Thesis

Smith

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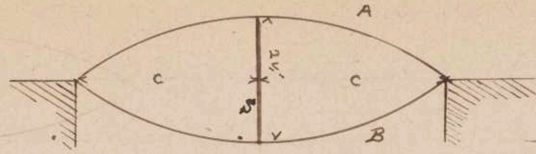
Upon a kind of Truss or
built beam.

My attention has been directed to a mode of constructing truss beams, of timber or iron, applicable to small spans; which is by no means new, but has some claims to notice, in regions where labor is scarcer than material, as in any newly settled country, or where time in construction is limited as in military works, repairing, and rebuilding; and for temporary structures.

Two beams are placed, the one above the other and fastened together at the ends while the central portions are kept apart by short struts.

The only gain, evidently is in the number and kind of the joints, and as in some cases these are matters of great importance, I endeavored to make an investigation of the truss, with the desire of comparing its strength with the more common forms.



We see at once that this differs from the ordinary truss beams, by having an element of internal stress introduced. The pieces A and B are evidently held apart by the reaction of the strut S and are each subjected to a certain amount of strain as if loaded with a weight; and this internal stress must modify all other stress in the truss, whether produced by the weight of the truss and permanent load, or by any load upon it.

In any wooden structure of course after the lapse of some time, the elasticity of the timber would become so much impaired as virtually to render that element of very little effect. In an iron truss, much longer time would be required to produce such a deterioration in the material, and in fact from Dr. Fairbairn's experiments on the effect of vibration and longcontinued sudden changes of load on beams and girders; it may be doubted,—under some circumstances,—if such loss of elasticity would take place.

In either case however it becomes necessary to know how this condition of internal stress affects the strength, for when the truss is new its elasticity is in its normal amount, and if it should materially weaken the sustaining power, as such weakening is the preceeding and not the following state, the truss might give way, or suffer material injury, before its computed load—if computed as for the common forms,—should be placed upon it. This consideration alone makes it desirable to know all about it.

Let us suppose that the two beams are of the same dimensions, and are kept asunder by a single

strut, hence the deflections of the two beams will be equal.
Let us call the length of ~~the truss~~ $2c$, and the length of the separating strut, or the distance apart at the middle $2v_1'$; and each beam is therefore, as if deflected an amount v_1' by a force and is subject to the same internal stress; we will call the amount of force, which would produce the deflection v_1'; P. This notation was adopted as being in accordance with that of Prof. W. J. M. Rankine, in his "Manual of Civil Engineering" by the aid of which this investiga- was first carried out; and which will be used here to check results.

Starting with the fundamental equation.

$$\frac{1}{r} = \frac{M}{EI} \qquad (1)$$

Where r = the radius of curvature
M = " bending moment
E = " modulus of direct elasticity
I " moment of inertia of the beam

Taking the value of $r = \frac{ds^3}{dx\,dy^2}$

and remembering that in all practical cases $ds = dx$ and $r = \frac{dx^2}{dy}$ and therefore (1) becomes

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \qquad (2)$$

Taking the origin at the middle and calling distances upward, or rather toward the deflecting force positive. In any beam supported at both ends and loaded in the middle $M = \frac{P(c-x)}{2}$ and (2) becomes

$$\frac{d^2y}{dx^2} = \frac{P}{2EI}(c-x) \qquad (3)$$

Integrating we have

$$\frac{dy}{dx} = \frac{P}{2EI}\left(cx - \frac{x^2}{2}\right) \qquad (4)$$

the constant of integration is 0, for when $x = 0$ $\frac{dy}{dx} = 0$ for there is no slope in the middle. Integrating (4) we have,

$$y = \frac{P}{2EI}\left(\frac{cx^2}{2} - \frac{x^3}{6}\right) \qquad (5)$$

and again the constant of integration is 0, for when $x = 0$, $y = 0$, and (5) may be put in this shape for $x = c$

$$y = v_1' = \frac{Pc^3}{6EI} \qquad (6)$$

We get the same result, more readily by taking a formula found on Page 273 of "A Manual of Civil Engineering" already mentioned.

