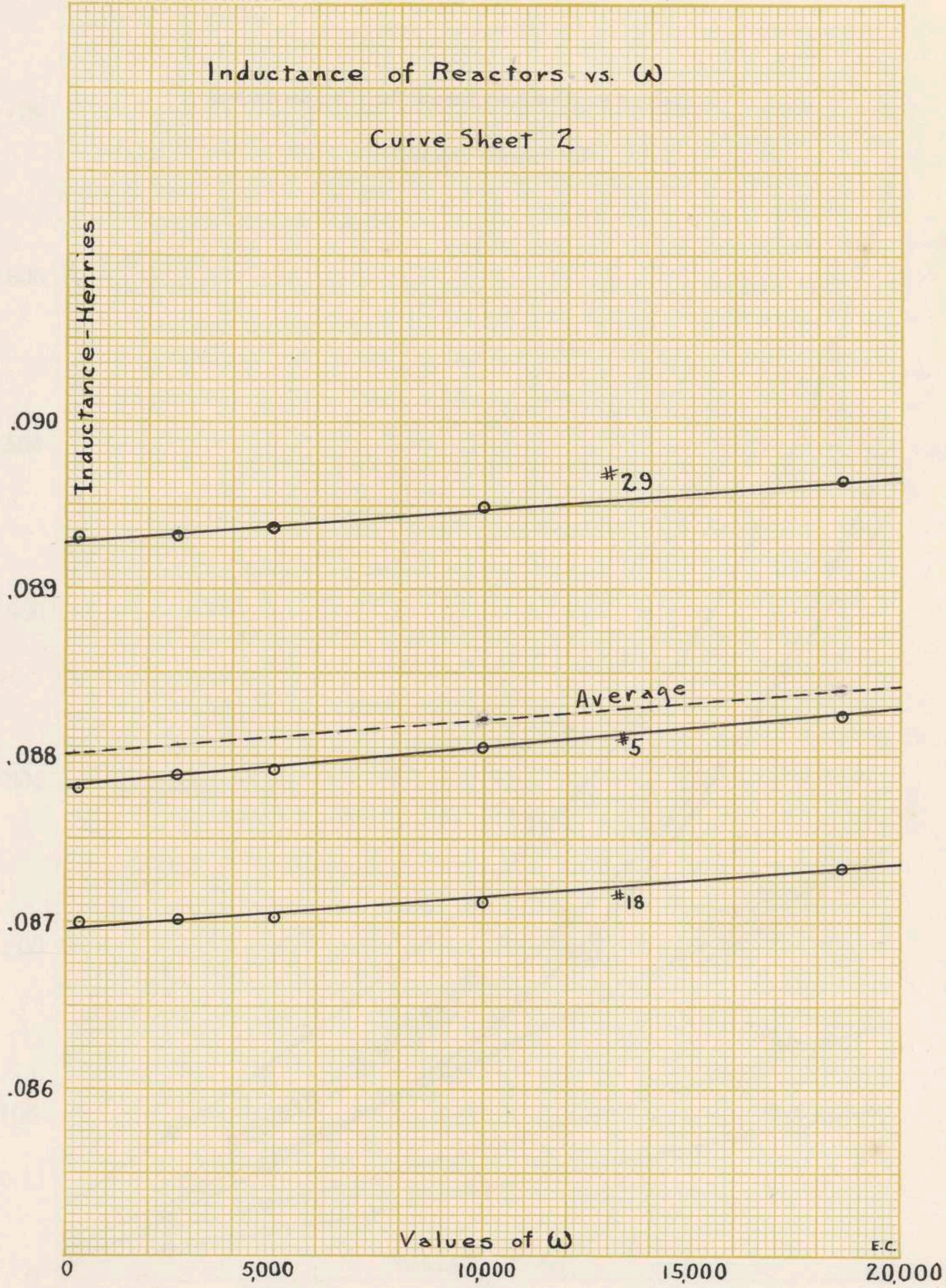
+ 90° to 0°. The slope of Y' lies between 180° and 90°. G' is always negative, C always positive. C' has a minimum value near the critical frequency but increases indefinitely as the frequency is raised.

The above statements would be somewhat modified when R and G are not neglected, particularly in the neighborhood of the critical frequency.



