

Pricing Strategies for Continuous Replenishment Perishable Goods

By

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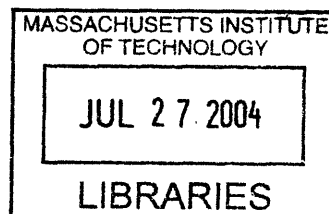
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ABSTRACT

This thesis investigates the application of Dynamic Pricing strategies at a manufacturer of continuous replenishment perishable goods. I begin with a discussion of Dynamic Pricing models, and select a mixed integer programming formulation as most applicable to the available systems and data of the target company. Cost formulations are built through a detailed analysis of current cost allocations within the company and actual costs when available. Revenue and price elasticity models are built from existing formulations. The continuous functions are then discretized through piece-wise approximations and input into a mixed integer program using production and pricing as the decision variables. The results were not entirely conclusive as sensitivity around the base values, particularly the price elasticity value, can create very different price path solutions. Greater stability is achieved through tightening the price ranges, but the suggested policy of always charging the maximum allowable price is not practical within the company's existing policies. For actual implementation, a much more thorough understanding of the price elasticity mechanism would be required.

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CHAPTER 1:

INTRODUCTION

Pricing decisions are one of the most critical and difficult decisions that a manufacturer must make. The framework that a manufacturer applies in making those decisions is equally critical. Will prices be changed daily, weekly, or annually? Will prices be set to maximize sales volume so as to dominate the market? Will prices be set to maximize profitability? Will changes in prices be used to drive promotions? Dynamic Pricing is an area of research that seeks to address many of these questions or lay the framework through which they can be answered. Dynamic Pricing studies how prices can be changed to actively adapt to the market, the parameters of adaptation largely separate lines of research.

My objective with this thesis is to examine the current pricing strategies used at a large manufacturer, see how Dynamic Pricing might improve the pricing strategies, and to develop insights into how they might go about developing and implementing Dynamic Pricing strategies.

1.1 MOTIVATION

This thesis was initially proposed when a large Fortune 500TM manufacturer of high sales volume perishable consumables, Company X, questioned the logic of current pricing strategies that did not consider supply chain capacities, costs, and the multiple burdens of

fluctuations in production schedules. Company X has highly seasonal demand patterns that place heavy strain on production capacities during peaks and allow for inefficient underutilization of assets during off peak seasons. The problem is extremely common in industries with seasonal demand patterns and can often be exaggerated or created within the supply chain.

In common supply chain parlance, “The Bullwhip Effect” refers to the effect by which small perturbations in demand signals can be exaggerated through levels of the supply chain [20]. The effect can be minimized by information sharing through all levels of the supply chain. In the case of Company X, the supply chain is completely integrated all the way to the customer shelf. Another means to minimize the effect is to minimize the initial perturbations that can be exaggerated through the supply chain.

Within Company X, the demand instability is further compounded by promotional activity that concentrates low price promotions around times of heightened demand. This thesis was proposed to investigate whether more prudent pricing decisions could allow for greater total profits by steadying volumes in relation to production costs and capacities. Traditionally, two competing incentive structures have come to a head where this thesis intends to build a bridge. Marketing and sales wish to continually drive up total sales volume. Supply chain managers wish to operate as efficiently as possible. Stability allows greater efficiency. Variability in sales drives the need for excess capacity and inefficiencies in the supply chain.

Pricing is not the only lever to smooth the production variability. Inventory is a standard buffer that can be prudently used to smooth out demand variability. Company X often builds inventory volumes for several weeks before peak demand periods. Building

volume before demand allows consistent service during seasonal demand peaks that exceed total production capacity. Consequently, build-up periods can be identified by high inventories and resultantly high carrying costs. Other methods of temporarily adjusting to demand spikes include interplant shipments of goods and incurring of overtime labor. The overtime consideration is tricky to formulate as labor allocation is highly dependent on the labor policies at the plants and is not simply correlated with total production.

On top of the lower per unit costs during off-peak production periods, a steadier demand signal allows production and distribution to both operate more efficiently. With perfectly predictable demand, production schedules can be consistent and consistently optimized. Distribution and production can be tailored to a steady demand allowing all excess capacity in manufacturing, distribution, and warehousing used as a buffer against demand spikes to be eliminated. Currently Company X must carry extra capacity during the majority of the year when demand is at average levels.

Many researchers have come to realize that capacity and inventory problems cannot be separated from pricing and promotional activities [6]. Company X currently operates with a pricing strategy is sufficiently poor for supply chain operations that management within supply chain intuitively understood the possible benefits of a better price path. Dynamic Pricing strategies have already yielded huge dividends in other industries, including revenue increases of as much as five percent in the airline industry [4]. This thesis was proposed as the first step in a possible Dynamic Pricing initiative. I investigate how Dynamic Pricing may be implemented, what the potential rewards are, and what the current limitations towards implementation are.

1.2 LITERATURE REVIEW

In the sum of all works on Dynamic Pricing, models that consider combined inventory and pricing decisions are only a small portion. Many of the papers that are not directly relevant to this thesis may prove useful in formulating future extensions or further work.

As a matter of definitions, I use the terms flexible pricing and Dynamic Pricing interchangeably. I have divided the literature review into two sections; Section 1.2.1 covers general Dynamic Pricing theory. Section 1.2.2 concentrates on models that are more directly applicable to Company X and the motivation of this thesis.

1.2.1 Dynamic Pricing

Much of the initial thrust of Dynamic Pricing literature concentrates on finite sales horizons, stochastic demand patterns effected by pricing decisions, and fixed initial inventories without replenishment. Such problems are often categorized under the general field of Revenue Management. This thesis does not borrow heavily from this line of Dynamic Pricing literature because the problem as framed allows for continuous replenishment of goods and a presumed deterministic correlation between price and demand. The bulk of the Dynamic Pricing literature treats supply processes as fixed and exogenous or determined statically. Conversely, most inventory planning research assumes that demand processes are fixed and exogenous [13]. The papers bridging the two are discussed in the next section.

Revenue Management has found broad application in the fashionable apparel goods industries and airline ticket sales. The study of airline ticket allocation and pricing has largely driven the field of Revenue Management. Initially, Revenue Management focused on the problem of allocating tickets within predetermined pricing categories, see Belobaba [4]. Looking at fashionable and durable goods sales models, Lazear [17] first proposed a multi-period pricing model with two periods and one opportunity to alter pricing but a single fixed initial inventory. The model makes several simplifying assumptions that limit its practical applicability, including an assumption that customers have a continuous distribution of reservation prices that is identical for all customers. Gallego and van Ryzin [14] developed a continuous time model using intensity control theory. They use a deterministic formulation to provide an upper bound and then develop a set of heuristics generating solutions that prove to be asymptotically optimal. Gallego and van Ryzin allow the customer arrival rate to vary as a function of price. Bitran and Mondschein [7] contribute a more detailed model of customer behavior by assigning a reservation price distribution to customers and fixing the customer arrival rate to exogenous factors such as advertising. Bitran, Caldentey and Mondschein [6] examine pricing strategies across multiple stores and the possibility of inventory reallocations. Awad, Bitran and Mondschein [2] look at perishable products that fall into a family of substitutable products. Extensions into multiple product models have also been made by Gallego and van Ryzin [15]. See Bitran and Caldentey [6] for a more thorough review of Revenue Management work.

The products of Company X are high volume and in heavy competition for shelf space. In this thesis, I do not consider the shelf inventory levels as a factor in demand. No

consumer choice model is constructed and only price is considered. A more thorough analysis might attempt to study the purchasing behavior caused by the constantly varying inventory levels. A detailed analysis would require availability of point of sale data to calculate actual inventory levels at any given time. Any practical application would also have to consider substitution effects. Smith and Achabel [21] have developed a single product model that considers the effects of diminishing inventory and reduced product assortment on consumer behavior. The larger focus of their research is on pricing strategies for clearance items. Of note, Smith and Achabel, like many before them, use a multiplicative exponential price sensitivity of the form $e^{-\gamma p}$. This form yields an elasticity that is proportional to price but constant across a set increment, where a reduction from \$1.00 to \$0.90 has the same effect on demand as a reduction from \$100 to \$99.90. In the formulation, p is the price and γ is the elasticity factor. Company X uses the form $p(t)^{-\eta}$. This allows the same change in price for any proportional change in prices, and allows accurate aggregation across pricing categories. In that, a 10% lift in prices will yield the same proportional lift for all price levels. More detail on the exact application within Company X is provided with the model formulation. Such a pricing model has some precedent in the work of Bass [3].

The study of Dynamic Pricing frequently crosses paths with price differentiation. Price differentiation is the practice of charging different prices to different buying segments, in other words, “differentiating” the market. Dynamic price differentiation has been put into practice by many online retailers, often resulting in customer backlash. Both Dell and Amazon.com have attempted to actively include market segmentation in their Dynamic Pricing algorithms [12]. These retailers attempt to segment their markets

by generating comprehensive user profiles or attempting to guide different segments to different areas of the website. Intertemporal price differentiation studies how markets can be differentiated by varying prices through time and closely ties to Revenue Management [6]. Stokey [22] laid much of the groundwork by attempting to determine whether observable intertemporal price declines were driven by an attempt to differentiate the market or a result of competition and production cost declines, she concludes that the latter is the more convincing cause. Company X offers a consistent product mix on store shelves only, so any retail price differentiation is ruled out, but it is still an open issue when dealing with wholesale prices to the retailers.