$$v_1'' = \frac{n''' W c^3}{E n' b h^3}$$

which by putting for $n' b h^3$, I, and for W the deflecting force P and taking from ~~the~~ table on page 274 the value $n''' = 1/6$, becomes the same expression

$$v_1' = \frac{Pc^3}{6EI} \qquad (6)$$

But we wish to have this in terms of the greatest bending moment, which we will call M_0 and which we know to equal $\frac{Pc}{2}$ therefore separating (6) we have $v_1' = \frac{Pc}{2} \cdot \frac{c^2}{3EI} = \frac{M_0 c^2}{3EI}$ and therefore

$$M_0 = \frac{v_1' 3EI}{c^2} \qquad (7)$$

This equation expresses the greatest bending moment on the beam, — at the centre, — produced by a deflecting force, in terms of the deflection, and this bending moment acts equally but in opposite directions upon the upper and lower beams of our truss.

Hence if a central load W, be placed upon the truss, supposing half of it to be borne by each beam in the truss, then the greatest bending moment, due to this load alone, is $M = \frac{Wc}{4}$, but with this we must combine the moment due the deflection, and calling M_u the greatest bending moment on the upper beam, and M_2 that on the lower beam.

Because the load and deflection act in opposite ways on the upper beam their combined effect will be the difference of their respective moments, that is to say

$$M_u = \frac{Wc}{4} - \frac{V_1' 3EI}{c^2} \qquad (8)$$

And on the lower beam because they act in the same way $M_2 = \frac{Wc}{4} + \frac{V_1' 3EI}{c^2}$ (9)

To obtain the forces which produce these bending moments, we divide the moments by their respective lever arms, and obtain $\frac{M_u}{c}$, & $\frac{M_2}{c}$, for the forces. Now considering one half of the truss as nearly a triangle, and c the half span, as the half length of the beam, we have approximately, by the triangle of forces,

$$\left.\begin{array}{l} V_1' : c = \frac{M_u}{c} : H_u \\ V_1' : c = \frac{M_2}{c} : H_2 \end{array}\right\}$$ and therefore

$$\left\{\begin{array}{l} H_u = \frac{M_u}{V_1'} \\ H_2 = \frac{M_2}{V_1'} \end{array}\right\} \qquad (10)$$

H_u being the longitudinal stress upon the upper beam, and H_2 that upon the lower.

Putting (8) and (9) into (10) we obtain

$$\left.\begin{array}{l} H_u = \frac{Wc}{4V_1'} - \frac{3EI}{c^2} \\ H_2 = \frac{Wc}{4V_1'} + \frac{3EI}{c^2} \end{array}\right\} \qquad (11)$$

The difference between H_u and H_2 or $\frac{6EI}{c^2}$, is obviously the longitudinal shear at the ends of the truss, and which must be resisted at the joints, by proper fastenings.

If we had supposed a load $w = \frac{W}{2c}$ to be uniformly distributed along the beam, w being the load upon a unit of length, we should have for the bending moment due to that alone $M = \frac{Wc}{8}$, and combining as we did before, we arrive at this result.

$$\left.\begin{aligned} H_u &= \frac{Wc}{8 V_1'} - \frac{3EI}{c^2} \\ H_2 &= \frac{Wc}{8 V_1'} + \frac{3EI}{c^2} \end{aligned}\right\} \quad (12)$$

As before the difference between H_u and H_2 is the shearing force at the extremities, and is the same $\frac{6EI}{c^2}$.

We see in (12) that if $\frac{Wc}{8}$ be greater than $\frac{3EI}{c^2}$, H_u is positive and a thrust. H_2 is in any case a tension and is greater in amount than H_u.

This method of analysis may be applied to any sizes of beams relatively to each other. In cases where there are more than one strut between the beams some difficulty would be found in getting the forces producing flexure.

We might seek to provide for the unequal stress upon the beams, by making the lower one of larger dimensions than the upper, but although this is often done in practice, yet any mathematical solution, although doubtless possible, yet would be very complex. For the deflections of beams of equal length are as the cubes of the depths inversely, and the areas varying only as the depths, the areas must be altered in a ratio which itself must be changed with every change in the areas. As the solution of this question appears to have little practical value, I have not attempted it.

The best arrangement of the two beams seems to be when they are equally deflected; in any other case the stress either longitudinal or transverse, will be increased in one beam, while it will be diminished in the other. The beam which is least deflected, while the transverse stress is diminished the longitudinal is augmented, while in the other beam the bending moment is increased.