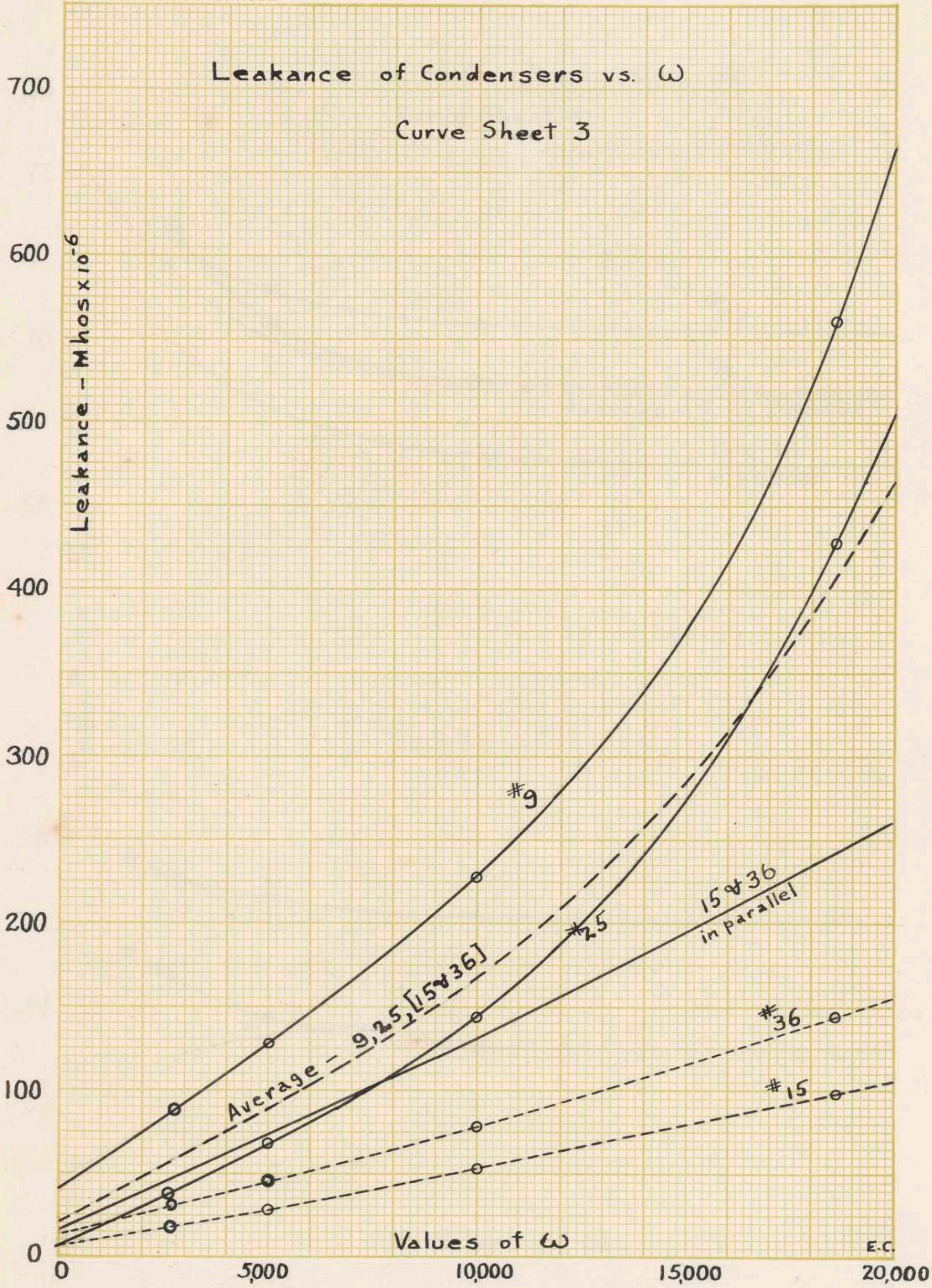
Effective Resistance of Reactors vs. ω

Curve Sheet 1



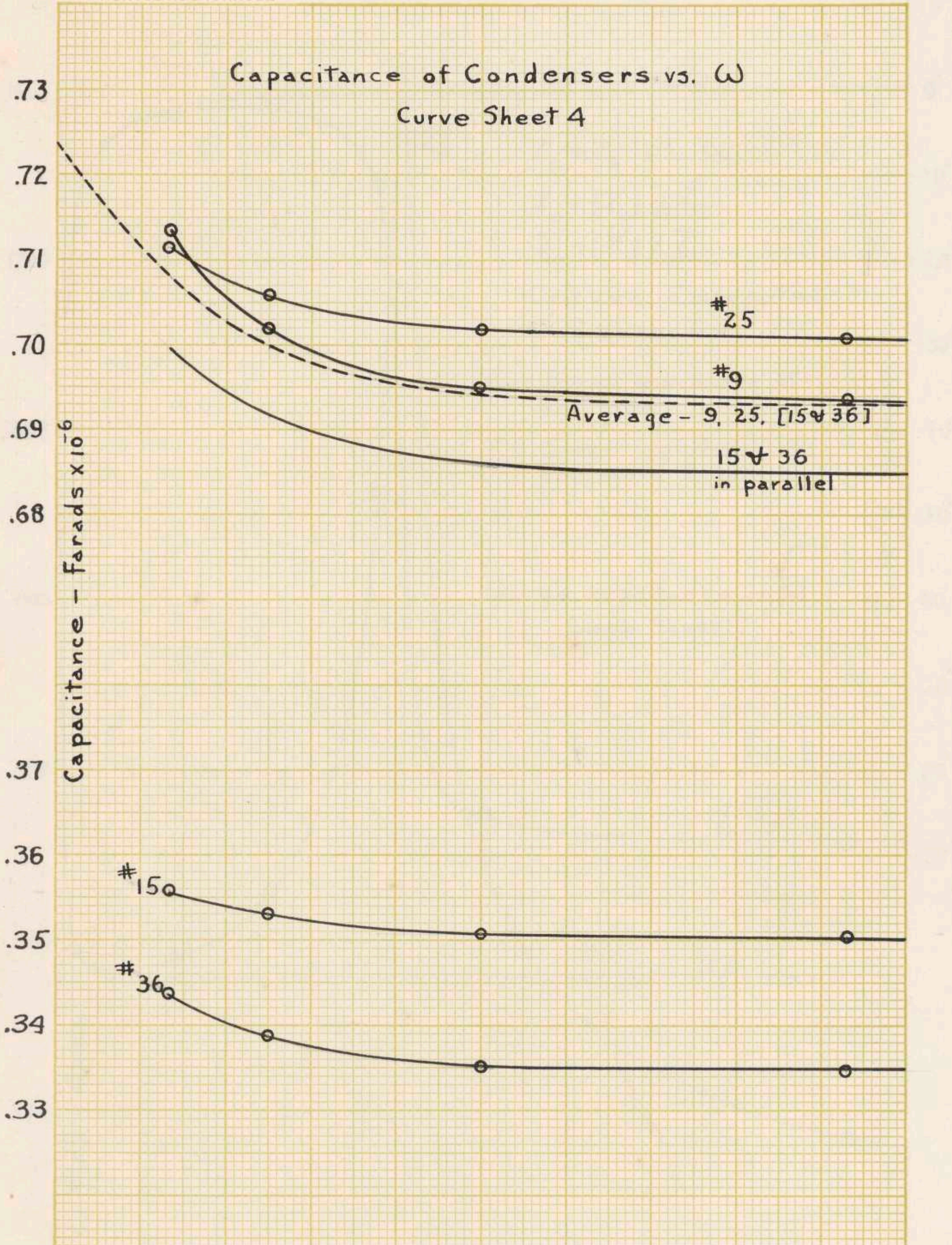
Inductance of Reactors vs. ω

Curve Sheet 2

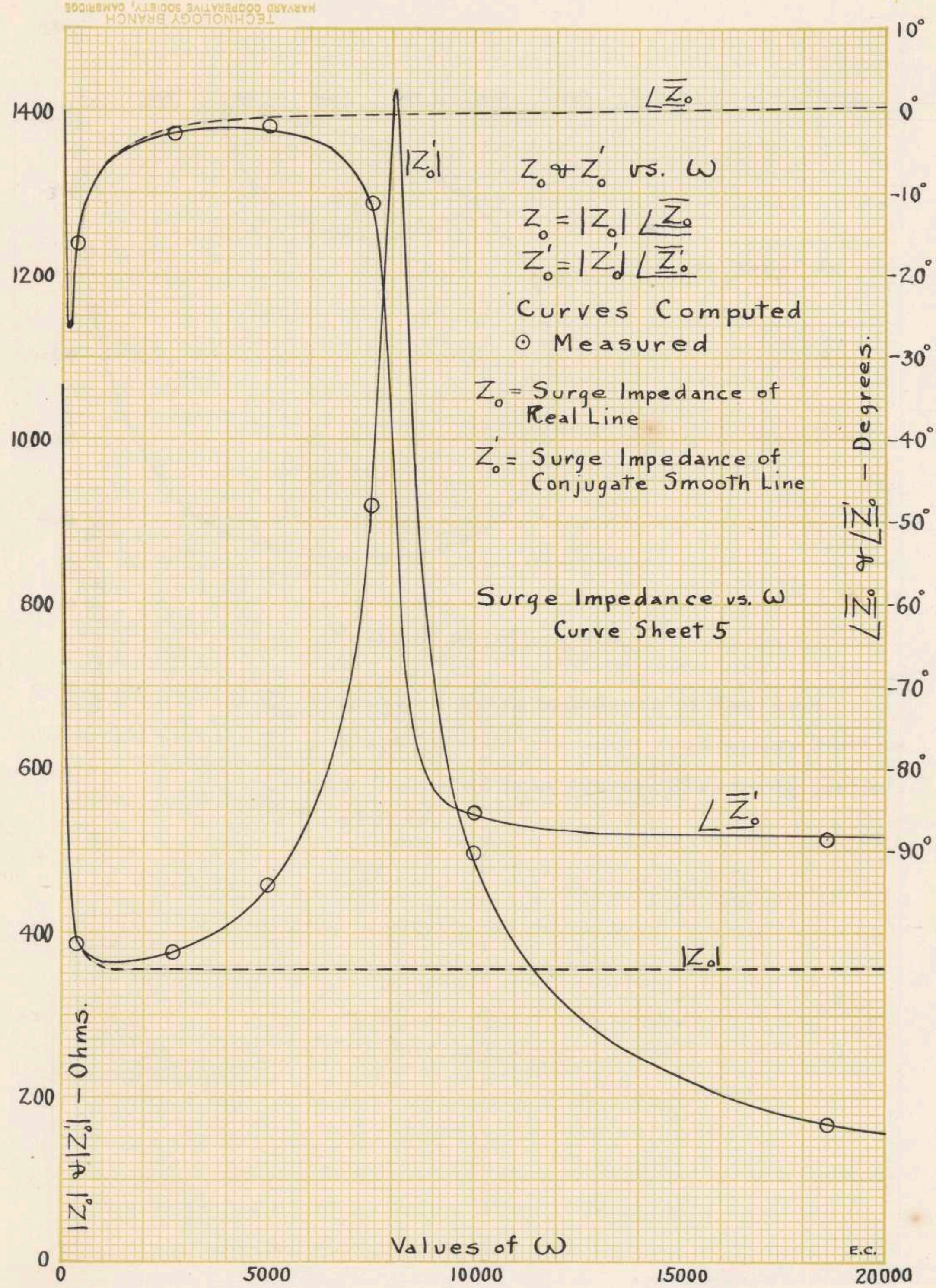


Leakance of Condensers vs. ω

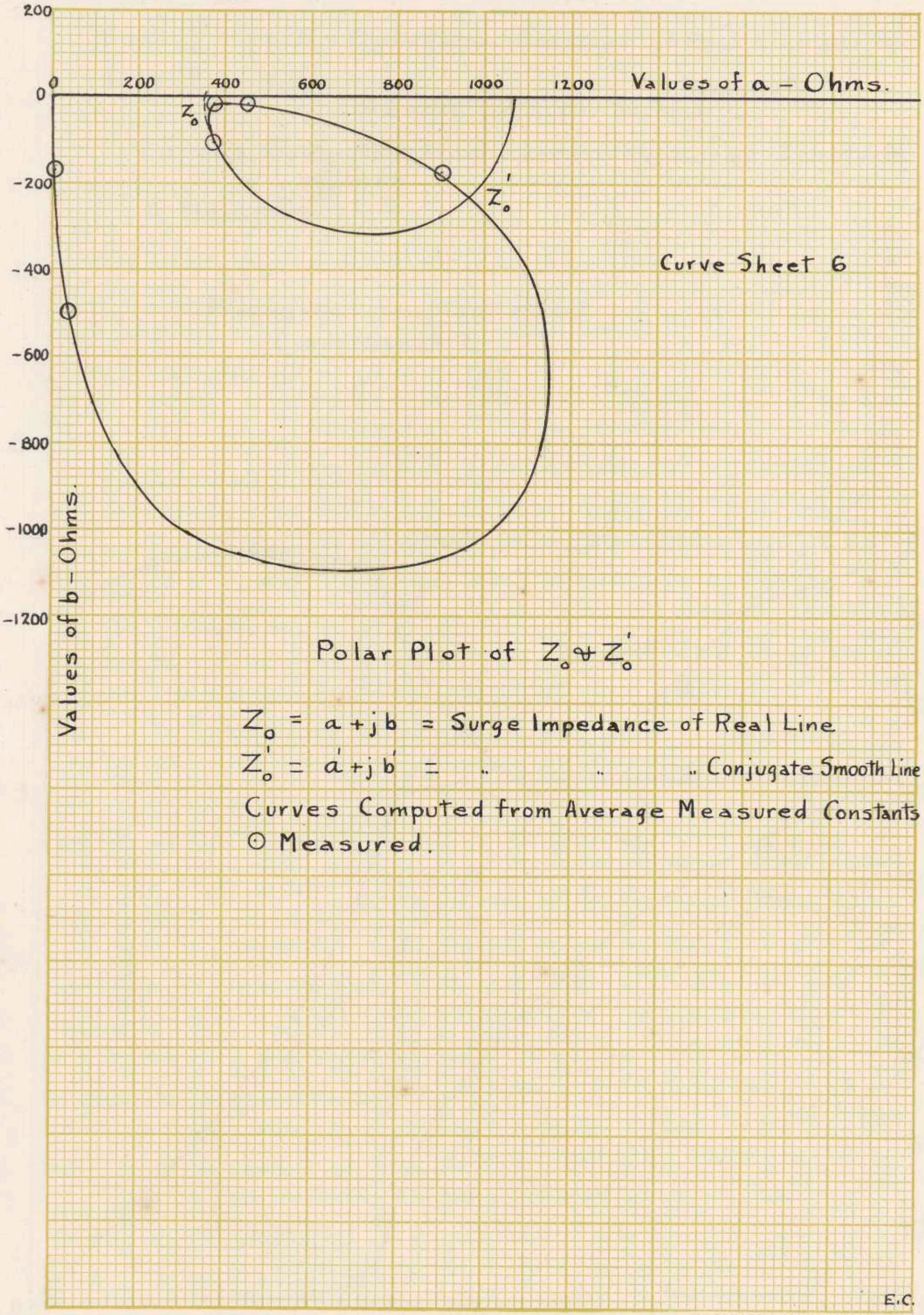
Curve Sheet 3



Capacitance of Condensers vs. ω
Curve Sheet 4
.73
.72
.71
.70
.69
.68
.37
.36
.35
.34
.33
Capacitance - Farads x 10⁻⁶
#25
#9
Average - 9, 25, [15 & 36]
15 & 36
in parallel
#15
#36
Values of ω
0
5000
10,000
15,000
20,000
E.C.

Z₀ & Z₀' vs. ω
Z₀ = |Z₀| ∠Z̄₀
Z₀' = |Z₀'| ∠Z̄₀'
Curves Computed
⊙ Measured
Z₀ = Surge Impedance of Real Line
Z₀' = Surge Impedance of Conjugate Smooth Line
Surge Impedance vs. ω
Curve Sheet 5
∠Z̄₀
|Z₀'|
∠Z̄₀'
|Z₀|
|Z₀| & |Z₀'| – Ohms.
∠Z̄₀ & ∠Z̄₀' – Degrees.
Values of ω
E.C.
0
200
400
600
800
1000
1200
1400
0
5000
10000
15000
20000
10°
0°
-10°
-20°
-30°
-40°
-50°
-60°
-70°
-80°
-90°

Curve Sheet 6



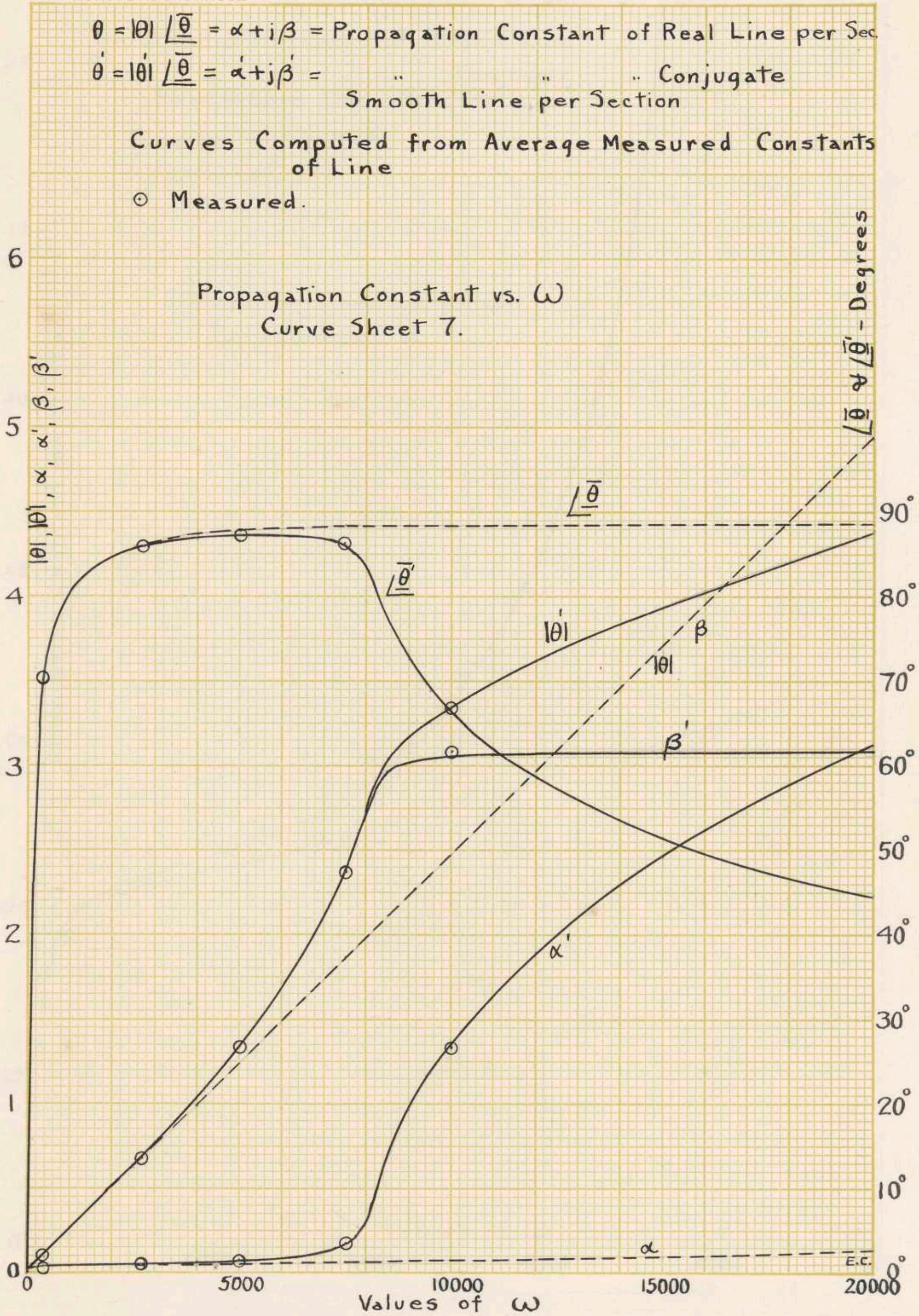
Polar Plot of Z_o & Z_o'