1.2.2 Pricing With Production and Inventory Decisions

This thesis aims to look at the efficacy of optimum combined production and pricing decisions. Whitin [27] first proposed formulations correlating inventory control with pricing decisions. He was motivated by the simple yet previously unwritten assertion that cost minimizing inventory control decisions and profit maximizing pricing decisions did not yield an optimum solution when made independently. His model simply combines the optimum order quantity formulation:

$$Q = \sqrt{\frac{2DS}{IC}}$$

Q = the optimum quantity; D = the annual demand. S = the ordering cost;

I = holding cost as a percent of production cost C .

With simple linear function of demand relative to price:

$$D = ap + b$$

a and b are constants; p = price

The model is strictly theoretical and primarily useful for laying the groundwork for further work. The formulation makes no consideration for time. Whitin also formulates a model to account for a probability distribution of expected sales with a lost sales cost and a liquidation value for remaining inventory. Again, time is not accounted for so a single decision must be made for both price and initial inventory levels by using expected sales at each price. The results are single period instantaneous optimums, similar to the classic Newsboy problem. For a summary of further extensions of Whitin's basic framework see Chan [8].

Kunreuther and Richard [16] assume a fixed demand that is varied with price, but the decision point is the demand quantity, through which price is then correlated. They examine the problem with a deterministic fixed and steady demand pattern, linear production costs, linear inventory holding costs, and a fixed cost per order. Their results exclude any intertemporal effects of price changes on demand, and generally serve more to provide a format to look at how the balance of cost factors might influence inventory and pricing decisions. When decisions are not combined, they assume that marketing will first make pricing decisions. In which case, pricing decisions that result in frequent orders or that do not consider particularly high holding or ordering costs will be furthest from optimal. Kunreuther and Richard try to expand the model to manufacturing environments in which setup costs replace ordering costs, but the extension is somewhat false. At Company X, as in many perishable goods manufacturing companies, the number and frequency of setups is relatively stable, often driven by single weekly batches. Setup

costs, being primarily labor, are therefore wrapped up into general fixed costs or linear variable costs and independent of the production volume decisions. In my formulation, any product with active demand will be produced during every period of production, so setup costs do not vary. Rajan et al [19] look at the possibility of continuous price changes over the inventory cycle with the prospects of perishability or decreasing value affecting the price path.

Thomas [25] first proposed formulating the problem as a mixed integer program and many of the concepts carry forward to this research. Thomas does include a set-up charge per period of production with a simple binary decision variable. The formulation maximizes profits over a fixed horizon of discrete time buckets with a deterministic demand and price elasticity formulation with discounted future profits. Thomas [26] later expands the model to account for random demand. He shows that an (s,S,p) heuristic policy is optimal under almost all conditions, meaning that for some optimum price p there exists an inventory level s under which an order should be placed to return inventory to the level S . This results in a combined peak inventory, pricing, and safety stock result.

More recently, Chen and Simchi-Levi [10,11] have extended the work of Thomas to better qualify the optimality conditions of the (s,S,p) policy. They conclude that with an additive stochastic demand function, the (s,S,p) policy is always optimal, but with multiplicative or other general forms of the demand function, the (s,S,p) policy is not always optimal. Where $D_t(p_t)$ is a deterministic formulation of demand as a function of price, an additive demand function takes the form $D_t(p_t)+\beta$. β is a random variable with $E\{\beta\}=0$. The multiplicative formulation takes the form $\alpha D_t(p_t)$. α is a random variable

with $E\{\alpha\}=1$. Chen and Simchi-Levi find similar results for both finite and infinite time horizons.

Researchers have only recently begun to consider the problem of joint pricing and production decisions given constrained production. Federgruen and Heching [13] consider pricing and production decisions for a single product with a stochastic demand signal and no lost sales. They consider the cases of unidirectional price changes, downward only, as well as bidirectional price changes. They also build the model within the contexts of both finite and infinite time horizons.

Building on the work of Federgruen and Heching, Chan et al [8] introduce a model using deterministic demand but allowing for the possibility of lost sales. They frame the problem as a resource allocation problem and develop a model to consider both inventory and pricing decisions given replenishments at a limited but variable capacity. The base formulation is very similar in structure to the formulation used in my research, so I include it for reference:

$$\begin{aligned}
 \text{Max: } & \sum_{t=1}^T (R_t(D_t) - h_t I_t - k_t X_t) \\
 \text{S.T.: } & I_0 = 0 \\
 & I_t = I_{t-1} + X_t - D_t, t = 1, 2, \dots, T \\
 & X_t \leq Q_t, t = 1, 2, \dots, T \\
 & I_t, X_t, D_t \in Z^+, t = 1, 2, \dots, T
 \end{aligned}$$

Here I_t is the inventory at time t . X_t is production, D_t : satisfied demand, production, Q_t : production capacity limits, $R_t(D_t)$: revenue function, h_t : holding costs, and k_t : production costs. They assume price and revenue to be one-for-one functions of demand

satisfied, so by calculating the satisfied demand, price should be set at the highest level that yields a demand higher than D_t . This provides for the potential for lost sales only if the available range of prices is not continuous. Through computational analysis, they find that optimal flexible pricing strategies generally reduce total production variability. Three interesting general results are drawn in comparison to setting a single optimal fixed price: flexible pricing increases average sales, lowers average inventory, and lowers the average price. They also find that a greedy algorithm is optimal, so the immediate optimum price at each decision point also defines the optimum total price path.

At the bequest of General Motors, Biller et al [5] apply the model presented by Chan [8] to retail pricing in the automotive industry. The model is applied to various demand scenarios. They find that profit capture of Dynamic Pricing schemes over fixed pricing schemes is on the order of 1-18% and that higher demand variability leads to greater potential profit capture. They also find that the higher the ratio between demand and capacity, the greater the potential profit capture of a Dynamic Pricing scheme over a static one.

Chan et al [9] later extend the model to include a stochastic formulation of demand. They also include seasonality of demand, although the formulation is still limited to a single product. The large state space of the Dynamic Pricing formulation makes the problem computationally infeasible; as a result, two general heuristics are developed. The Delayed Production Heuristic starts from a schedule of prices and then determines the production amounts at each period. The Delayed Pricing Heuristic generates a production schedule using a deterministic approximation of demand and then solves the pricing decision at each period before customer orders are known. Under

computational analysis, the Delayed Pricing Heuristic performed better under most circumstances than the Delayed Production Heuristic. They find that as capacity becomes more constrained, the benefits of Dynamic Pricing increase, and that greater seasonality increases the benefits of Dynamic Pricing. See Swann [24] for further research.

This research is closest in formulation and approach to the work of Chan et al [8] and Biller [5] et al in that I construct a linear program to solve joint pricing and production decisions and apply it within an existing company with real data. The complexity of the formulation and constraints necessitates deterministic assumptions so that the problem can be manageable and practically applied.

1.3 METHODOLOGY

This research was motivated by a need for more understanding, so the appropriate method is the one that builds a solid foundation for general understanding of the benefits and potential implementation of Dynamic Pricing strategies at Company X. This thesis presents a model formulation based around Company X's systems and data as a test case. If Dynamic Pricing solutions were going to be applied in practice, there are several deficiencies in the available systems and data that were brought to light during research. These are discussed in presentation of the final results. One limiting constraint in the framing and presentation of the thesis is Company X's desire to keep their identity concealed and the data disguised. In practice, the constraint proved very minor. Given the limitations, I aim to prove the efficacy of potential pricing decisions among a broad range

of possible input parameters. The general framework of the thesis is as a research study on real data applying and modifying existing formulations to generate answers that are relevant and insightful to the vested parties.

The model used is a mixed integer program, chosen for greater ease of formulation given the complexity of the inputs. I decided that a deterministic formulation would yield solid general results, as demonstrated by Biller [5]. Great effort was taken to make the model flexible to multiple sets of data as available and presented to Company X across plants. The selected data presents a simpler configuration than may be encountered within Company X's full network of production facilities.

The two core solution structures center around aggregate cost and demand functions. A cost function is constructed that relates costs to production decisions and inventory. The demand function takes a predetermined base demand and calculates a lift factor based on pricing decisions. The demand function is extended to provide the revenue function. Both core functions can only be optimized intertemporally, so they are combined into a mixed integer program comparing costs and sales to maximize total profits over the selected production period.

