Thesis of E. K. Turner.
1870

I have the honor to present to you the following paper. with the accompanying drawings

Respectfully
E. K. Turner

Drawings

Three plates of drawings accompany this paper.

No. 1 shows graphically the strains in one girder.

No. 2 shows the arrangement of plates in the flanges.

No. 3 has the elevation, plan and section of the bridge, with enlarged sections of details

Wrought Iron Girder Bridge

Description

This bridge has three I shaped girders, each one hundred and eight feet long, one hundred feet between supports. The girders on each side are designed to sustain one fourth of the entire load, the middle one to sustain one half the load. Above the girders are placed timbers forming the floor of the bridge and upon these the rails.

Each girder is composed of iron plates of various forms fastened together with rivets. The principal parts are the web, the lower flange, the upper flange, fish plates and stiffening plates.

The Web is composed of plates three eighths inch thick, eight feet high and six feet long, these dimensions are the same throughout the entire length of the girder

The lower flange is composed at the centre, of two plates two feet broad and three eighths inch thick. the upper layer continues the same to the end. the lower layer. has three plates each side of the centre. each twelve feet long. the first three eighths inch thick, the second one fourth of an inch thick. and the last one eighth of an inch thick

The upper flange is composed of plates two feet wide. in three layers. the plates at the centre are eighteen feet long. the others are twelve feet long. all the plates in the lower layer are three eighths inch thick. and continue to the end. The middle layer has a thickness of three eights inch and extends thirty six feet from the centre. The upper layer extends thirty six feet from the centre, with a thickness of one fourth inch, followed by a plate one eighth inch thick and twelve feet long.

The plates in both flanges are arranged to break joints

The flanges are joined to the web by L shaped angle irons, each arm is four inches broad, and one half inch thick. The rivets which fasten these to the web and flanges are one inch in diameter and four inches apart, from centre to centre.

The plates of the web are joined together with fish plates, these being made T shaped also act as stiffening plates to resist the tendency of the web to bend or buckle under its load. The tendency to bend is also resisted by angle irons placed midway between the fish plates. The flange of the T irons is eight inches broad and three eighths inch thick, the rib is six inches broad, with the same thickness. The angle iron is three eighths inch thick, its arms are four and six inches broad, the six inch arm being perpendicular to the web.

The rivets in the T iron are four inches apart, those in the angle iron eight inches. The flange of the T iron is bent over the longitudinal angle iron and riveted to the flange of the girder, forming a bracket, giving additional strength to the connection between the web and flanges. The rib of the iron is cut off at the first bend in its flange and is connected with the flange of the girder by a plate riveted to both.

The stiffening angle irons extend from the upper to the lower flange and are riveted to both and to the web.

Both kinds of stiffening plates act as struts or pillars to sustain the weight of the load and keep apart the flanges.

Each girder has at one end a piece of iron five feet long, two feet broad and two inches thick, bolted to the lower flange. This plate rests upon five steel rollers, each five inches in diameter, the rollers are connected by an iron frame, forming a sort of truck, the object of this frame is to prevent either of the rollers from turning, and to preserve the distance between them. The rollers rest upon another iron plate of the same dimensions as the upper one. This plate in its turn rests upon a block of hard wood, a little larger than the iron plates and six inches thick.

The girders are connected by diagonal braces at intervals of six feet. Each of these braces is formed of two flat plates, six inches broad and one half inch thick, the plates are connected by a small angle iron riveted to both, they form a brace fitted to resist extension or compression. The manner of joining to the flanges and T irons is shown in section

The above description applies to the side girders, the middle one has its flanges of nearly double the dimensions already given. the web and all other parts remain the same as in the other girders. In addition it has a rib composed of two angle irons, each having four inches on the flange and six inches above it. riveted together and to the flange. this rib prevents movement of the timbers. ~~and~~ also acts as part of the upper flange.