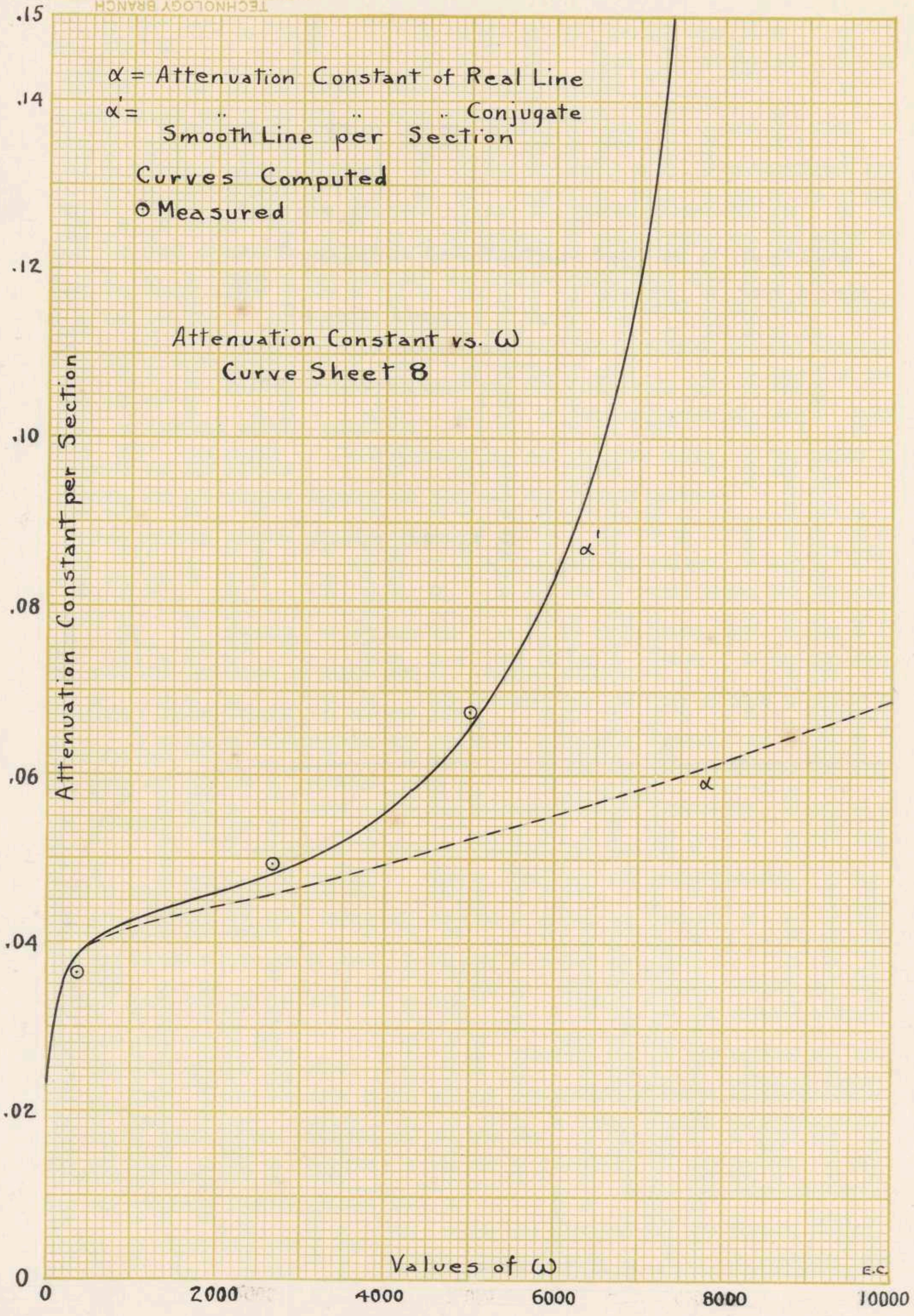
$Z_o = a + jb$ = Surge Impedance of Real Line

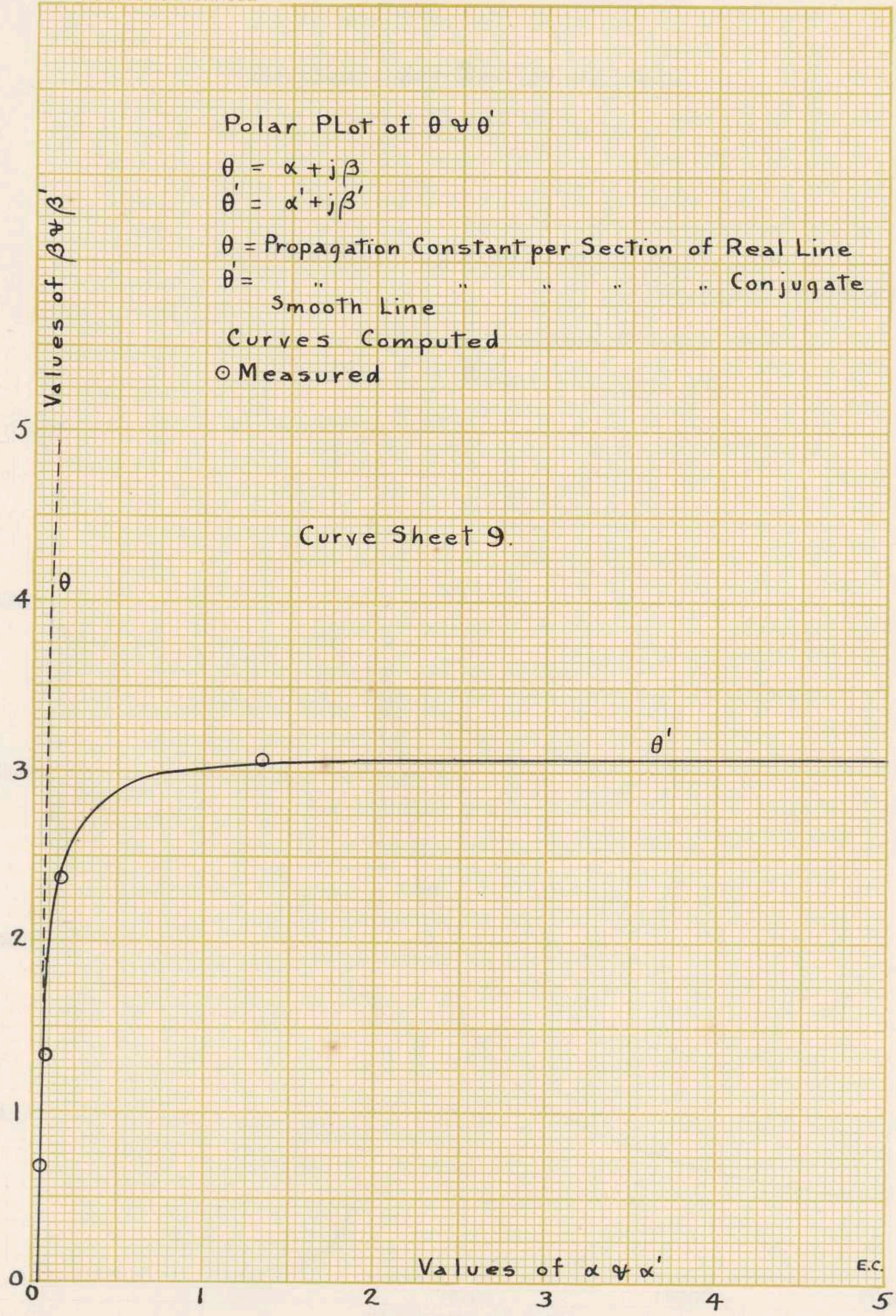
$Z_o' = a' + jb'$ = " " " Conjugate Smooth Line

Curves Computed from Average Measured Constants

⊙ Measured.


θ = |θ| ∠θ̄ = α + jβ = Propagation Constant of Real Line per Sec.
θ' = |θ'| ∠θ̄' = α' + jβ' = " " " Conjugate Smooth Line per Section
Curves Computed from Average Measured Constants of Line
⊙ Measured.
Propagation Constant vs. ω
Curve Sheet 7.
|θ|, |θ'|, α, α', β, β'
∠θ̄ & ∠θ̄' - Degrees
∠θ̄
∠θ̄'
|θ'|
|θ|
β
β'
α'
α
E.C.
0
1
2
3
4
5
6
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
0
5000
10000
15000
20000
Values of ω


α = Attenuation Constant of Real Line
α' = " " " Conjugate Smooth Line per Section
Curves Computed
⊙ Measured
Attenuation Constant vs. ω
Curve Sheet 8
Attenuation Constant per Section
Values of ω
α'
α
.15
.14
.12
.10
.08
.06
.04
.02
0
0
2000
4000
6000
8000
10000
E.C.