Company X operates in all 50 states with plants spread evenly among population centers. Competition varies throughout the country, so to simplify the analysis and allow for monopolistic assumptions, I have chosen to concentrate on a set of four plants that are geographically isolated and serve a region in which Company X has its most dominant market share. The model considers pricing decisions among five sizes within a single platform of product. Of the four plants within the selected region, only two produce the platform under consideration. The platform represents approximately 30% of combined

production. With no consumer choice model incorporated into the analysis, the assumption of monopolistic competition does not affect the final formulation. Increased competition could easily be reflected by changing the price elasticity. The assumption only serves to focus expectations.

Within the platform are two broad categories of products, large and small. This is key as small products are not promoted to the same extent that large sizes are. Despite a lack of price promotion, small and large sizes show similar standard deviations as a percent of mean, 0.22 and 0.20 respectively. The dissimilarity of the demand patterns is evidenced by a low correlation coefficient of 0.14. Exhibit 1 shows 52 weeks of demand for both categories.

The distribution strategies vary significantly by product category, and, as a result, the distribution channels limit how much pricing promotion can occur with the smaller sizes. Within the selected region small sizes represent only 23% of the total volume. Company X is only interested in pricing strategies for the large sizes, so for purposes of this thesis, the small size demand will be fixed. It must be accounted for within the formulation as it limits the amount of production available to the large sizes, so the production capacity available to large sizes will vary per time period. The large sizes are separated into 5 sizes, given here as $J = \{1,2,3,4,5\}$, which also represent homogeneously priced products.

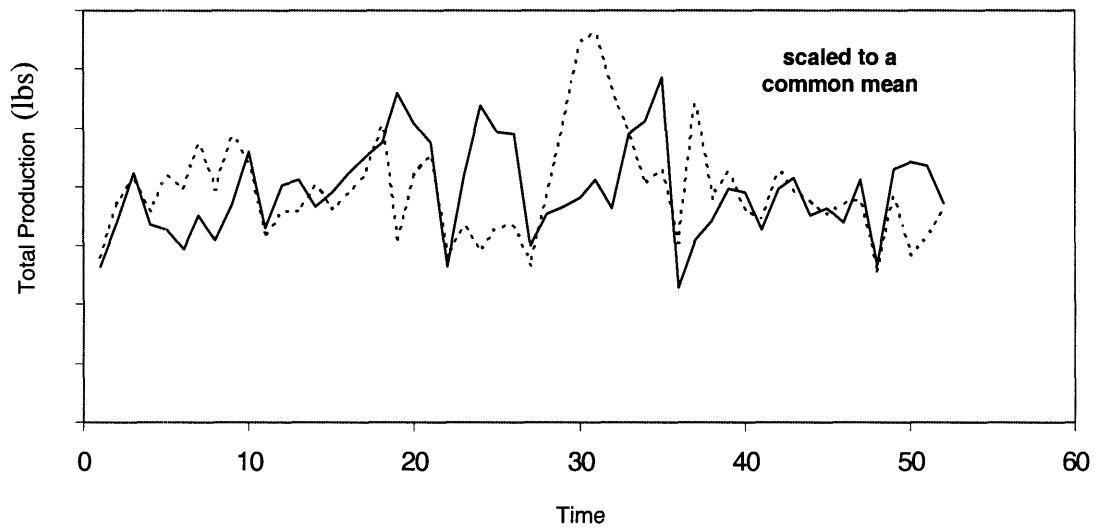
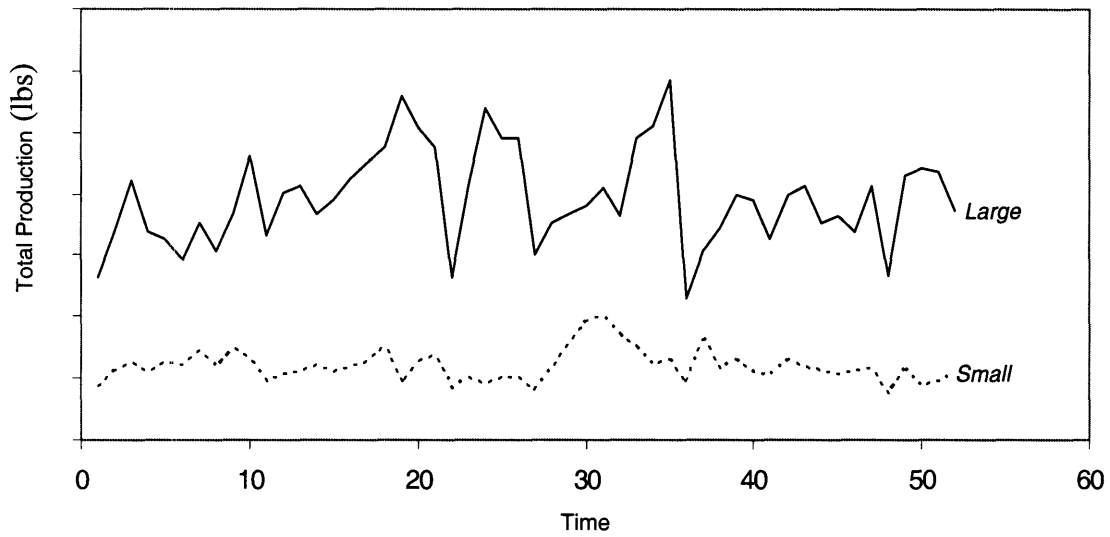


Exhibit 1

In order to generate useful results from any Dynamic Pricing framework, several key data points and descriptive functions must be known, including: operating cost functions, capacity restrictions, customer valuations, and projected demand. Company X already devotes considerable resources to the understanding of consumer choice and the forecasting of demand. Rather than reinventing the wheel, I used company X's forecasted demand signal as the base demand signal and then applied a price elasticity function upon the demand signal. For price elasticity, I also used company X's elasticity framework.

Cost formulations are similarly derived from existing data starting with the chart of accounts. Charts of accounts for manufacturing, warehousing, and transportation are used. Company X has already performed the allocation of costs as fixed and variable. In addition to the simple linear per-unit variable costs, I include several other linear costs and piece-wise linear approximations applied to cost components such as interplant shipments during peak seasons, inventory holding costs, and a consideration of overtime labor. The plant fixed costs are not a decision component and hence, are left out of the formulation, although I do consider them in the final analysis.

Several demand signals are evaluated based on historical data during key seasonal events. The key analysis is around periods of significant seasonal influence. Demand data is available by product by day at every plant. For this thesis it is aggregated by size. Historical production and pricing decisions are available weekly. This proved key in formulating the model and in providing the “base case” formulation.

At the two plants under consideration, hereafter known as Plant 1 and Plant 2, are three production lines, two at Plant 1 and one at Plant 2. Each production line has capacity constraints that are a function of product mix, per hour production capacity, and maximum run hours per week. All of these figures are well known and easily available. The largest portion of linear variable costs correlates well to actual production volumes in observed data. Unfortunately, the observed cost data is limited and cannot be relied on for conclusive analysis. The data is calculated on a monthly basis and only available in detail for a limited number of months. The small sample size of historical cost data is a key reason for relying on Company X’s existing formulations. The exceptions are in the addition of warehousing, holding costs and interplant transportation costs.

Within warehousing, the largest costs are fixed. Within the context of this research I assume that facilities will not be expanded or sold based on the variance in demand incurred by various price paths. Within warehousing there are still considerable variable costs as each case must be shipped or stored. At Company X, 23% percent of all produced cases are never stored. The rest of production is only stored briefly, as almost all production arrives to retail within seven days of production. The variable costs are assigned per case. Holding costs are an opportunity cost on inventory as it could be cash reinvested in other ventures. Holding costs are applied at a standard discount rate assuming little risk in the accumulation of short-term inventory. Holding costs are applied as a percent on average production cost of inventory. With a deterministic demand formulation, no safety stock is added to the solution. Properly accounted for, safety stock varies according to seasonality, demand variability, and base demand rate, so it is a cost that is conspicuous in its absence.

Production and demand are both assumed to occur instantaneously, so holding costs are paid on the ending inventory. No stock-outs are allowed. Inventory cannot decrease below zero and serviced demand is actual demand.

Data came from several sources within Company X, but most significantly, all previous cost data, formulations, and allocations came from the team that allocates production capacity between plants as a long term scheduling role. Cost accounts were taken directly from finance, and the pricing data and formulations came from the demand forecasting department. The key problem with the data is that it was not collected with Dynamic Pricing in mind. More detail about specific data problems is given in the following sections.

CHAPTER 2:

MATHEMATICAL FORMULATION AND INPUT DATA

Development of the mathematical formulation was an iterative process that relied heavily on what data was available and how it was structured. Limitations in the model are often the result of limitations in data.

2.1 GENERAL INPUT DATA

2.1.1 Conversion Factors

Production is measured in pounds. Warehousing costs are allocated by cases, and transportation costs are measured in volumes. A proper costs allocation includes proper conversion of products from one category to the next. Each product can vary in pounds per unit, units per case, and cubic feet per unit. These figures were readily available by product. In this thesis, products were aggregated by size, so the aggregated conversion factors and standard deviations are given in the following sections.