The deflections will not be equal unless the beams are equal,—if they are of similar section,—or unless the solution of the before mentioned difficult problem has been obtained either by investigation or by accident. With any variation in the depth or breadth of one beam the deflection of that one, —and as the distance between the beams is constant,—that of the other must vary correspondingly; these two deflections will adjust themselves to each other, and to the dimensions of the beams.

In designing a truss of this form, I should aim to have the deflections equally great. In any existing structure I should pursue the same analysis to obtain the stresses.

In any of the ordinary forms of truss uniformly loaded, we have for the bending moment $\frac{Wc}{4}$ and for the longitudinal stress $\frac{Wc}{4h}$ where h is the height of the truss; by equation (12)

$$H_2 = \frac{Wc}{8v_1'} + \frac{3EI}{c^2}$$ now $2v_1' = h$ therefore

$$H_2 = \frac{Wc}{4h} + \frac{3EI}{c^2}$$ and we see that the increase of horizontal stress upon the lower timber or beam, is $\frac{3EI}{c^2}$. For ordinary spans this is quite a large quantity and would be sufficient to condemn the use of such a truss in all ordinary cases.

As an example of the analytical work preceding, let us take the case of a foot bridge over the Merrimack Canal in Lowell Massachusetts, as a very good case of practical work. This bridge was erected in 1862, and is in apparently as good condition now as when first put up. But in order to arrive at satisfactory results it was necessary to obtain the original deflections.

Taking a plan of this bridge, in the possession of the Proprietors of Locks and Canals on Merrimack River, on a scale of one inch to a foot, and measuring thereon the deflections as closely as practicable, computing for each beam the deflecting force, and knowing that this must be the same for each beam, (which, as a matter of course, the calculations did not exactly agree upon,) taking the mean of these for the deflecting force, and calculating therefrom the deflections, both at the centre, and two other points where there are struts, I found that the deflections were wholly due to the centre strut, my results agreeing as closely with the measurements, —which were made first,— as could be expected.

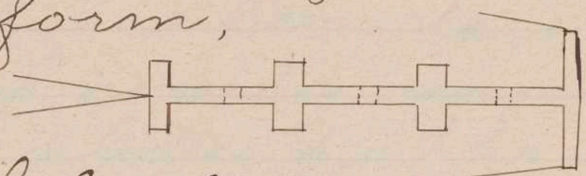
The dimensions are as follows,—all being taken from the before mentioned plan.

Span	48 ft.	
Width Upper Timber	0.75 ft.	
" Lower "	0.75 "	
Depth Upper "	9.625 inches	
" Lower "	9.875 "	
Central Deflection	Measured.	Computed.
Upper Timber	9.25 inches	9.35
Lower Timber	8.75 "	8.66
Deflection 6 ft 11 in from centre		
Upper Timber	8.125 "	8.30
Lower Timber	7.625 "	7.68
Deflection 13 ft 10 in. from centre		
Upper Timber	5.00 "	5.59
Lower Timber	4.75 "	5.17
Breadth of Struts at above points		9.00 inches
Width of all but central struts		7.75 "
" " central strut		9.00 "

The agreement, between the computed and measured deflections, ~~are~~ very close, toward the ends there appears to be a tensile force drawing the timbers together, so that the word strut is misapplied to the end pair of post. All these posts have a one inch iron bolt passing through them.

from top to bottom, and I think, in the case of the outer pair, they have pulled the beams in.

The joint at the ends is made by placing an iron plate, of this form,



between the beams: bolts are passed through the beams, in three places indicated by the dotted lines.

By computation I find that;– the thrust upon the upper timber with a load, being assumed to be less than the full on the lower,— a load of about 12000 lbs will produce a stress of 1000 lbs per square inch upon the lower timber, H_2 being about 88875 lbs. Now the horizontal stress in a truss, of any ordinary form, having the same span and depth, from the same load is about 40000 lbs less than this.

and for the shear at the joint we have about 80000 lbs.

I have now stated a method of analyzing the stresses in a truss of this kind, which is I think original; at this day, and more especially in my ignorance of what has been done, the claim of novelty would be simply absurd, I have however long searched for any discussion upon the subject, and have not as yet found anything thereunto pertaining.

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