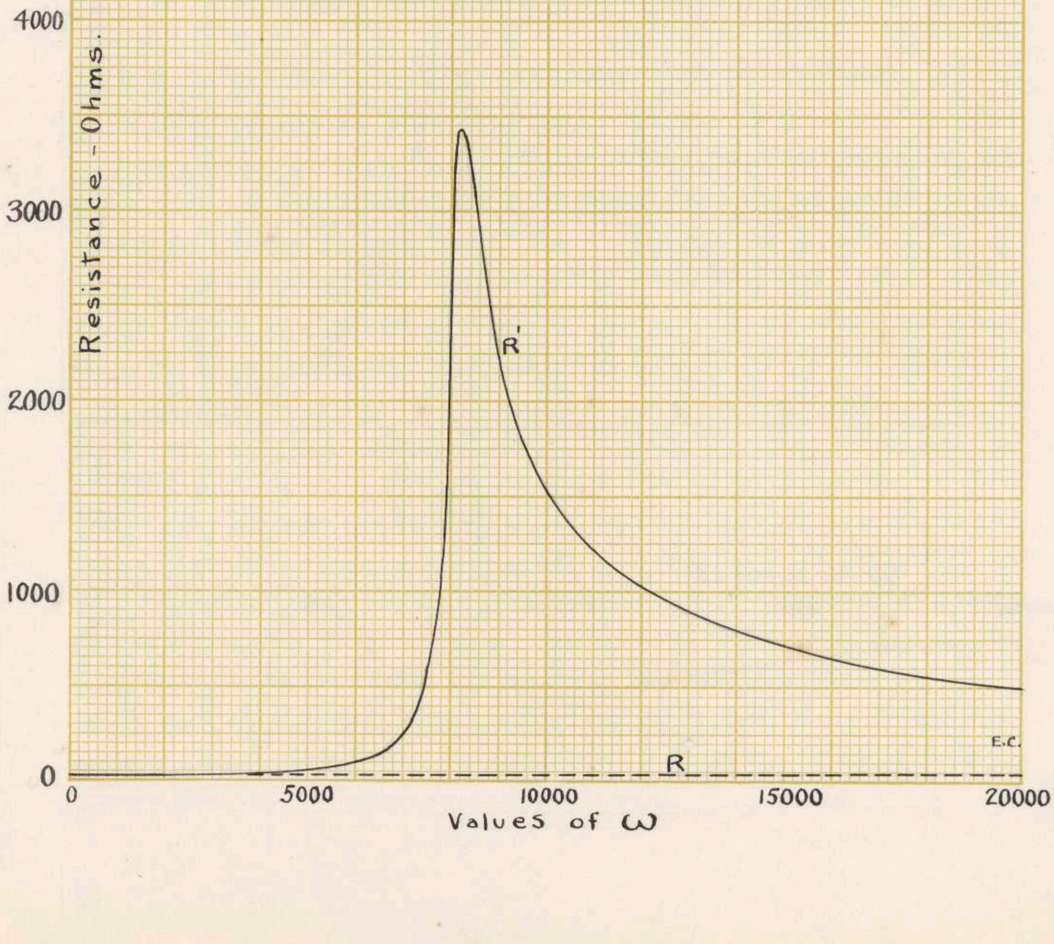

Curve Sheet 9.