2.1.2 Capacity Constraints

Total weekly production capacity is limited by the total number of hours that any one line can run. Company X pegs this number at around 135 hours per line. Then to calculate total capacity, I simply multiplied the line throughputs by the hours per line. This is discussed in more detail with the discussion of overtime in section 2.2.3

2.2 COST DATA

Manufacturing is a continuous flow operation. Output can be measured in cases, pounds, or individual units. For production, the most relevant unit is pounds per hour, as the production capacity is measured as such. For transportation and warehousing, costs are more easily assigned by case. Production capacity is allocated along three tiered product attributes platform, color, and size. For purposes of this model, the key attribute of any product is the pricing category that it falls into. Products can be aggregated along similar pricing categories. In this case, size is assumed to be the level of aggregation at which homogenous pricing strategies occur.

For reasons of simplicity, I will only consider production decisions to be constrained by pounds per hour of platform capacity and the available hours of production. The assumption is reasonable since I am aggregating an entire weeks worth of production. In day-to-day practice, total platform capacity is divided among individual color lines which each have capacity constraints. From the color lines split separately configured packaging lines, each with a capacity constraint. The configuration places natural limits on product mix, the most notable of which is the mix of smaller sized products. Smaller packaging machines have a lower throughput capacity, and as a result, no platform line is configured to produce 100% small sizes. A typical constraint is a maximum of 40% small sizes. The constraint does not enter into this formulation, as small size demand is fixed at below ultimate small size capacity.

To model all key costs, I have divided costs among manufacturing, transportation, and warehousing. Fixed costs will simply be aggregated together as a single value β , but the fixed costs have no bearing on the optimum pricing and production decisions and are left out of the model. Fixed costs only affect the solution if they can be avoided, such as set-up costs on a line. The profit maximizing price path will be optimum regardless of whether or not fixed costs are added during the solution process or afterwards.

The costs formulation takes the general standard material input formulation of: $x_1Q_1 + \dots + x_nQ_n$, where x represents the cost factor and Q represents the quantity used. The inputs in this case are aggregated to the four general rate categories per product (Standard, Overtime, Interplant Standard, Interplant Overtime). Production within each category represents one of four available inputs.

2.2.1 Manufacturing and Warehousing Costs

Manufacturing, warehousing, and transportation costs are all converted to a single variable cost allocated by the pound. All variable costs were calculated using available cost and production histories that included the last four months of 2002. Starting with the historical cost allocations, weighted averages are used to generate a base cost factor for each size. This is simply the sum of all production costs for a given product size estimated using the historical cost factors divided by total production for a given size. See Exhibit 2 for the base variable cost factors. Note that a value of 100 is used when the product is not produced at the plant.

BASE COST FACTORS		
Size	Plants	
	1	2
1	0.73	0.70
2	0.96	100.00
3	0.71	0.64
4	0.66	100.00
5	0.69	0.62

Exhibit 2

Limitations: Manufacturing and warehousing costs were taken from existing allocations used by Company X. The allocations are not an accurate reflection of production costs, and are biased by existing pricing strategies. Total dollar sales (D_j) per product are used to form the basis for assigning costs at the same proportion per product that total dollar sales occurred. For example, if J represents the set of products, in this case only two, C represents the total variable costs, and U_j represents total unit sales of product j . The costs per unit (c_j) can be calculated as follows:

$$c_j = \frac{\left(\frac{D_j}{\sum_{j \in J} D_j} \right) C}{U_j}$$

A simple way to increase the accuracy of the model results would be to construct a more thorough evaluation of the cost assignments. The current allocation method biases costs based on current pricing strategies, as simply raising the base price increases the proportional cost allocation of a product. The strength of the effect depends on the elasticity of demand. The effect being exaggerated when revenue is a nondecreasing function of price and somewhat curbed when revenue is a nonincreasing function of price. More detail and clarification on price elasticity and revenue curves is presented in the following sections

2.2.2 Transportation Costs

Transportation costs are not included in the cost factors used by Company X, as the cost factors are used only to allocate production capacity efficiently. The manufacturing and warehousing cost factors are allocated from the chart of accounts using a table of allocation factors. I simply used the same allocation factors to come up with a variable transportation cost per case shipped.

The chart of accounts is only constructed for the traffic center that schedules shipments. The costs are not allocated to the production facilities. The chart of accounts maintains the total amount of cases shipped. As a result, only one variable cost per case shipped is available. This can then be allocated per product pound based on the average cases per pound. Traffic costs do not include the costs of interplant shipments. These are modeled separately. See Exhibit 3 for the base variable cost factors adjusted with transportation costs.

Avg Trans Cost per Case 0.25

BASE COST WITH TRANSPORT						
Size	Costs		Avg Case/Lb		Std Dev	
	1	2	1	2	1	2
1	0.78	0.75	0.20	0.20	0.01	0.01
2	1.00	100.00	0.15	0.00	0.00	0.00
3	0.75	0.68	0.16	0.16	0.02	0.01
4	0.70	100.00	0.17	0.00	0.00	0.00
5	0.74	0.68	0.20	0.21	0.05	0.05

Exhibit 3

Limitations: A more thorough accounting of costs would require lane shipment histories, destination demand histories, and lane rates per cubic feet. From this data, a network model could be constructed matching origin and destination pairs to linear cost factors including production costs and the costs on each individual lane. This data was not

available at Company X. As a result, the model makes the assumption that the percent allocation of volumes per lane remains constant and each plant maintains its own distribution area.

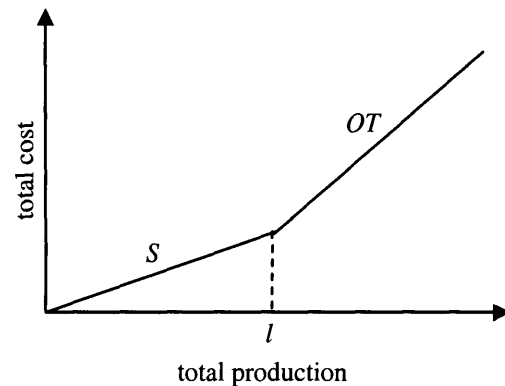
2.2.3 Overtime

For purposes of this thesis, I assume that overtime will apply within a threshold model.

Each plant has a threshold production rate l over which overtime labor rates apply.

Exhibit 4 shows the slopes of standard costs (S) and overtime costs (OT) as total production (X) increases.

$$TC(X) = \begin{cases} S * X & X \leq l \\ S * l + OT * (X - l) & X > l \end{cases}$$



Given this assumption the x-intercept of a linear regression of overtime costs versus total production should yield the threshold point. Another approach would be to look at the total cost ratios of regular labor and overtime, assuming a 1.5 multiplier for overtime, and using the average production at standard costs as the threshold point. A simple linear regression on the data available for both plants 1 and 2 reveal line equations of $0.0244x - 3.383$ and $0.0363x - 3.375$, respectively. These yield threshold points of 138,821 lbs/wk and 92,890 lbs/wk. Using the cost ratio method, threshold points of 266,635 lbs/wk and 120,074 lbs/wk are obtained. To relate these thresholds to capacities,

using a maximum of 135 run hours per line, peak production capacities of 439,809 lbs/wk and 148,318 lbs/wk are obtained for plants 1 and 2, respectively. Against total capacity, the second set of numbers seems more reasonable.

The threshold levels translate into 82 hours per line at plant 1 and 109 hours on the single line at plant 2, but in truth, line 2 is often left idle and shares production with other platforms. Historical data shows that the primary lines at both plants run close to full capacity, so I have decided to set 109 hours as the threshold level for the primary lines at both plants generating the results in Exhibit 5. Line 2 at plant 1 has been given twice the overtime hours to allow it to work as an overflow line while recognizing that its total hours are limited due to shared capacity.

Plant	Line	Total Capacity	Max Regular Hours	Max OT Hours
1	1	2316	109	26
1	2	942	15	52
2	3	1099	109	26

Exhibit 5

Using the given threshold levels, the slopes can be backed out of the non-overtime cost table. Assuming a total per period non overtime production of P^s and a total actual production of P . Overtime production P^o is simply: $P^o = P - P^s$. This can be allocated to products based on historical production allocations. Given this and assuming that overtime is applied at a rate of 1.5 times the standard labor rate and knowing the actual overtime costs (v) and total production costs (C), we can then calculate what the production costs would be if all labor was charged at the standard rate. The result can then be divided by total production to generate a standard production cost (a). This is performed for each product.

The calculation is as follows:

$$a = \frac{C - (v - (v/1.5))}{P}$$

The overtime rate (*b*) can then quite easily be calculated by extension:

$$b = \frac{C - (a * P^s)}{P^o}$$

The results of such calculations reveal the cost table expressed in Exhibit 6.