Upon the girders is placed the floor of the bridge. it is composed of timbers eight inches by ten and ten inches apart, length twenty four feet. These may be in one piece with a notch at the centre to receive the rib. or may be in two parts joined at the centre. These timbers are much stronger than is necessary to sustain the track with its load. also much stronger than are usually used.

they are intended to sustain a locomotive or cars in case they are thrown from the track. The timbers being near enough to prevent the wheels from dropping between them

On the ends of the transverse floor timbers are placed longitudinal timbers: ten inches square. bolted at intervals to the floor timbers

The rails are spiked to the floor timbers.

Calculations

The proportions of the depth to the length $\frac{1}{12.5}$ are nearly those given by Prof. Rankine. C.E. Page 276. deduced from the relation between the deflection and span. According to Humber (Strains in Girders) the central depth of a straight independent girder may be from $\frac{1}{10}$ to $\frac{1}{16}$ of the span. The greatest economy of material is obtained at $\frac{1}{12}$.

In the calculations for the strains in the girders, it is considered that the entire shearing force is borne by the vertical web, while the bending moment is taken by the flanges. (considering the angle irons which connects the web with the flanges as part of the flanges) Although this is not strictly true, still it errs on the side of safety and therefore can lead to no dangerous results.

"The greatest shearing force at a given cross section occurs when the longer of the two segments into which the beam is divided is loaded with the travelling load as well as the permanent load, and the shorter with the permanent load only."

"The greatest bending moment at each section occurs when the travelling load extends over the whole length of the beam" Rankine C.E.P. 248

I have calculated the greatest forces acting at intervals of ten feet. in accordance with the above principles

The loads taken are, for permanent load 500 lbs per foot, for travelling load 1000 lbs per foot, the latter being the weight due to a train of locomotives

The shearing forces are obtained by using the formula

$$F' = w\left(\frac{l}{2} - x\right) + \frac{w'(l-x)^2}{2l}$$

F' is the greatest shearing force
x is the distance of the section from the origin
l = length of the bridge
w = permanent load and w' = travelling load

Supplying numerical data in the formula we have the following

Sec	P. Load		T. Load		
0	500 × 50	+	5 × 10000 =	25000 +	50000
10	500 × 40	+	5 × 8100 =	20000 +	40500
20	500 × 30	+	5 × 6400 =	15000 +	32000
30	500 × 20	+	5 × 4900 =	10000 +	24500
40	500 × 10	+	5 × 3600 =	5000 +	18000
50	500 × 0	+	5 × 2500 =	0 +	12500
60			5 × 1600 =		8000
70			5 × 900 =		4500
80			5 × 400 =		2000
90			5 × 100 =		500
100			5 × 0 =		0

To resist the above forces we have the web. thickness 3/8" depth 96" this gives an area of 36". Taking the lowest value for resistance to shearing, of wrought iron, 36000 lbs per inch, and the highest factor of safety 5. 3 for permanent load and 6 for travelling load

$36000 \div 5 = 7500$ $\quad 7500 \times 36 = 270000$

The greatest shearing force acting at any section was found to be 75000. therefore the web is strong enough to resist all forces which will be brought upon it

Since the girders are symmetrical at each side of the centre, the above calculations as well as those which follow apply to either end.

The bending moments due to the travelling load are as follows, supplying numerical data to the formula

$$M = \frac{wx'(l-x')}{2}$$

Sec		
10	500 X 10 X 90 =	450000
20	500 X 20 X 80 =	800000
30	500 X 30 X 70 =	1050000
40	500 X 40 X 60 =	1200000
50	500 X 50 X 50 =	1250000

The formula for greatest bending moment $M' = \frac{wl^2}{8} \quad \frac{1000 \times 10000}{8} = 1250000.$

The bending moment due to the permanent load, 500 lbs per foot, is obtained by using the same formula

$$M = \frac{w x (l - x)}{2}$$

The following are the numerical results

Sec		
10	250 X 10 X 90 =	225000
20	250 X 20 X 80 =	400000
30	250 X 30 X 70 =	525000
40	250 X 40 X 60 =	600000
50	250 X 50 X 50 =	625000

The bending moments act as couples producing extension in the lower flange and compression in the upper one. These forces are equal in ~~a~~amount and are found by dividing the moment of the couple by its arm, that is, by dividing the bending moment by the depth of the girder

The factor of safety mentioned above is the ratio in which the breaking load exceeds the safe working load. In these calculations I have used as factors three for permanent load and six for travelling load. The factor of safety six is used because the strains in a girder caused by a load suddenly applied, as a swiftly moving train moving on to the bridge, are twice as great as those produced by the same weight in a permanent load or one applied slowly.