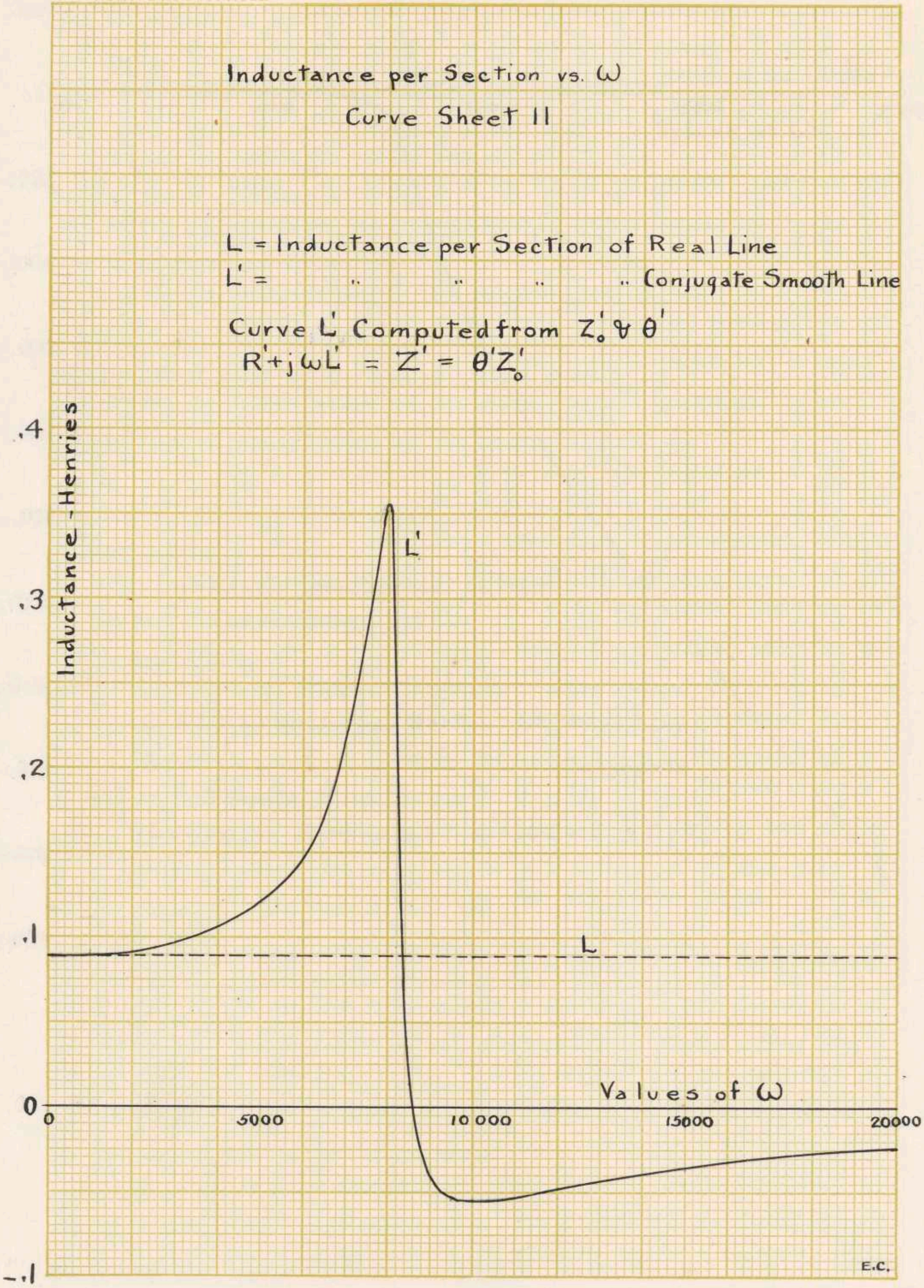
Resistance per Section vs. ω

Curve Sheet 10

R = Resistance per Section of Real Line
R' = " " " " Conjugate Smooth Line

Curve, R', Computed from θ' & Z_0'
$R' + j\omega L' = Z' = \theta' Z_0'$





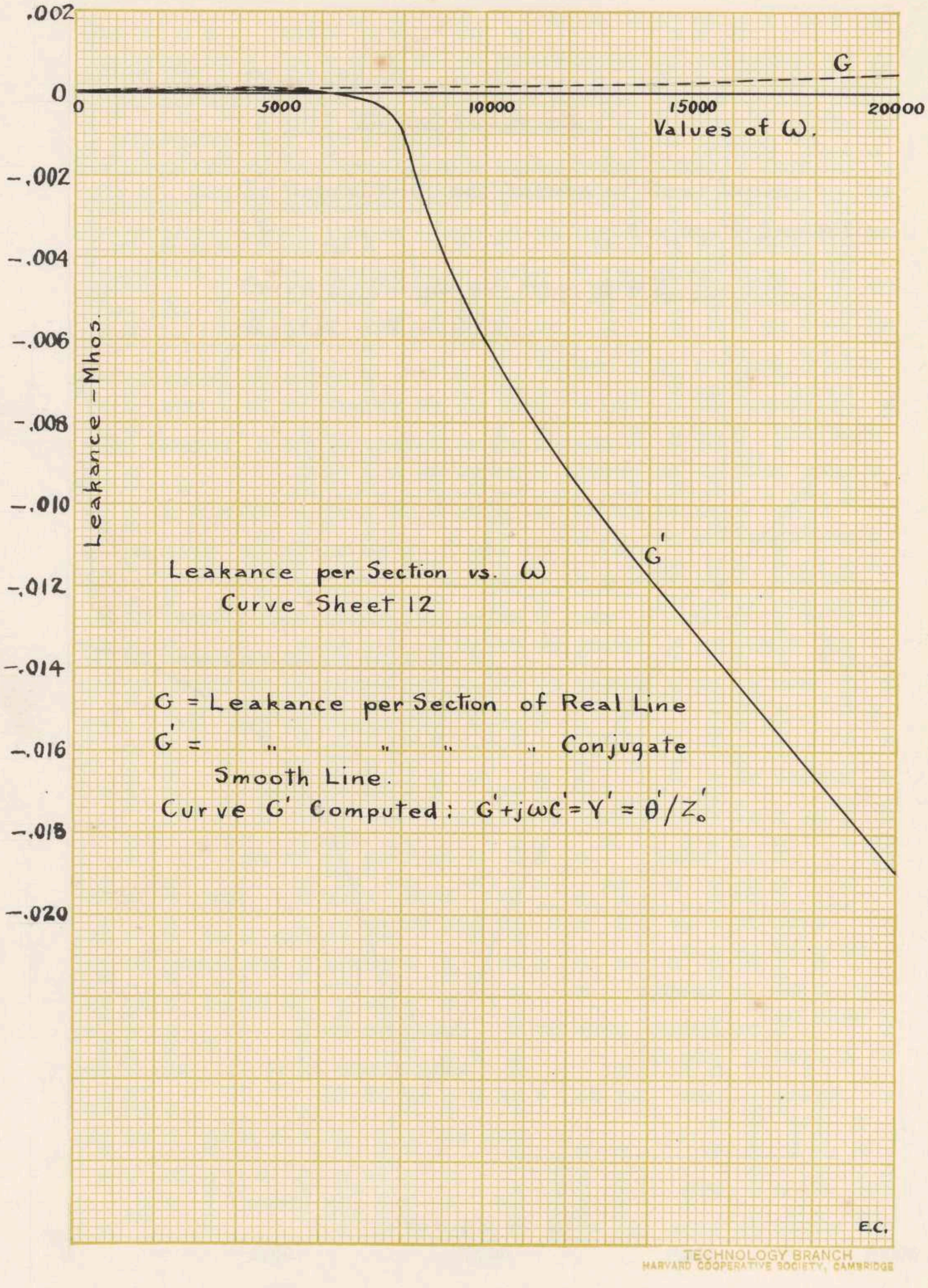
Inductance per Section vs. ω

Curve Sheet 11

L = Inductance per Section of Real Line
L' = " " " " Conjugate Smooth Line

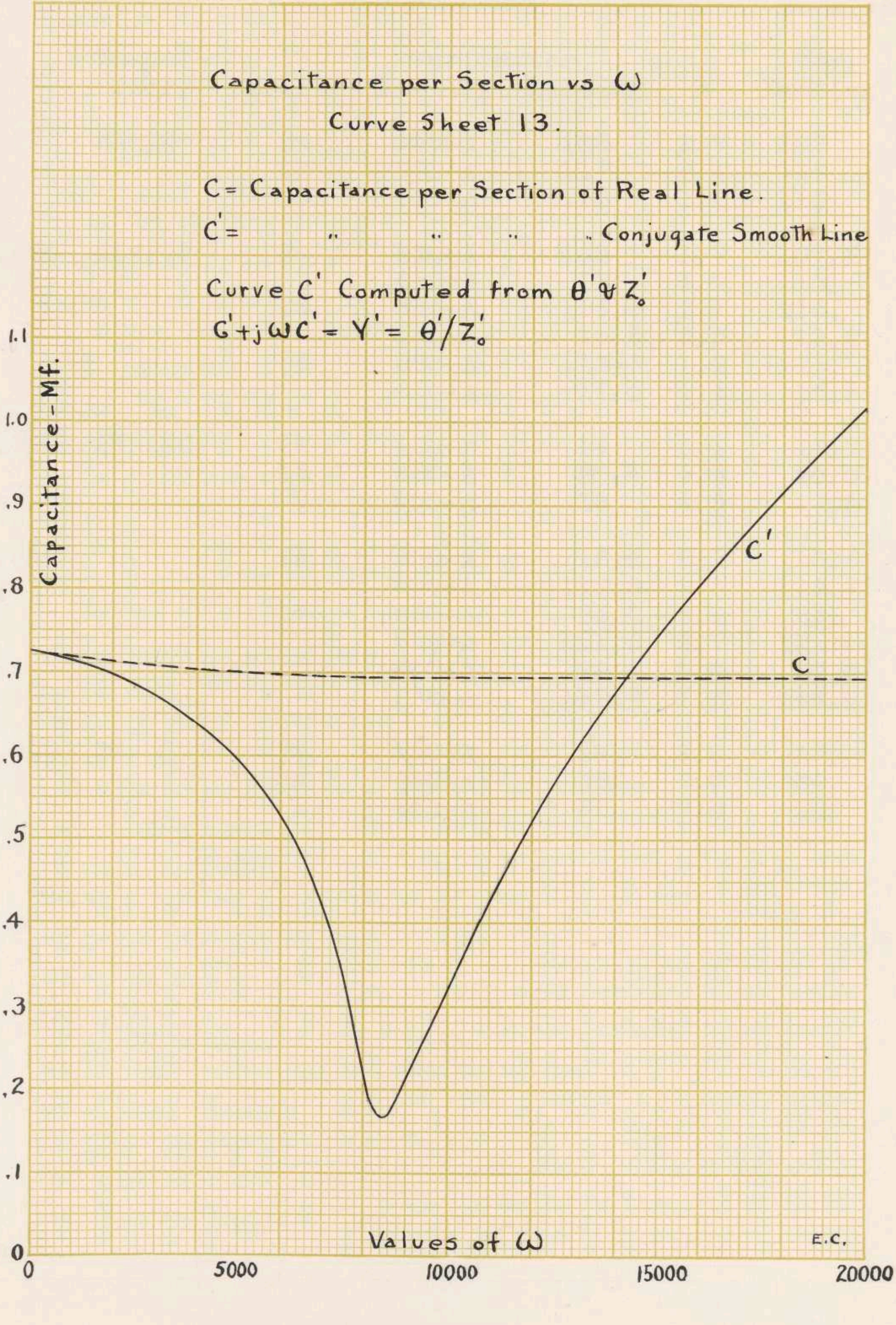
Curve L' Computed from Z_0' & θ'

$R' + j\omega L' = Z' = \theta' Z_0'$



Leakance per Section vs. ω
Curve Sheet 12

G = Leakance per Section of Real Line
G' = " " " " Conjugate Smooth Line.
Curve G' Computed: $G'+j\omega C'=Y'=\theta'/Z_0'$



Capacitance per Section vs ω

Curve Sheet 13.

C = Capacitance per Section of Real Line.

C' = " " " Conjugate Smooth Line

Curve C' Computed from θ' & Z_0'

$G'+j\omega C' = Y' = \theta'/Z_0'$

OBSERVED DATA: RAYLEIGH BRIDGE MEASUREMENTS - REACTORS.

ω	Temperature	Coil #	Variometer			Bridge Arms Reversed	
			Setting	Reading	R	Var. Reading	R
377	23°.C	29	550-A	108	1.7	107	1.7
		18	550-A	117	2.4	116	2.5
		5	550-A	113	1.2	113	1.2
2,680	21°.5C	29	550-A	108	1.7	107.2	1.7
			450-A	13	4.8	13.	4.8
		18	550-A	116.8	2.4	116.	2.5
			450-A	19.5	5.5	19.2	5.6
		5	550-A	113-5	1.2	113.	1.2
			450-A	17.0	4.3	16.8	4.4
5,000	24°.5C	29	450-A	14	5.1	13.2	5.2
		18	450-A	20.4	5.6	20.0	5.8
		5	450-A	18	4.5	17.2	4.7
10,000		29	450-A	16.2	5.7	16.5	6.1
			550-A	112.5	2.5	112.7	2.9
		18	450-A	23.0	5.9	23.	6.3
			550-A	121.0	2.7	121.	3.1
		5	450-A	20.0	5.0	20.5	5.3
			550-A	117.7	1.8	118.0	2.1
18,600	26. 0	29	450-A	28.	8.0	27.	8.3
		18	450-A	35.	7.0	34.	7.0
		5	450-A	31.	7.0	32.	7.4

OBSERVED DATA: CAMPBELL-MUTUAL INDUCTANCE BRIDGE MEASUREMENTS-REACTORS.

				Bridge Arms Reversed	
ω	Coil #	M + m	R	M + m	R
2,680	29	.005400	2.6	.005290	2.6
	18	.006550	3.3	.006440	3.3
	5	.006110	2.1	.006000	2.1
5,000	29	.005360	2.8	.005272	2.8
	18	.006540	3.4	.006430	3.4
	5	.006090	2.4	.006000	2.3
10,000	29	.005330	5.0	.005180	5.0
	18	.006500	5.0	.006385	5.1
	5	.006050	4.6	.005900	4.4
18,600	29	.005290	13.	.005070	12.5
	18	.006420	12.	.006260	11.0
	5	.006000	12.	.005770	11.0

Equal Bridge Arms

L = 0.100 henry
r = 22.2 - D.C.

OBSERVED DATA: RAYLEIGH BRIDGE MEASUREMENTS - CONDENSERS.

ω	Temperature	Condenser			Bridge Arms Reversed	
	C	#	R	C	R	C
2,680	23°.0	15	19.3	.3562	19.5	.3556
		36	36.3	.3436	36.5	.3430
		9	23.8	.7140	24.0	.7126
		25	10.3	.7117	10.5	.7106
5,000		15	9.0	.3533	9.2	.3526
		36	15.7	.3390	15.9	.3383
		9	10.3	.7025	10.5	.7011
		25	5.3	.7062	5.4	.7050
10,000		15	4.2	.3510	4.4	.3504
		36	6.8	.3353	7.0	.3347
		9	4.6	.6953	4.8	.6941
		25	2.8	.7024	3.0	.7010
18,600		15	2.2	.3505	2.4	.3500
		36	3.6	.3348	3.8	.3344
		9	3.3	.6940	3.4	.6930
		25	2.4	.7014	2.5	.7000

OBSERVED DATA: HOME END IMPEDANCE MEASUREMENTS DISTANT END GROUNDED AND FREE.

ω = 377 Temp. 23°.5

Sections		Var. Setting	Var. Reading	R Ohms	C Mf.	Extra Inductance (henry)
1	Z_g	550-A	100	0		
" (rev)	Z_g	"	105	0		
	Z_f			20	.74	
" (rev)	Z_f			20	.75	
2	Z_g	750-A	65	20		
" (rev)	Z_g	"	65	22		
	Z_f			100	1.45	
" (rev)	Z_f			100	1.55	
3	Z_g	750-A	90	27.		0.1
" (rev)	Z_g	"	80	27.		0.1
	Z_f			180	2.25	
" (rev)	Z_f			180	2.30	
ω = 2,680 Temp. 22°.0						
1	Z_g	550-A	27.5	18.6		
	"	"	27.0	18.6		Bridge Arms Rev.
" (rev)	Z_g	550-A	31.	18.7		
	"	"	30.	18.7		" " "
	Z_f		21.5		.789	
	"		21.7		.787	" " "
" (rev)	Z_f		20.3		.798	
	"		20.5		.796	" " "
2	Z_g	750-A	95	706		Coils #15,3,6,28
	"	"	93	708		Bridge Arms Rev.
" (rev)	Z_g	"	126	714		Coils #15,3,6,28
	"	"	125	714		Bridge Arms Rev.
	Z_f	550-A	35.7	13.	1.0	
	"	"	35.9	13	1.0	" " "
" (rev)	Z_f	"	35.0	11.9	1.0	
	"	"	35.0	12.0	1.0	" " "
3	Z_g			233.2	.557	
	"			233.7	.557	" " "

Sections		Var. Setting	Var. Reading	R Ohms	C Mf.	Extra Inductance (henry)
ω = 2,680 Temp. 22°.0						
	Z_f	450-A	99	54.5		
	"	"	99	54.5		Bridge Arms Rev.
3 (rev)	Z_f	"	100	56.4		
	"	"	99.5	56.5		" " "
ω = 5,000 Temp. 23°.5						
1	Z_g	750-A	40	517		Coils #15 & #3
	"	"	37	518		Bridge Arms Rev.
" (rev)	Z_g	"	152	431		Coils #15 & #3
	"	"	150	432		Bridge Arms Rev.
	Z_f	450-A	149.5	5.8	0.5	
		"	149.5	6.0	0.5	" " "
" (rev)	Z_f	"	138.5	3.3	0.5	
			139	3.5	0.5	" " "
Temp. 24°.0						
2	Z_g			80.9	.906	
	"			81.1	.905	" " "
2 (rev)	Z_g			79.7	.937	
	"			79.9	.935	" " "
	Z_f	750-A	99	286.6		
	"	"	98	287.4		" " "
" (rev)	Z_f	"	112	272.2		
	"	"	111	273.2		" " "
Temp. 24°5						
3	Z_g	550-A	46.5	246.6		
	"	"	46.	247.6		" " "
" (rev)	Z_g	"	83	238.7		
	"	"	82	239.7		" " "
	Z_f			150	.532	
	"			150	.532	" " "
" (rev)	Z_f			127.4	.574	
	"			128.0	.572	" " "

ω = 7,500 Temp. 24.5

Sections		Var. Setting	Var. Reading	R Ohms	C Mf.	Extra Inductance (henry)
1	Z_g			81.5	.151	
	"			81.5	.151	Bridge Arms Rev.
" (rev)	Z_g			124	.137	
	"			124	.137	" " "
	Z_f	550-A	79	315		
	"	"	78	315		" " "
" (rev)	Z_f	"	40	332		
	"	"	38	327		" " "
2	Z_g			2300	.103	
	"			2300	.103	" " "
" (rev)	Z_g	50-D	Anywhere	2600		
	"	"	"	"		
	Z_f	450-A	97	263	.2	
	"	"	98	265	.2	
" (rev)	Z_f	550-A	104	304	.2	
	"	"	105	304	.2	
3	Z_g	350-A	37	1030		
	"	"	"	"		Bridge Arms Rev.
" (rev)	Z_g	450-A	88	858		
	"	"	"	"		" " "
	Z_f			500	.172	
	"			"	"	" " "
" (rev)	Z_f			600	.219	
	"			"	"	" " "

ω = 10,000

1	Z_g			15.3	.237	
	"			"	"	" " "
" (rev)	Z_g			22.3	.222	
	"			"	"	" " "
	Zf			46.2	.181	
	"			"	"	" " "

$\omega = 10{,}000$

Sections		Var. Setting	Var. Reading	R Ohms	C Mf.	Extra Inductance (henry)
1 (rev)	Z_f			57	.169	
	"			"	"	
2	Z_g			26.8	.213	
	"			"	"	Bridge Arms Rev.
" (rev)	Z_g			43.	.195	
	"			"	"	" " "
"	Z_f			40.6	.212	
	"			"	"	" " "
" (rev)	Z_f			57.	.194	
	"			"	"	" " "
3	Z_g			32.7	.212	
	"			"	"	" " "
" (rev)	Z_g			47.5	.195	
	"			"	"	" " "
	Z_f			32.7	.212	
	"			"	"	" " "
" (rev)	Z_f			47.5	.195	
	"			"	"	" " "

$\omega = 18{,}600$ Temp. 26°

Sections		Var. Setting	Var. Reading	R Ohms	C Mf.	Extra Inductance (henry)
1	Z_g			3.0	.318	
	"			3.3	.318	" " "
" (rev)	Z_g			4.7	.302	
	"			4.9	.302	" " "
	Z_f			3.2	.315	
	"			3.4	.314	" " "
" (rev)	Z_f			4.9	.300	
	"			5.1	.299	" " "
2	Z_g			3.2	.317	
	"			3.4	.316	" " "
" (rev)	Z_g			4.8	.300	
	"			5.1	.300	" " "
	Z_f			3.2	.317	
	"			3.4	.317	" " "

ω = 18,600 Temp. 26°

Sections		Var. Setting	Var. Reading	R Ohms	C Mf.	Extra Inductance (henry)
2 (rev)	Z_f			4.9	.300	
	"			5.1	.300	Bridge Arms Rev.
3	Z_g			3.1	.317	
	"			3.4	.317	" " "
" (rev)	Z_g			4.8	.300	
	"			5.1	.300	" " "
	Z_f			3.1	.317	
	"			3.4	.317	" " "
" (rev)	Z_f			4.8	.300	
	"			5.1	.300	" " "
ω = 2,680						
Coils #15 & #3		750-A	58	19.3		
" # <u>6</u> & #28		"	59.3	20.0		
ω = 5,000						
" #15 & #3		750-A	60-.5	20.1		

RESISTANCE OF VARIOMETER AT 22° C. CONNECTION:

A ROT. 50 + 100.