Standard and Overtime Costs				
Size	Standard		OT	
	1	2	1	2
1	0.77	0.74	0.90	0.84
2	0.98	100.00	1.11	100.00
3	0.74	0.67	0.87	0.77
4	0.68	100.00	0.81	100.00
5	0.72	0.67	0.85	0.76

Exhibit 6

Limitations: The key problem in realizing overtime in the formulation is that within the available data and in discussion with staff at Company X, it is apparent that overtime cannot be easily correlated to total production numbers. Overtime data is available over only a very limited time horizon, and it is aggregated to the month. Meaning that any hope of statistical accuracy is lost, and the aggregation will work to dampen any observable effects. An additional complication is that overtime is often more of a function of personal policies at the various plants and individuals own motivations for incurring overtime to increase their effective salary. Reducing overtime often becomes a policy issue, not a matter of leveling production or making better use of capacity. If

accounting were done with an Activity Based Costing (ABC) approach, constructing an overtime model would be a much easier approach. The ABC approach assigns costs to actual activities as they are relevant to the given product. Such an approach would be generally better for allocating costs in general as will be discussed in more detail in the conclusion.

Despite these assumptions, the staggered non-linear component of overtime is very important to production considerations, and the assumption of a threshold model for overtime seems to be visible in the very limited data available. Rather than leave overtime out of the formulation, I chose to make do with the data available and simply adjust the baseline parameters to account for possible errors in the data. This included running the model with no overtime. With such limited data it is impossible to have confidence in a quantitative analysis, so a bit of subjective analysis and intuition is also involved. This is the most common problem with attempting to build such a detailed quantitative model in a real world environment. Without significant time and resources devoted to such a project for the express purpose of creating a more inviting data environment, the limited data that is available will always force compromises in the formulation. Exhibit 7 shows total overtime costs as a function of total production volume over a four-week period.

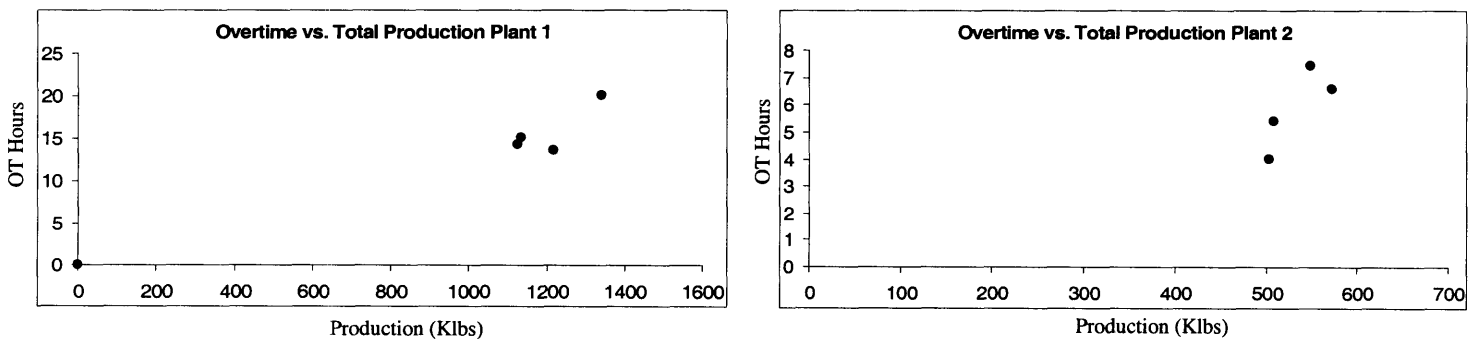


Exhibit 7

2.2.4 Interplant shipments

When plant 1 is running at full capacity and plant 2 has capacity to spare, plant 2 can support plant 1 and simply ship support to plant 1. The costs are easy to calculate as only a single transportation lane is considered. Allowing interplant shipments supports the assumption that the plants will continue to service the same routes. Rather than plant 2 supporting plant 1's customers, plant 2 will ship product to plant 1 for distribution. In the formulation, interplant shipments cannot be made from inventory. They can only be allocated from production.

Limiting the model to two plants allows the formulation to exclude information on the destination of production for interplant shipments, as all production at plant 1 destined for interplant shipments is going to plant 2. The solution for more than two plants would be to use a network formulation. Instead of J representing the set of single plants, it would represent the set of all possible plant-to-plant routes, including same to same. Standard and overtime rates would be calculated for all possible routes, and in the formulation the sum of all production starting at a network node would need to be less than that node's capacity. The complexity of the formulation increases exponentially with the number of nodes.

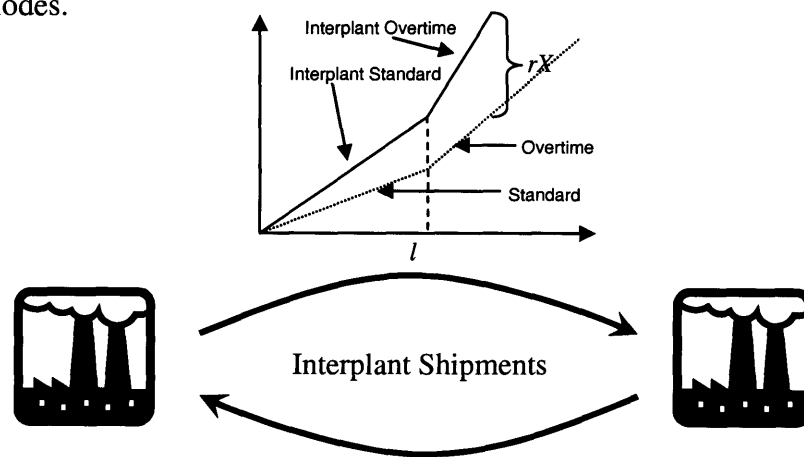


Exhibit 8

Within the model, interplant shipments are treated as another cost factor. Any plant can produce product itself or service its demand through interplant shipments. The cost of interplant shipments is a linear function of the amount of production at plant 1 allocated to plant 2. The rate is simply the cost to produce one pound of product j at plant 1 (c_j), be it standard rate or overtime rate, plus the cost to ship one cubic foot of product j converted to a cost per pound (r_j) using the cubic feet per pound conversions for each product j . The extra costs in the whole system will simply be the interplant shipping rate times total product shipped. Given the nature of the product, shipping rates are only available as a rate per cubic foot, so the only step in building the rate table is to take the table of standard and overtime rates per product and add a converted interplant shipment rate per pound. The results of the allocation are shown in Exhibit 9.

Interplant Shipment Cost per cubic foot 0.155

Interplant Shipment Costs								
Size	IP Standard		IP OT		Cubic Ft/Lb		Std Dev	
	1	2	1	2	1	2	1	2
1	0.82	0.80	0.95	0.90	0.37	0.39	0.09	0.09
2	1.03	100.00	1.16	100.00	0.27	0.00	0.00	0.00
3	0.79	0.73	0.92	0.82	0.33	0.35	0.06	0.04
4	0.72	100.00	0.85	100.00	0.24	0.00	0.00	0.00
5	0.78	0.72	0.91	0.82	0.35	0.37	0.04	0.05

Exhibit 9

2.2.5 Inventory costs

The variable component of warehousing costs is already accounted for, and the largest costs with holding inventory are the recurring fixed costs of the facilities and equipment. Large inventories are most easily accounted for as an opportunity cost. The capital

invested in inventory could have been reinvested in other projects returning a rate r . The correct assignment of r involves comparing the risk and returns profile of the capital investment under analysis with marketable securities with similar risk and return ratios.

Within Company X, capital could be reinvested into other projects, so a useful point of measure is to look at their required rate of return on projects. In this case, it is 20%. In the context of inventory, the discount rate is applied as a holding cost on the inventory. Meaning that each period, the potential return is lost and therefore accounted as a cost of holding the inventory. In the final results, discount rate manipulation proved to have little effect.

Holding costs do present a problem in the formulation. Holding costs are applied against the produced cost of the inventory. With four production costs at two separate plants, there are eight possible production costs for each product. Inventory will only be tracked at the plant level, reducing the possible costs of inventory to four but not eliminating the problem. When demand is served from inventory, there is no way, within the model formulation, to track which cost of inventory should be applied to the demand serviced. The easiest solution, and the one I have applied, is to create one holding cost per plant per product. I simply applied the variable costs assuming no overtime and no interplant shipments to produce a per period per pound charge for each product at both plants. This was derived from the base rates as shown in Exhibit 3, Exhibit 10 shows the resulting rates.

Discount Rate r 20% per annum
Discount Rate per Period 0.385%

Holding Costs per Period				
Size	Base Rates		Holding Costs	
	1	2	1	2
1	0.73	0.70	0.00281	0.00270
2	0.96	100.00	0.00369	1.00000
3	0.71	0.64	0.00274	0.00247
4	0.66	100.00	0.00253	1.00000
5	0.69	0.62	0.00264	0.00239

Exhibit 10

2.3 COST FORMULATION

Within the model formulation, the different cost factors are simply formulated using piecewise approximation, the only exception being the inventory holding costs. Any product j can be produced at four different cost factors (a_j : Standard, b_j : Overtime, c_j : Interplant Standard, and d_j : Interplant Overtime) with various total production constraints at each plant.