For both flanges the forces are as follows.

Sec	T. Load	P. Load
50	156250	78125
40	150000	75000
30	131250	65625
20	100000	50000
10	56250	28125

To find the required sectional area it is necessary to divide each of these by the values for strength of material, divided by the propper factors of safety.

For tension I have taken the value 54000 lbs per inch. this divided by 6 gives 9000 for resistance to Travelling load. divided by 3 gives 18000 for resistance to permanent load.

The following table gives the required sectional areas.

Sec.	T. Load	P. Load	Total
50	17.36	4.34	21.70
40	16.67	4.16	20.83
30	14.58	3.64	18.22
20	11.11	2.78	13.89
10	6.25	1.56	7.81

These areas are required after making deduction for rivet holes. The greatest number of rivets found at any section is four. two in fish plate and two in angle iron. Each rivet being one inch in diameter, the area to be deducted for these is $\frac{1}{2}'' \times 2 = 1''$

$\frac{3}{4} \times 2 = 1\frac{1}{2}''$. $2\frac{1}{2}''$ total

The angle irons have each a sectional area of 4" two give 8" $8'' - 2\frac{1}{2}'' = 5\frac{1}{2}''$ this can be subtracted from 21.70 leaving 16.20 as the required sectional area of the lower flange plates at centre.

The following table gives the required sectional areas and those used.

Sec	Req areas	Areas Used
50	16.20	18.00
40	15.33	18.00
30	12.72	15.00
20	8.39	12.00
10	2.31	9.00

The Upper Flange or Compression Member

The forces acting on this member tend to crush the material of which it is composed From the nature of the material the limit of elasticity may be approached nearer with safety in a compressed member than in an extended one

When iron is subjected to any strain either of tension or compression the material is lengthened or shortened, when released from the strain it resumes its original length, unless permanent set has occured. In case permanent set has occured when the piece is extended the diameter is reduced. and the ability to sustain a repetition of the strain is lessened.

When the piece is shortened the conditions are reversed. the diameter is increased and the ability to sustain the same force repeatedly applied. is increased.

In a long beam secured only at the ends. the compressing force acts upon its member as upon a long strut and the member gives way by bending. but in this girder has its compressed member firmly connected with its own tension member. also with that of the next girder, the tension members being in stable equilibrium will counteract the tendency of the compression member to bend. Therefore the only force to be taken into account is the crushing force.

The sectional areas of the upper flange will not have to be increased to allow for rivet holes. since if the rivets are properly inserted they will resist crushing as well as the plates themselves

Using 36000 lbs per inch resistance to crushing The following table gives the required sectional areas.

Sec.	T. Load	P. Load	Total
50	26.04	6.51	32.55
40	25.00	6.25	31.25
30	21.21	5.47	26.68
20	16.67	4.17	20.84
10	9.37	2.34	12.74

The areas after subtracting 8" area of angle irons, and the areas used are as follows.

Sec	Req. Areas	Areas Used
50	24.55	27.00
40	23.25	24.00
30	18.68	21.00
20	12.84	15.00
10	4.71	9.00

All results already obtained are for the side girders. Those for the middle girder will be nearly twice as great, since it must bear a travelling load twice as great, and a permanent load of 700 lbs per foot.

The following table gives the bending moments, forces of extension and compression, and the areas necessary to resist these forces

Sec	B.M.	Force	Area Tension	Area Comp
50	875000	109412	6.08	9.12
40	840000	105000	5.83	8.75
30	735000	91875	5.10	7.66
20	560000	70000	3.89	5.83
10	315000	39375	2.18	3.28

The required areas, after deducting the effective areas of the angle irons and rib on the upper flange. are as follows

Tension Member.