Fixed	Values of ω						
	0	2680	3140	800	7500	10,000	18,600
750	29.4	30.4		31.0			56
650	26.3	27.0		27.9			49
550	23.0		23.6	23.9	24.9	25.5	34
450	19.9			20.7	21.4	22.3	29
350	15.5					16.4	
200	10.8						

RESISTANCE OF STANDARD 0.1 HENRY COIL.

	0	2680	3140	800	7500	10,000	18,600
	22.2	22.6		23.1			24.5

PRELIMINARY CALCULATIONS - Z_o θ, Z'_o AND θ'
FROM GIVEN CONSTANTS.*

	Z (R+jωL)	$Y10^6$ $(G+j\omega C)10^6$	θ $\sqrt{ZY}$	Z_o $\sqrt{Z/Y}$
377	24.1+j34.1 41.7/54°.8	9.6+j283 283/88°.0	.0347+j1030 .1087/71°.4	284/16°.6
2500	25.6+j226 227/83°.5	9.6+j1876 1876/89°.7	.0385+j652 .6531/86°.6	348/3°.1
5000	26.3+j452 452/86°.67	9.6+j3752 3752/89°.85	.0394+j1.302 1.303/88°.3	347/1°.6
7500	27.3+j678 678/87°.7	9.6+j5628 5628/89°.9	.0409+j1.954 1.954/88°.8	347/1°.1
10000	29.+j904 904/88°.2	9.6+j7504 7504/89°.9	.0432+j2.605 2.605/89°.0	347/0°.9

	$\theta'/2$ $\sinh^{-1}\theta/2$	Cosh $\frac{\theta'}{2}$	θ'	Z'_o $Z_o/\cosh\theta'/2$
377	.05435/71°.4	.999/0°.1	.0347+j.1030 .1087/71°.4	384/16°.7
2500	.3327/86°.5	.946/0°.4	.0408+j664 .665/86°.5	368/3°.5
5000	.710/87°.9	.760/1°.3	.0520+j1418 1.420/87°.9	457/2°.9
7500	1.341/86°.2	.247/20°.4	.1770+j2.676 2.682/86°.2	1405/21°.5
10000	1.724/63°.8	.835/88°.0	1.524+j3.09 3.45/63°.8	415/88°.9

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* "Artificial Electric Lines", page 201,
by A. E. Kennelly.

CONSTANTS OF THE LINE - CALCULATED FROM MEASUREMENTS.

ω	Coil #	R	L
377	29	24.8	.0893
	18	25.6	.0870
	5	24.3	.0878
	Average	24.9	.0880
2680	29	25.2	.08931
	18	25.9	.08701
	5	24.7	.08789
	Average	25.3	.08807
5000	29	25.9	.08937
	18	26.5	,08703
	5	25.4	.08791
	Average	25.9	.08810
10000	29	28.2	.08949
	18	28.4	.08712
	5	27.5	.08805
	Average	28.0	.08822
18600	29	37.2	.08964
	18	36.0	.08732
	5	36.2	.08823
	Average	36.5	.08840

CONSTANTS OF THE LINE - CALCULATED FROM MEASUREMENTS

ω	Condenser Number	R	C	G
2680	15	19.4	.3559	17.6
	36	36.4	.3433	30.8
	9	23.9	.7133	87.2
	25	10.4	.7112	37.8
	Average*		.7079	57.8
5000	15	9.1	.3530	28.4
	36	15.8	.3386	45.4
	9	10.4	.7018	128.
	25	5.4	.7056	67.2
	Average		.6997	89.7
10000	15	4.3	.3507	52.7
	36	6.9	.3350	77.4
	9	4.7	.6947	227.
	25	2.9	.7017	143.
	Average		.6940	166.7
18600	15	2.3	.3502	98.3
	36	3.7	.3346	144.
	9	3.35	.6935	560.
	25	2.45	.7007	427.
	Average		.6930	410.

* With Condensers #36 and #15 in parallel.

CALCULATING OF Z_0, θ, Z'_0 and θ' FROM THE AVERAGE MEASURED CONSTANTS OF THE LINE.

ω	Z $(R+j\omega L)$	$Y10^6$ $(G+j\omega C)10^6$	$\theta = \sqrt{ZY}$	$Z_0 = \sqrt{Z/Y}$
0	$24.9+j0$	$22+j0$	$.0234+j0$ $.0234\underline{/0^\circ}$	$1065+j0$ $1065\underline{/0^\circ}$
10	$24.9+j.880$ $24.9\underline{/2^\circ.03}$	$22+j7.24$ $23.2\underline{/18^\circ.40}$	$.0236+j.0043$ $.0240\underline{/10^\circ.21}$	$1024-j148$ $1035\underline{/8^\circ.2}$
25	$24.9+j2.20$ $25.0\underline{/5^\circ.05}$	$22+j18.1$ $28.5\underline{/39^\circ.43}$	$.0247+j.0101$ $.0267\underline{/22^\circ.24}$	$895-j277$ $936\underline{/17^\circ.2}$
50	$24.9+j4.40$ $25.3\underline{/10^\circ.04}$	$22.5+j36.18$ $42.5\underline{/58^\circ.1}$	$.0280+j.0189$ $.0338\underline{/34^\circ.0}$	$705-j314$ $771\underline{/24^\circ.0}$
100	$24.9+j8.80$ $26.4\underline{/19^\circ.45}$	$23+j72.3$ $75.6\underline{/72^\circ.36}$	$.0311+j.0321$ $.0446\underline{/45^\circ.9}$	$530-j265$ $591\underline{/26^\circ.5}$
377	$24.9+j33.3$ $41.5\underline{/53^\circ.22}$	$26+j272$ $273\underline{/84^\circ.52}$	$.0384+j.0993$ $.1065\underline{/68^\circ.87}$	$375-j105$ $390\underline{/15^\circ.65}$
1,000	$25.0+j88.0$ $92.3\underline{/74^\circ.15}$	$35.0+j717$ $718\underline{/87^\circ.20}$	$.0419+j.255$ $.258\underline{/80^\circ.68}$	$357-j34.2$ $359\underline{/6^\circ.52}$
2,500	$25.3+j220$ $222\underline{/83^\circ.45}$	$55+j1770$ $1770\underline{/88^\circ.21}$	$.0454+j.625$ $.626\underline{/85^\circ.83}$	$353-j14.7$ $354\underline{/2^\circ.38}$
4,000	$25.6+j352.5$ $354\underline{/85^\circ.85}$	$76+j2810$ $2810\underline{/88^\circ.45}$	$.0492+j.996$ $.996\underline{/87^\circ.15}$	$355-j8.1$ $355\underline{/1^\circ.30}$
5,000	$25.9+j441$ $441\underline{/86^\circ.62}$	$90+j3500$ $3500\underline{/88^\circ.52}$	$.0525+j1.243$ $1.243\underline{/87^\circ.57}$	$355-j5.9$ $355\underline{/0^\circ.95}$
6,500	$26.4+j573$ $574\underline{/87^\circ.35}$	$112+j4530$ $4530\underline{/88^\circ.59}$	$.0568+j1.613$ $1.613\underline{/87^\circ.97}$	$356-j3.8$ $356\underline{/0^\circ.62}$
7,500	$26.8+j662$ $662\underline{/87^\circ.68}$	$126+j5225$ $5225\underline{/88^\circ.62}$	$.0600+j1.860$ $1.860\underline{/88^\circ.15}$	$356-j2.9$ $356\underline{/0^\circ.47}$
7,800	$26.9+j688$ $688\underline{/87^\circ.76}$	$130+j5430$ $5430\underline{/88^\circ.62}$	$.0612+j1.935$ $1.935\underline{/88^\circ.19}$	$356-j2.7$ $356\underline{/0^\circ.43}$
8,000	$27.0+j705$ $705\underline{/87^\circ.80}$	$132+j5560$ $5560\underline{/88^\circ.64}$	$.0614+j1.980$ $1.980\underline{/88^\circ.22}$	$356-j2.6$ $356\underline{/0^\circ.43}$

CALCULATING OF Z_o, θ, Z'_o and θ'

FROM THE AVERAGE MEASURED CONSTANTS OF THE LINE.

ω	$\theta'/2$ $\sinh^{-1}\theta/2$	$\cosh\theta'/2$	θ'	Z'_o $Z_o/\cosh\theta'/2$
0	$.0117\underline{/0^\circ}$	$1.000\underline{/0^\circ}$	$.0234+j0$ $.0234\underline{/0^\circ}$	$1065+j0$ $1065\underline{/0^\circ}$
10	$.0120\underline{/10^\circ.21}$	$1.000\underline{/0^\circ}$	$.0236+j.0043$ $.0240\underline{/10^\circ.21}$	$1024-j148$ $1035\overline{/8^\circ.2}$
25	$.0133\underline{/22^\circ.24}$	$1.000\underline{/0^\circ}$	$.0247+j.0101$ $.0267\underline{/22^\circ.24}$	$895-j277$ $936\overline{/17^\circ.2}$
50	$.0169\underline{/34^\circ.0}$	$1.000\underline{/0^\circ}$	$.0280+j.0189$ $.0338\underline{/34^\circ.0}$	$705-j314$ $711\overline{/24^\circ.0}$
100	$.0223\underline{/45^\circ.9}$	$1.000\underline{/0^\circ}$	$.0311+j.0321$ $.0446\underline{/45^\circ.9}$	$530-j265$ $591\overline{/26^\circ.5}$
377	$.0532\underline{/68^\circ.8}$	$.999\underline{/0^\circ}$	$.0385+j.0993$ $.1064\underline{/68^\circ.8}$	$375-j103$ $390\overline{/15^\circ.7}$
1,000	$.130\underline{/80^\circ.6}$	$.990\underline{/0^\circ.2}$	$.0425+j.256$ $.260\underline{/80^\circ.6}$	$360-j42$ $362\overline{/6^\circ.7}$
2,500	$.317\underline{/85^\circ.7}$	$.950\underline{/0^\circ.4}$	$.0475+j.632$ $.634\underline{/85^\circ.7}$	$372-j18$ $373\overline{/2^\circ.8}$
4,000	$.52\underline{/86^\circ.9}$	$.868\underline{/0^\circ.9}$	$.056+j1.04$ $1.04\underline{/86^\circ.9}$	$409-j16$ $409\overline{/2^\circ.2}$
5,000	$.671\underline{/87^\circ.2}$	$.783\underline{/1^\circ.5}$	$.0655+j1.341$ $1.342\underline{/87^\circ.2}$	$452-j19$ $453\overline{/2^\circ.4}$
6,500	$.938\underline{/87^\circ.0}$	$.594\underline{/3^\circ.8}$	$.098+j1.875$ $1.876\underline{/87^\circ.0}$	$599-j46$ $600\overline{/4^\circ.4}$
7,500	$1.190\underline{/86^\circ.1}$	$.383\underline{/11^\circ.2}$	$.162+j2.38$ $2.38\underline{/86^\circ.1}$	$910-j189$ $929\overline{/11^\circ.7}$
7,800	$1.30\underline{/85^\circ.0}$	$.295\underline{/21^\circ.8}$	$.226+j2.59$ $2.60\underline{/85^\circ.0}$	$1120-j456$ $1210\overline{/22^\circ.2}$
8,000	$1.37\underline{/83^\circ.7}$	$.254\underline{/35^\circ.6}$	$.301+j2.72$ $2.74\underline{/83^\circ.7}$	$1130-j825$ $1400\overline{/36^\circ.0}$

CALCULATION OF θ, Z_o, θ' and Z'_o FROM AVERAGE MEASURED CONSTANTS OF THE LINE.