With the following notation:

- J = set of all product sizes, excluding small sizes
- F = set of manufacturing lines
- P = set of manufacturing plants
- T = set of all discrete time periods $\{0,1,\dots,T-1, T\}$
- r_t = compounded discount factor at time $t, t \in T$
- w_{fp} = 1 if line f belongs to plant p , 0 otherwise, $\forall f \in F, p \in P$
- a_{jp} = standard production cost of size j at plant $p, \forall j \in J, p \in P$
- b_{jp} = overtime production cost of size j at plant $p, \forall j \in J, p \in P$
- c_{jp} = interplant standard, production cost of size j at plant $p, \forall j \in J, p \in P$
- d_{jp} = interplant overtime production cost of size j at plant $p, \forall j \in J, p \in P$
- u_{jft} = production of size j on line f at cost a in period $t, \forall f \in F, j \in J, t \in T$
- v_{jft} = production of size j on line f at cost b in period $t, \forall f \in F, j \in J, t \in T$
- x_{jft} = production of size j on line f at cost c in period $t, \forall f \in F, j \in J, t \in T$
- y_{jft} = production of size j on line f at cost d in period $t, \forall f \in F, j \in J, t \in T$
- h_{jp} = holding cost of inventory of product j at plant $p, \forall j \in J, p \in P$
- i_{jpt} = inventory of product j at plant p in period $t, \forall j \in J, p \in P, t \in T$

Total costs can be formulated as follows:

$$TC = \sum_{t=1}^T r_t \sum_{j \in J} \sum_{p \in P} \left(a_{jp} \sum_{f \in F} w_{fp} u_{jft} + b_{jp} \sum_{f \in F} w_{fp} v_{jft} + c_{jp} \sum_{f \in F} w_{fp} x_{jft} + d_{jp} \sum_{f \in F} w_{fp} y_{jft} \right) + \sum_{j \in J} \sum_{p \in P} h_{jp} \sum_{t=1}^T i_{jpt} r_t$$

2.4 DEMAND AND PRICE ELASTICITY MODEL

2.4.1 Demand Data

Demand is assumed to be deterministic and is an exogenous input into the formulation.

For each period, a base demand signal d_t is adjusted according to the pricing decision made in each period. For a given base demand d_t , adjusted demand is a continuous function of price. The adjusted demand d_t' function takes the form: $d_t' = D(d_t, p_t)$.

Historical sales figures are used to provide the base demand signal for all large product sizes. The historical demand is an adjusted demand signal, as it has been influenced by the previously selected price paths. Since prices were not steady during the historical demand signal, the demand must be adjusted to a base price level as a preprocessing step. Historical demand must be converted to a base demand signal. The historical demand is converted to base by reversing the price elasticity formulation as described in the following sections. For example if the historical demand is 125, but given the actual price, the price elasticity factor was 1.25 then the base demand signal is 100. All demands must be normalized to price elasticity factors of 1. A detailed explanation of how the price elasticity factors are calculated is provided in the following section.

Three twelve-week demand signals were used in the final analysis. Demand was selected over periods of high seasonality. The unaltered signals are given in Appendix A. The efficacy of a deterministic assumption is heavily dependent on the accuracy of the demand forecasts, but demand forecasts are not available historically by the week.

Comparing results against a generated forecast of all average demands created a partial solution.

Limitations: Demand at the plant is known through orders. Point of purchase data is not available, so the data input is already at least once removed from the customer. Final implementation should use point of sales data or a good approximation thereof. This is a natural improvement of the forecasting method as well and would enable more accurate price elasticity estimation.

2.4.2 Elasticity Model

Company X uses a very simple price elasticity formulation. Every product has a base price p^* . Price elasticity is measured the same for all products and is applied by measuring the ratio of the period price p_t to the base price p^* . The ratios are assigned ranges, and a single multiplicative lift factor (price factor) is assigned to each range using a price elasticity function of the form $N(p)^{-\eta}$, where $N(p_t)$ is the price at time t normalized to the base price (p^*). $N(p_b) = 1$ and η is the elasticity.

$$N(p_t) = \frac{p_t}{p_b}$$

As an example, if the base price was 2 ($p_b=2$), the period price was 3 ($p_t=3$), and the elasticity was 1.5 ($\eta=1.5$), the price factor would be calculated as: $(3/2)^{-1.5}$, and the adjusted demand would be $d_t' = d_t(3/2)^{-1.5}$.

This approach produces a simple table and discrete stepped elasticity curve as demonstrated in exhibit 11. In the exhibit, the percent ranges represent ranges of the

function $N(p_t)$. 90-100% indicates that for any period price in the range of 90-100% of the base price, a price factor of 1 should be used. This model is applied by company X using a series of if/then logic statements. For purposes of this thesis, I needed to formulate a piecewise approximation of the continuous form of the elasticity function $N(p)^{-\eta}$, so that demand will be a continuous function of price. The stepped function used by Company X yields an approximate elasticity of $\eta=1.19$.

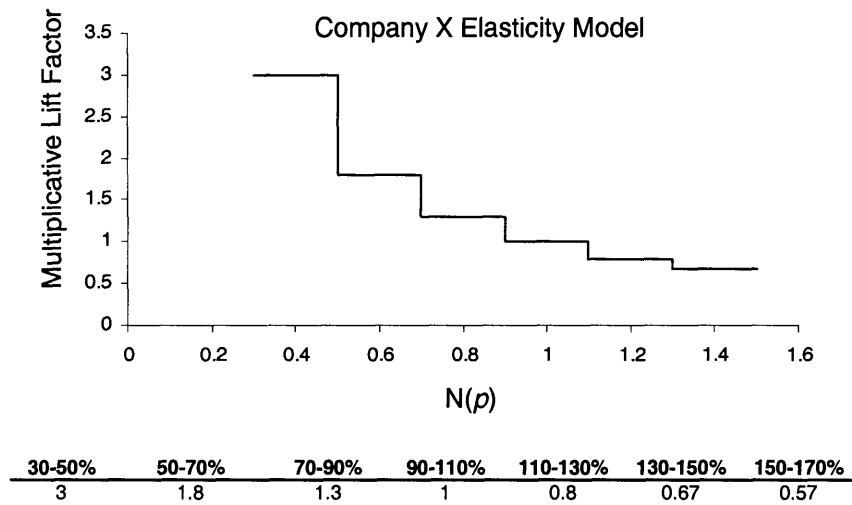


Exhibit 11

Company X's price elasticity function is applied to a base demand signal normalized to the base price p^* , so prerequisite to applying a similar formulation is isolating the base demand signal. When using historical data, this involves dividing the demand signal by the multiplicative lift factors generated by the actual price path. At this point, I should note one sticking point of the elasticity formulation; the price elasticity formulation used by Company X has no intertemporal effects. A price changed in period t will only affect demand in period t . A partial intuitive justification rests on the stability of

the demand and the perishability of the product, but it is largely a function of the forecasting model.

Company X sees minimal year after year growth (1-2%) in the pounds of product demanded, so the market is extremely mature. It is a product that the majority of the target population has already sampled and is quite familiar with. While a lower price may affect the quantity purchased, within a reasonable range it is unlikely to change the market size or consumer valuations, and hence the base demand signal. In fact, if you look at the nature of the elasticity function, an elasticity of $\eta=1$ would simply mean that the consumer was paying a fixed amount and buying the amount of product that fit within that fixed amount. This is also significant when calculating the revenue function. A price elasticity of $\eta=1$ is the point at which revenue switches between being a nondecreasing function of price to a nonincreasing function, as shall be demonstrated in section 2.7. The perishability of the product means that price changes will not result in hoarding or delayed purchasing. The product is meant to be consumed.

The forecasting model used by Company X is a triple exponential smoothing model. Factors are estimated for seasonality, trend, and base. All three factors are then adjusted continually through exponential smoothing. Within the forecasting model, any intertemporal pricing effects would be seen through a change in trend. It would be very difficult to isolate market driven trend from pricing driven trend if the effects were significantly intertemporally displaced or diluted. Given that there is an observable negative correlation between prices and seasonal demand patterns, it would be very difficult to add in intertemporally adjusted price elasticity, as many of the historical effects of price changes are captured in the seasonality component of the forecast, so the

instantaneous application of demand is driven by the construction of the forecasting model.

The price elasticity formulation used in this model is calculated by converting the stepped function used by Company X into a continuous function using piecewise approximations. Prices will be limited to the range of 40% to 160% of the base price p^* . The function is divided into three linear piecewise approximations of ranges 40-80%, 80-120%, and 120-160%. I represent each segment by q , where q indicates one linear approximation in the set of linear approximations $Q = [1,2,3]$, See Exhibit 12 for clarification.