Sec	Req Area	Area Used
50	35.30	36.00
40	33.67	36.00
30	28.76	33.00
20	20.61	24.00
10	9.18	15.00

Compression Member.

Sec	Req Area	Area Used
50	43.20	45.00
40	40.70	42.00
30	32.08	33.00
20	21.17	24.00
10	4.02	9.00

Area of angle irons 8"
Area of rib 10"

T irons and stiffening plates considered as struts

These irons and plates have sectional areas of 10."5 and 7."5 respectively, and are capable of sustaining 378000 lbs and 253800 lbs, much greater than the heaviest weight which will be brought upon them, viz the driving wheel of a locomotive, weight 10000 lbs.

Weight of Girder

Upper flange		Lower flange	
108 × 2 × 3/96 =	6.75	108 × 2 × 3/96 =	6.75
48 × 2 × 3/96 =	3.	24 × 2 × 3/96 =	1.50
48 × 2 × 2/96 =	2.	24 × 2 × 2/96 =	1.
36 × 2 × 1/96 =	.75	24 × 2 × 1/96 =	.50
	12.50		9.75

Web 108 × 8 × 3/96 =	27.00	feet
Upper flange	12.50	"
Lower flange	9.75	"
S plate	7.50	"
T irons	9.91	"
Ang. irons	12.00	"
End irons	1.00	"
	80.26 × 480 = 38524.8 lbs.	

The weights given include the weight of rivets since no deduction has been made for rivet holes.

Fish plates, rivet heads &c will make the weight 21 tons. One half of this rests on each end of the girder. The travelling load is 54 tons, 27 tons on each end of the girder. Making total load on each end, 37.5 tons.

At each end of the girder there is a bearing surface four feet in length. The part above this may be considered as a strut, which must be able to sustain the load on one end of the girder. the sectional area is 45.5 inches. this with a value 6000 lbs per inch resistance to crushing, will sustain 136.5 tons

Force of Wind on the Bridge

For each foot in length nine and one half feet of surface, exposed to the wind. Taking fifty pounds per square foot as the force of the wind we have four hundred and sixty pounds pressure per foot in length. The bending moment due to this pressure is 670680

If we consider the flanges taken together as a rectangular beam, its moment of rupture is found by the formula

$$M^o = n f b h^2$$

$$= \frac{9000 \times 3 \times 576}{6 \times 2} = 1296000$$

The greatest shearing force is found by using the formula

$$F = w' \times \frac{l}{2}$$

$$460 \times 50 = 23000$$

$$23000 \div 6000 = 3.83 \text{ inches}$$

The sectional area is much greater than is necessary to sustain the shearing force.

Since one girder is able to resist the force of the wind, it is unnecessary to calculate the strength of the three connected girders

Rivets

The number of rivets at the joints is regulated by the rule that the joint sectional area of the rivets shall be equal to the area of the plate left after punching the rivet holes.

The number of rivets and the length before clenching, measured from the head is as follows

Length	number
3 inches	864
3½ "	7850
4 "	2580
4½ "	2316
Total	13610

In the following table of weights of materials no deduction of weight is made for rivet holes, which are very numerous and reduce the weight of the bridge considerably.

Materials Required

Iron

Web	81.	feet	38880.	lbs
Upper flange side girders	25.	"	12000.	"
" " middle girder	19.06	"	9148.8	"
Lower flange side girders	19.50	"	9360.	"
" " middle girder	19.50	"	9360.	"
S. plates	22.50	"	10800.	"
T Irons	29.73	"	14270.4	"
Angle irons	36.00	"	17280.	"
End plates	3.00	"	1440.	"
Fish plates	25.02	"	12009.6	"
Rivets	145.56	"	69868.8	"
Diagonal braces	14.40	"	6912.0	"
Total	440.27	"	211329.6	"

Wood

Floor timbers	960	feet
Longitudinal timbers	150	"
Blocks under end of girder	30	"
Total	1140.	"

Iron bolts in timbers (not included above) 760 lbs.