ω	Z	Y	θ	Z_o
8,050	27.05+j710 $710\underline{/87^\circ.82}$	133+j5600 $5600\underline{/88^\circ.64}$	.0615+j1.995 $1.995\underline{/88^\circ.23}$	356-j2.5 $.356\overline{/0^\circ.41}$
8,075	27.05+712.5 $712.5\underline{/87^\circ.82}$	133+j5610 $5610\underline{/88^\circ.64}$	.0616+j2.000 $2.000\underline{/88^\circ.23}$	356-j2.5 $356\overline{/0^\circ.41}$
8,100	27.05+j715 $715\underline{/87^\circ.83}$	134+j5640 $5640\underline{/88^\circ.63}$	.0620+j2.005 $2.005\underline{/88^\circ.23}$	356-j2.5 $356\overline{/0^\circ.40}$
8,200	27.1+j723 $723\underline{/87^\circ.85}$	136+j5700 $5700\underline{/88^\circ.63}$	.0621+j2.03 $2.03\underline{/88^\circ.24}$	356-j2.4 $356\overline{/0^\circ.39}$
8,500	27.25+j750 $750\underline{/87^\circ.92}$	140+j5910 $5910\underline{/88^\circ.64}$	.0635+j2.106 $2.106\underline{/88^\circ.28}$	356-j2.2 $356\overline{/0^\circ.36}$
9,000	27.5+j795 $795\underline{/88^\circ.02}$	150+j6250 $6250\underline{/88^\circ.62}$	.0655+j2.23 $2.23\underline{/88^\circ.31}$	357-j1.9 $357\overline{/0^\circ.30}$
9,500	27.7+j838 $838\underline{/88^\circ.10}$	157+j6590 $6590\underline{/88^\circ.62}$	.067+j2.35 $2.35\underline{/88^\circ.36}$	357-j1.6 $357\overline{/0^\circ.26}$
10,000	28.0+j882 $882.\underline{/88^\circ.18}$	167+j6940 $6940\underline{/88^\circ.62}$	.0690+j2.47 $2.47\underline{/88^\circ.40}$	357-j1.4 $357\overline{/0^\circ.22}$
12,000	29.4+j1059 $1059\underline{/88^\circ.40}$	210+j8320 $8320\underline{/88^\circ.54}$	.0790+j2.97 $2.97\underline{/88^\circ.47}$	357-j0.04 $357\overline{/0.07}$
15,000	32.0+j1325 $1.325\underline{/88^\circ.61}$	283+j10400 $10400\underline{/88^\circ.43}$	.0956+j3.71 $3.71\underline{/88^\circ.52}$	357+j0.03 $357\underline{/0^\circ.05}$
20,000	39.0+j1768 $1768\underline{/88^\circ.73}$	465+j13840 $13840\underline{/88^\circ.07}$	.138+j4.94 $4.94\underline{/88^\circ.40}$	357+j2.1 $357\underline{/0^\circ.33}$
40,000*	80+j3550 $3550\underline{/88^\circ.7}$	1500+j27600 $27600\underline{/86^\circ.9}$	.38+j9.88 $9.88\underline{/87^\circ.8}$	359+j5.6 $359\underline{/0^\circ.9}$

* Constants estimated.

CALCULATION OF θ, Z_o, θ' and Z_o' FROM AVERAGE MEASURED CONSTANTS OF THE LINE.

ω	$\theta'/2$	Cosh $\frac{\theta'}{2}$	θ'	Z_o'
8,050	$1.40\underline{/83^\circ.1}$	$.250\underline{/41^\circ.1}$	$.337+j2.78$ $2.80\underline{/83^\circ.1}$	$1065-j948$ $1426\overline{/41^\circ.5}$
8,075	$1.40\underline{/82^\circ.9}$	$.250\underline{/43^\circ.6}$	$.346+j2.78$ $2.80\underline{/82^\circ.9}$	$1027-j992$ $1426\overline{/44^\circ.0}$
8,100	$1.41\underline{/82^\circ.3}$	$.251\underline{/47^\circ.1}$	$.378+j2.79$ $2.82\underline{/82^\circ.3}$	$960-j1046$ $1420\overline{/47^\circ.5}$
8,200	$1.44\underline{/81^\circ.0}$	$.274\underline{/57^\circ.0}$	$.451+j2.85$ $2.88\underline{/81^\circ.0}$	$702-j1095$ $1300\overline{/57^\circ.4}$
8,500	$1.52\underline{/77^\circ.1}$	$.356\underline{/74^\circ.6}$	$.682+j2.97$ $3.04\underline{/77^\circ.1}$	$259-966$ $1000\overline{/75^\circ.0}$
9,000	$1.58\underline{/72^\circ.4}$	$.500\underline{/82^\circ.0}$	$.956+3,01$ $3.16\underline{/72^\circ.4}$	$96-j706$ $714\overline{/82^\circ.3}$
9,500	$1.63\underline{/68^\circ.9}$	$.624\underline{/84^\circ.6}$	$1.175+j3.04$ $3.26\underline{/68^\circ.9}$	$53-j570$ $573\overline{/84^\circ.7}$
10,000	$1,665\underline{/66^\circ.1}$	$.730\underline{/85^\circ.5}$	$1.35+j3.05$ $3.33\underline{/66^\circ.1}$	$36.5-j486$ $488\overline{/85^\circ.7}$
12,000	$1.805\underline{/58^\circ.3}$	$1.10\underline{/87^\circ.5}$	$1.90+j3.07$ $3.61\underline{/58^\circ.3}$	$14-j324$ $325\overline{/87^\circ.6}$
15,000	$1.97\underline{/51^\circ.4}$	$1.56\underline{/88^\circ.0}$	$2.46+j3.08$ $3.94\underline{/51^\circ.4}$	$8-j229$ $229\overline{/88^\circ.0}$
20,000	$1.56+j1.54$ $2.19\underline{/44^\circ.6}$	$2.26\underline{/88^\circ.0}$	$3.12+j3.08$ $4.38\underline{/44^\circ.6}$	$4.7-j158$ $158\overline{/88^\circ.3}$
40,000*	$2.28+j1.533$ $2.75\underline{/34^\circ}$	$4.85\underline{/87^\circ.7}$	$4.14+j3.08$ $5.5\underline{/34^\circ}$	$2-j74$ $74\overline{/88^\circ.4}$

* Constants estimated.

CALCULATION OF THE CONSTANTS OF THE CONJUGATE SMOOTH LINE

ω	Z' θ'Z₀'	Y'10^{+6} θ'/Z₀'10^{6}	R'	L'	G' 10^{6}	C' 10^{6}
0	24.9+j0	22+j0	24.9		22.	
100	24.9+j8.80 26.4/19°.4	23+j72.3 75.6/72°.4	24.9	.0880	23.	.723
377	24.9+j33.3 41.5/53°.1	26+j272 273/84°.5	24.9	.0880	26	.721
1000	26.1+j90.5 94.1/73°.9	34+j717 718/87°.3	26.1	.0905	34	.717
2500	29.3+j235 236/82°.9	44.5+j1700 1700/88°.5	29.3	.0940	44.5	.680
4000	40+j424 425/84°.6	44.5+j2550 2550/89°.0	40.0	.1060	44.5	.637
5000	55+j606 608/84°.8	20.6+j2970 2970/89°.6	55.0	.1210	20.6	.594
6500	145+j1115 1125/ 82°.6	-76.5+j3130 3130/ 91°.4	145	.1715	-76.5	.481
7500	595+j2130 2210/74°.4	-350+j2540 2570/ 97°.8	595	.284	-350	.339
8000	2580+j2840 3840/47°.7	-970+j1700 1955/119°.7	2580	.355	-970	.213
8100	3290+j2290 4000/34°.8	-1270+j1520 1980/129°.8	3290	.283	-1270	.188
8200	3430+j1500 3740/23°.6	-1655+j1470 2210/138°.4	3430	.183	-1655	.179
8500	3040+j111 3040/2°.1	-2690+j1425 3040/ 152°.1	3040	.0131	-2690	.168
9000	2230-j389 2260/9°.9	-4000+j1900 4430/154°.7	2230	-.0432	-4000	.211
9500	1800-j510 1870/15°.8	-5100+j2530 5700/ 153°.6	1800	-.0537	-5100	.266
10,000	1530-j545 1625/ 19°.6	-6000+j3220 6820/151°.8	1530	-.0545	-6000	.322

	Z' $O'Z_o'$	$Y'10^{-6}$ O'/Z_o'	R'	L'	$G'10^6$	$C'\ 10^6$
12,000	1025-j57 1172/29°.3	-9200+j6240 11,100/145°.9	1025	-.0480	-9200	.520
15,000	724+j538 900/36°.6	-13100+j11200 17,200/139°.4	724	-.0359	-13,000	.746
20,000	500-j479 693/43.7	-18900+j20,300 27700/ 132°.9	500	-.0240	-18,900	1.015
40,000	237-j331 407/54°.4	-40000+j62,800 74,300/122°.4	237	-.0083	-40,000	1.57

CALCULATION OF θ' AND Z_o' FROM IMPEDANCE MEASUREMENTS.

$$Z_o' = \sqrt{Z_g Z_f} \qquad \theta' = \tanh^{-1} \sqrt{Z_g/Z_f}$$

ω=377

No. of Sections	X	R	L	C	Z	Z_o'	θ'
1	Z_g	23.	.091		23.+j34.3		
1(rev)	Z_g	23.	.090		23.+j33.9		
	Z_g(Ave)				23.+j34.1		
					41.2/56°.0		
	Z_f	43		.74	43-j3590		
"(rev)	Z_f	43		.75	43-j3540		
	Z_f(Ave)				43-j3565		
					3565/89.°3		
						383/16.°7	.107/72°5
2	Z_g	50	.175		50+j66.		
"(rev)	"	52	.175		52+j66.		
	"(Ave)				51+j66		
					83.5/52.°3		
	Z_f	100		1.45	100-j1830		
"(rev)	Z_f	100		1.55	100-j1710		
	Z_f(Ave)				100-j1770		
					1775/86.°7		
						384/17.°2	.107/70°.2
3	Z_g	79	.267		79+j101		
"(rev)	Z_g	79	.271		79+j102		
	Z_g(Ave)				79+j101.5		
					128.5/52°.1		
	Z_f	180		2.25	180-j1180		
"(rev)	Z_f	180		2.30	180-j1150		
	Z_f(Ave)				180-j1165		
					1180/81°.2		
						389/14°.6	.107/68°.0