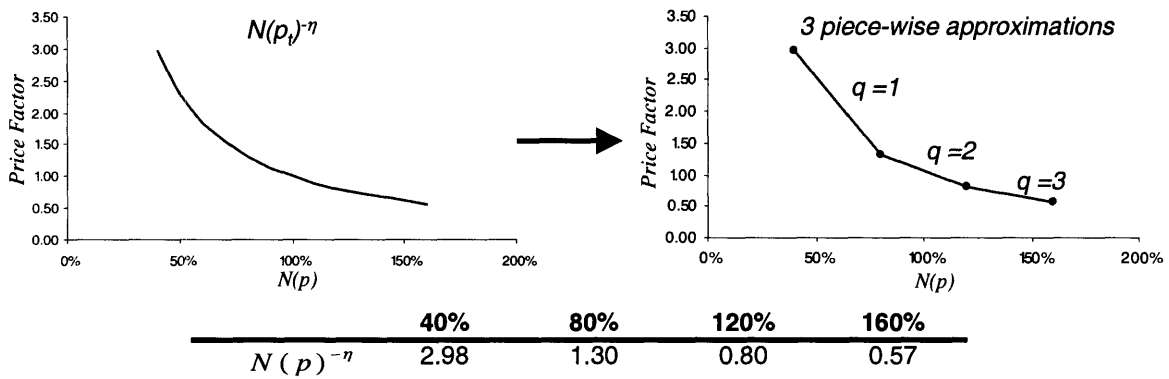


Exhibit 12

Since $N(p_t)$ is non-linear, scaled functions must be calculated for each pricing category. For example if the base price $p^* = 2$, the graph and input points would be as shown in Exhibit 13.

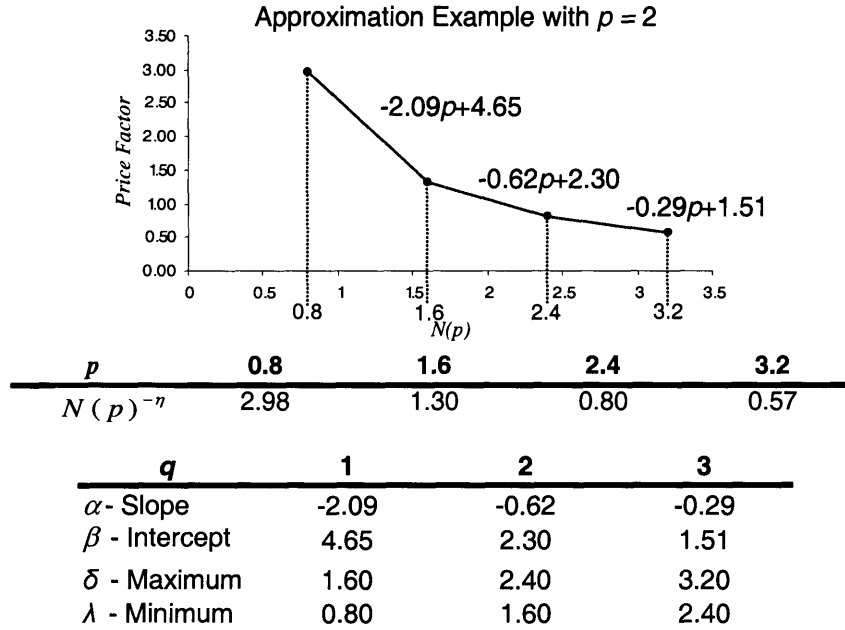


Exhibit 13

The calculation of slopes and intercepts are as follows:

$$\alpha_{qj} = \begin{cases} \frac{(0.80)^{-\eta} - (0.40)^{-\eta}}{p_j^*(0.8 - 0.40)}, q = 1 \\ \frac{(1.20)^{-\eta} - (0.80)^{-\eta}}{p_j^*(0.40)}, q = 2 \\ \frac{(1.60)^{-\eta} - (1.20)^{-\eta}}{p_j^*(0.40)}, q = 3 \end{cases} \quad \forall j \in J$$

Intercepts can be calculated easily by picking end points to generate the following:

$$\beta_q = \begin{cases} (0.8)^{-\eta} - 2((0.80)^{-\eta} - (0.40)^{-\eta}), q = 1 \\ (1.2)^{-\eta} - 3((1.20)^{-\eta} - (0.80)^{-\eta}), q = 2 \\ (1.6)^{-\eta} - 4((1.60)^{-\eta} - (1.20)^{-\eta}), q = 3 \end{cases} \quad \forall j \in J$$

It should be noted that the intercepts are not dependent on the product's base price, the intercepts are only dependent on the points at which the function is divided and the price elasticity. As a result, intercepts are identical for all products.

In the formulation, there are three possible pricing categories for each product within the spectrum of 40-160% of base price. The slope and intercept applied depend on which price is selected and which category the price falls into. To make it work in the formulation requires maximum and minimum constraints for each price category. The maximum and minimum values are simply the break points, 40%, 80%, and 120% for the minimums and 80%, 120%, and 160% for the maximums. This is illustrated in the final combined formulation.

A key metric to judging the strength of the approximation is by bounding the maximum error. This can be easily accomplished by, for each factor segment, taking the first derivative of the linear approximation minus the continuous formulation. This will yield the price at which maximum error occurs. Then plugging the price into both formulations and taking the difference will yield the maximum error. The base price can be seeded at one for simplicity since the key metric is a percent error. This would yield the following:

$$f_1 = (-4.18p_i + 4.65) - (p_i)^{-1.19} \quad \frac{df_1}{dp_i} = (p_i)^{-2.19} - 4.18 \quad p_i = (4.18)^{-(1/2.19)} = 0.521$$

$$f_2 = (-1.25p_i + 2.30) - (p_i)^{-1.19} \quad \frac{df_2}{dp_i} = (p_i)^{-2.19} - 1.25 \quad p_i = (0.62)^{-(1/2.19)} = 0.903$$

$$f_3 = (-0.58p_i + 1.51) - (p_i)^{-1.19} \quad \frac{df_3}{dp_i} = (p_i)^{-2.19} - 0.58 \quad p_i = (0.29)^{-(1/2.19)} = 1.278$$

Plugging these values in, the maximum error is achieved in the low price factor . The errors from low to high factors were 12%, 4%, and 1.7%. This indicates that high pricing strategies will yield the most accurate results. In practice, the base case scenario chose an all high price strategy. Increasing the number of segments would improve accuracy.

2.5 DEMAND AND PRICE ELASTICITY FORMULATION

In the context of this model and previous formulations of Company X, price elasticity is a multiplicative lift factor q applied to a base demand level d . Before the model is run, the demand input data is renormalized to a single base pricing level.

With the following notation:

Q = set of all price and revenue factors {1,2,3}

p_{qjt} = price charged at factor q for product j in period t , $\forall q \in Q \ j \in J \ t \in T$

z_{qjt} = binary decision variable used to assign a single price factor q for product j in period t , $\forall q \in Q \ j \in J \ t \in T$

α_{qj} = slope of price factor q for product j , $\forall q \in Q \ j \in J$

β_q = intercept of price factor $q \ \forall q \in Q$

d_{jpt} = base demand for product j in period t at plant p , $\forall j \in J \ t \in T$

Adjusted period demand can be formulated as follows:

$$D(p_{jt}, d_{jpt}) = d_{jpt} \left(\sum_{q \in Q} p_{qjt} \alpha_{qj} + \sum_{q \in Q} z_{qjt} \beta_q \right), \forall j \in J \ t \in T \ p \in P$$

z_{qjt} Explanation

The binary decision variable z_{qjt} plays two roles in the final formulation. It assures that only one price and revenue factor is selected and it activates the appropriate maximum and minimum price values for the given factor.

price factor intercepts constraint

$$z_{qjt} M \geq p_{qjt}, \forall t \in \{1, 2, \dots, T\} \ j \in J \ q \in Q$$

This constraint assures that if a price is chosen then z_{qjt} is turned on ($z_{qjt} = 1$).

single price constraint

$$\sum_{q \in Q} z_{qjt} = 1, \forall t \in \{1, 2, \dots, T\} \ j \in J$$

This constraint assures only one price factor can be selected for a given product and period.

binary constraint

$$z_{qjt} \in \{0,1\}, \forall t \in \{1,2,\dots,T\} \quad j \in J \quad q \in Q$$

This constraint simply assures that z_{qjt} is binary

max price per price factor constraint

$$p_{qjt} < z_{qjt} \delta_{qj}, \forall t \in \{1,2,\dots,T\} \quad j \in J \quad q \in Q$$

min price per price factor constraint

$$p_{qjt} \geq z_{qjt} \lambda_{qj}, \forall t \in \{1,2,\dots,T\} \quad j \in J \quad q \in Q$$

These constraints use z_{qjt} to activate the appropriate price maximum and minimums

2.6 REVENUE MODEL

Similar to price elasticity, revenue is a non-linear function that must be approximated through piecewise approximations. For any given time period t and product j , total revenue is simply serviced demand multiplied by price, or $p_{jt}(D(p_{jt}, d_{jpt}))$. Such a function can not be applied in the linear formulation as it is non-linear. To build the piecewise approximations, I started with the continuous formulation, $R =$

$p_{jt} N(p_{jt})^{-\eta} d_{jpt}$. As in the elasticity formulation, the point is to create piecewise functions of price that yield multiplicative revenue factors that can be applied to the demand. Removing demand, the continuous formulation of factors (Q) can be expressed as:

$$Q(p_{jt}) = \frac{p_j^{*\eta}}{p_{jt}^{\eta-1}}$$

It is important to note that for any price elasticity greater than $\eta=1$, the revenue function will be a non-increasing function of price. A price elasticity of one is the switching point between revenue being a nonincreasing or nondecreasing function of price. This is critical, as the end results are highly dependent on the accurate determination of elasticity, which I am not entirely convinced has been performed, as will be discussed in the results.