ω = 2680

No. of Sections	X	R	L	C	Z	Z_o'	θ'
1	Z_g	42.1	.1125		42.4+j301		
"(rev)	Z_g	42.2	.1113		42.2+j298		
	Z_g(Ave)				42.2+j300		
					303/ 82°.0		
	Z_f	21.6		.788	21.6-j475		
"(rev)	Z_f	20.4		.797	20.4-j469		
"	Z_f(Ave)				21.0-j472		
					474/87°.4		

ω = 2680

No. of Sections	X	R	L	C	Z	Z_o'	θ'
						$378\overline{/27^\circ.7}$	$.674\underline{/86^\circ.0}$
2	Z_g	836	.5215		836+j1395		
"(rev)	Z_g	842	.5100		842+j1365		
					839+j1380		
					$1615\underline{/58^\circ.7}$		
"	Z_f	36.5	.1093	1.0	36.5-j82		
"(rev)	Z_f	35.5	.1095	1.0	35.5-j81		
	Z_f(Ave)				36.0-j81.5		
					$89.2/66^\circ.1$		
						$379\overline{/3^\circ.7}$	$.678\underline{/85^\circ.6}$
3	Z_g	233.4		.577	233.4-j671		
"(rev)	Z_g	226.4		.555	226.4-j675		
	Z_g(Ave)				230-j673		
					$713/76^\circ.1$		
	Z_f	74.8	.0675		74.8+j180.5		
"(rev)	Z_f	76.8	.0673		76.8+j180.		
	Z_f(Ave)				75.8+j180.2		
					$195.5\underline{/67^\circ.2}$		
						$373\overline{/2^\circ.0}$	$.683\underline{/85^\circ.6}$
ω = 5000							
1	Z_g	568	.364		568+j1830		
"(rev)	Z_g	482	.320		482+j1610		
	Z_g(Ave)				525+j1720		
					$1800\underline{/73^\circ.0}$		
	Z_f	26.5	.0552	.5	26.5-j122		
"(rev)	Z_f		.0585	.5	24.1-j105		
	Z_f(Ave)				25.3-j113.5		
					$116.5/77^\circ.4$		
						$457\overline{/2^\circ.2}$	$1.325\underline{/87^\circ.4}$
2	Z_g	80.0		.905	80-j220		
"(rev)	Z_g	79.8		.936	79.8-j213		
"	Z_g(Ave)				79.9-j217		
					$231/69^\circ.8$		
	Z_f	318	.1640		318+j823		
"(rev)	Z_f	303.7	.1592		304+j800		
	Z_f				311+j811		
					$871\underline{/69^\circ.0}$		

ω = 5000

No. of Sections	X	R	L	C	Z	Z_o'	θ'
						$449/\overline{0^{\circ}.4}$	$1.345/\underline{86^{\circ}.9}$
3	Z_g	270	.1056		270+j530		
"(rev)	Z_g	263.	.0955		263+j480		
	Z_g(Ave)				266+j505		
					$571/\underline{62^{\circ}.2}$		
	Z_f	150		.532	150-j375		
"(rev)	Z_f	127.7		.573	128-j348		
	Z_f(Ave)				139-j361		
					$387/\overline{69^{\circ}.0}$		
						$470/\overline{3^{\circ}.4}$	$1.346/\underline{86^{\circ}.9}$

ω = 7500

No. of Sections	X	R	L	C	Z	Z_o'	θ'
1	Z_g	81.5		.151	81.5-j884		
"(rev)	Z_g	124		.137	124-j974		
	Z_g(Ave)				103-j929		
					$937/\overline{83^{\circ}.7}$		
	Z_f	340	.0965		340+j725		
"(rev)	Z_f	354	.1082		354+j813		
	Z_f(ave)				347+j769		
					$845/\underline{65^{\circ}.7}$		
						$890/\overline{9^{\circ}.0}$	$2.34/\underline{86^{\circ}.7}$
2	Z_g	2300		.103	2300-j1300		
"(rev)	Z_g	2600	.002		2600+j150		
	Z_g(Ave)				2450-j575		
					$2520/\overline{13^{\circ}.3}$		
	Z_f	285	.0678	.2	285-j158		
"(rev)	Z_f	329	.0902	.2	329+j10		
	Z_f(ave)				307-j74		
					$315/\overline{13^{\circ}.6}$		
						$890/\overline{13^{\circ}.4}$	$2.36/\underline{86^{\circ}.2}$
3	Z_g	1050	.0523		1050+j393		
"(rev)	Z_g	880	.0698		880+j525		
	Z_g(ave)				965+j459		
					$1070/\underline{25^{\circ}.4}$		
	Z_f	500		.172	500-j775		
"(rev)	Z_f	600		.219	600-j610		
	Z_f				550-j692		
					$886/\overline{51^{\circ}.5}$		
						$973/\overline{13^{\circ}.0}$	$2.38/\underline{86^{\circ}.1}$

ω = 10,000

No. of Sections	X	R	L	C	Z	Z_o'	θ'
1	Z_g	15.3		.237	15.3-j423		
"(rev)	Z_g	22.3		.222	22.3-j452		
	Z_g(ave)				18.8-j428		
					438/87°.5		
	Z_f	46.2		.181	46.2-j554		
"(rev)	Z_f	57.0		.169	57.0-j594		
	Z_f(ave)				51.6-j574		
					576/84°.8		
						502/86°.2	3.34/66°.5
2	Z_g	26.8		.213	26.8-j471		
"(rev)	Z_g	43.		.195	43.0-j515		
	Z_g(ave)				34.9-j494		
					495/86°.0		
	Z_f	40.6		.212	40.6-j473		
"(rev)	Z_f	57.0		.194	57.0-j517		
	Z_f(ave)				48.8-j495		
					498/84°.3		
						496/85°.2	3.10/67°.0
3	Z_g	32.7		.212	32.7-j473		
"(rev)	Z_g	47.5		.195	47.5-j515		
	Z_g(ave)				40.1-j494		
					496/85°.3		
	Z_f	32.7		.212	32.7-j473		
"(rev)	Z_f	47.5		.195	47.5-j515		
	Z_f(ave)				40.2-j494		
					496/85°.3		
						496/85°.3	

ω = 18600

No. of Sections	X	R	L	C	Z	Z_o'	θ'
1	Z_g	3.2		.318	3.2-j163		
"(rev)	Z_g	4.8		.302	4.8-j172		
	Z_g				4.0-j168		
					168/88°.6		
	Z_f	3.3		.315	3.3-j165		
	Z_f	5.0		.300	5.0-j173		
	Z_f				4.2-j169		
					169/88°.6		
						168/88°.6	

$\omega = 186.00$

No. of Sections	X	R	L	C	Z	Z_o'	θ'
2	Z_g	3.3		.316	3.3-j164		
"(rev)	Z_g	5.0		.300	5.0-j173		
	Z_g(ave)				4.2-j169 169/88°.6		
	Z_f	3.3		.317	3.3-j163		
"(rev)	Z_f	5.0		.300	5.0-j173		
	Z_f(Ave)				4.2-j168 168/88°.6		
						168/88°.6	
3	Z_g	3.3		.317	3.3-j163		
"(rev)	Z_g	5.0		.300	5.0-j173		
	Z_g(ave)				4.2-j168 168/ 88°.6		
						168/88°.6	

MEASURED AND COMPUTED VALUES OF Z_o', θ'

		Measured		Computed	
ω	sections	θ'	Z_o'	θ'	Z_o'
377	1	.107/72°.5	383/16°.7		
	2	.107/70°.2	384/17°.2		
	3	.107/68°.0	389/14°.6		
	Ave.	.107/70°.2	385/16°.2	.1064/68°.8	390/15°.7
2680	1	.674/86°.0	378/2°.7		
	2	.678/85°.6	379/3°.1		
	3	.683/85°.6	373/2°.0		
	Ave.	.678/85°.8	377/2°.8	.680/85°.8	377/2°.8
5000	1	1.325/87°.4	457/2°.2		
	2	1.345/86°.9	449/0°.4		
	3	1.346/86°.9	470/3°.4		
	Ave.	1.338/87°.1	459/2°.0	1.342/87°.2	453/2°.4
7500	1	2.34/ 86°.7	890/9°.0		
	2	2.36/ 86°.2	890/13°.4		
	3	2.38/ 86°.1	973/13°.0		
	Ave.	2.36/ 86°.3	918/11°.1	2.38/ 86°.1	929/11°.7
10000	1	3.34/ 66°.5	502/86°.2		
	2	3.10/ 67°.0	496/85°.2		
	3	-	496/85°.3		
	Ave.		498/85°.6	3.33/66°.1	488/85°.7
18,600	1		168/88°.6		
	2		168/88°.6		
	3		168/88°.6		
	Ave.		168/88°.6	4.25/46°.0	168/88°.2

R, L, G & C CALCULATED FROM AVERAGE MEASURED VALUE OF Z_0' AND θ'.

ω	Z' $Z_0'\ \theta'$	$Y'10^6$ $\theta'/Z_0'10^6$	$\frac{\sinh\theta'}{\theta'}$	$\frac{\tanh\theta'/2}{\theta'/2}$
377	$41.2\underline{/54^\circ.}$	$278\underline{/86^\circ.4}$	$.999\underline{/0^\circ.1}$	$1.00\overline{/0^\circ.1}$
2680	$256\underline{/83^\circ.0}$	$1800\underline{/88^\circ.6}$	$.926\underline{/0^\circ.6}$	$1.039\overline{/0^\circ.3}$
5000	$613\underline{/85^\circ.1}$	$2920\underline{/89^\circ.1}$	$.73\underline{/1^\circ.9}$	$1.18\overline{/1^\circ.0}$
7500	$2184\underline{/75^\circ.0}$	$2590\underline{/97^\circ.2}$	$.30\underline{/13^\circ.1}$	$2.1\overline{/9^\circ.}$
10000	$1676\overline{/19^\circ.7}$	$6650\underline{/152^\circ.7}$	$.528\underline{/108^\circ.5}$	$1.025\overline{/64^\circ.5}$

ω	$\frac{Z'\sinh\theta'}{\theta'}$	$\frac{10^6 Y'\tanh\frac{\theta'}{2}}{\theta'/2}$	R	L	G	C
377	$41.2\underline{/54^\circ.1}$ $24.1+j33.5$	$278\underline{/86^\circ.3}$ $18+j278$	24.	.0890	18.	.740
2680	$237\underline{/83^\circ.6}$ $26+j235$	$1870\underline{/88^\circ.3}$ $54+j1870$	26.	.0883	54	.700
5000	$448\underline{/87^\circ.0}$ $23.5+j448$	$3440\underline{/88^\circ.1}$ $112+j3440$	24.	.0893	112	.688
7500	$656\underline{/88^\circ.1}$ $21.8+j656$	$5440\underline{/88^\circ.2}$ $85+j2720$	22.	.0875	170.	.724
10000	$885\underline{/88^\circ.8}$ $17+j885$	$6820\underline{/88^\circ.2}$ $216+j6820$	17.	.0885	216.	.682