The partitioning of the function is identical to that of the price elasticity function, and is demonstrated for $p^* = 2$ in Exhibit 14.

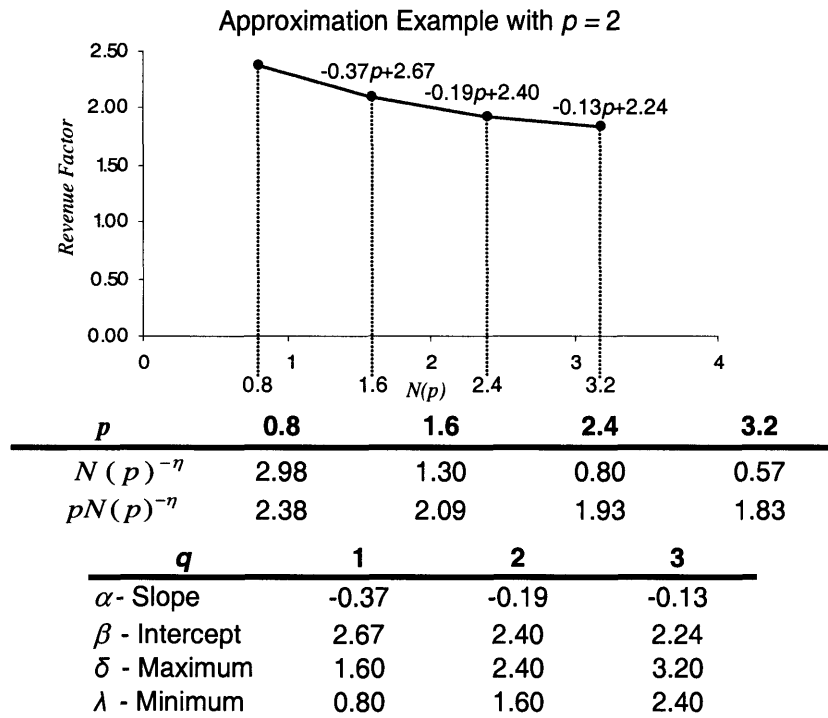


Exhibit 14

The calculation of slopes and intercepts are as follows:

$$\alpha_q = \begin{cases} \frac{(0.80)^{-\eta+1} - (0.40)^{-\eta+1}}{(0.8 - 0.40)}, q = 1 \\ \frac{(1.20)^{-\eta+1} - (0.80)^{-\eta+1}}{(0.40)}, q = 2 \\ \frac{(1.60)^{-\eta+1} - (1.20)^{-\eta+1}}{(0.40)}, q = 3 \end{cases} \quad \forall j \in J$$

Intercepts can be calculated easily by picking end points to generate the following:

$$\beta_{qj} = \begin{cases} p_j^* ((0.8)^{-\eta+1} - 2((0.80)^{-\eta+1} - (0.40)^{-\eta+1})), q = 1 \\ p_j^* ((1.2)^{-\eta+1} - 3((1.20)^{-\eta+1} - (0.80)^{-\eta+1})), q = 2 \\ p_j^* ((1.6)^{-\eta+1} - 4((1.60)^{-\eta+1} - (1.20)^{-\eta+1})), q = 3 \end{cases} \quad \forall j \in J$$

Note that with the revenue function, the slopes are identical for all base prices, but the intercepts vary. Using the same price ranges in the revenue function allows simpler

notation and formulation, as the only new variables needed are those for slope and intercept of the revenue function.

Similar to price elasticity, error bounds must be set for the revenue approximation. The calculations are as follows:

$$f_1 = (-0.37p_t^2 + 1.33p_t) - (p_t)^{-0.19} \quad \frac{df_1}{dp_t} = p_t^{-1.19} - 0.37p_t + 1.33 \quad p_t = 0.581$$

$$f_2 = (-0.19p_t^2 + 1.20p_t) - (p_t)^{-0.19} \quad \frac{df_2}{dp_t} = p_t^{-1.19} - 0.19p_t + 1.20 \quad p_t = 0.989$$

$$f_3 = (-0.13p_t^2 + 1.12p_t) - (p_t)^{-0.19} \quad \frac{df_3}{dp_t} = p_t^{-1.19} - 0.13p_t + 1.12 \quad p_t = 1.392$$

Again the maximum error is achieved at the lowest price factor, but in the revenue approximation, the errors are much smaller. The errors from low to high factors were 1.3%, 0.4%, and 0.2%. This indicates that revenue estimates will be relatively accurate.

2.7 REVENUE FORMULATION

With the following additional notation:

$\sigma_q =$ slope of revenue factor q for product $j, \forall q \in Q$
 $\rho_{qj} =$ intercept of revenue factor $q \forall q \in Q, j \in J$

Total revenue can be formulated as follows:

$$R = \sum_{t=1}^T r_t \sum_{p \in P} \sum_{j \in J} d_{jpt} \left(\sum_{q \in Q} p_{qjt} \sigma_q + \sum_{q \in Q} z_{qjt} \rho_{qj} \right)$$

2.8 MODEL FORMULATION

Putting the pieces together, the primary objective is to maximize profits (Π), or the difference between revenue (R) and total costs (TC).

With the following additional notation:

- s_f = maximum standard run hours allowed on line f per period, $\forall f \in F$
 t_f = maximum total run hours (incl. overtime) allowed on line f per period, $\forall f \in F$
 c_{ft} = pounds per hour of output of line f available at time t , $\forall f \in F \ t \in T$
 δ_{qj} = max price of price or revenue factor q for product j , $\forall q \in Q \ j \in J$
 λ_{qj} = min price of price or revenue factor q for product j , $\forall q \in Q \ j \in J$
 M = big M - used to set binary variables, $M = 1,000,000,000$
 Π = total profit

The combined model can be formulated as follows:

$$\text{Max } \Pi = R - \text{TC}$$

Subject To:

initial inventory set to zero

$$i_{jpt} = 0, \text{ for } t = 0 \ j \in J \ p \in P$$

inventory formulation constraint

$$i_{jpt} = i_{jpt-1} + \sum_{f \in F} w_{fp} u_{jft} + \sum_{f \in F} w_{fp} v_{jft} + \sum_{f \in F} (1 - w_{fp}) x_{jft} + \sum_{f \in F} (1 - w_{fp}) y_{jft} - D(p_{jt}, d_{jpt}), \forall t \in \{1, 2, \dots, T\} \ j \in J \ p \in P$$

all demand is serviced constraint

$$i_{jpt} \geq 0 \ \forall t \in \{1, 2, \dots, T\} \ j \in J \ p \in P$$

adjusted demand greater than zero

$$D(p_{jt}, d_{jpt}) \geq 0$$

total capacity constraint

$$\sum_{j \in J} u_{jft} + \sum_{j \in J} v_{jft} + \sum_{j \in J} x_{jft} + \sum_{j \in J} y_{jft} \leq c_{ft} t_f, \forall t \in \{1, 2, \dots, T\} \ f \in F$$

standard capacity constraint

$$\sum_{j \in J} u_{jft} + \sum_{j \in J} x_{jft} \leq c_{ft} s_f, \forall t \in \{1, 2, \dots, T\} \ f \in F$$

price factor intercepts constraint

$$z_{ajt}M \geq p_{ajt}, \forall t \in \{1,2,\dots,T\} \quad j \in J \quad q \in Q$$

single price constraint

$$\sum_{q \in Q} z_{ajt} = 1, \forall t \in \{1,2,\dots,T\} \quad j \in J$$

binary constraint

$$z_{ajt} \in \{0,1\}, \forall t \in \{1,2,\dots,T\} \quad j \in J \quad q \in Q$$

max price per price factor constraint

$$p_{ajt} < z_{ajt}\delta_{aj}, \forall t \in \{1,2,\dots,T\} \quad j \in J \quad q \in Q$$

min price per price factor constraint

$$p_{ajt} \geq z_{ajt}\lambda_{aj}, \forall t \in \{1,2,\dots,T\} \quad j \in J \quad q \in Q$$

CHAPTER 3:

COMPUTATIONAL ANALYSIS AND RESULTS

Computational analysis was performed using the Express MP optimization engine within Microsoft Excel implemented through Frontline System's Premium Solver Platform. Company X has access to all three licenses. All arrays were simply expressed as written. The only constraint not included in the model implementation was the *inventory formulation constraint*, as this was expressed through cell formulas. It should be well noted that the results were obtained using the available data, so final conclusions should not be drawn. The results provide solid insights and serve to highlight areas where further investigation may be warranted.

3.1 BASELINE RESULTS

The baseline parameters were those given in chapter 2. With the assumed elasticity, the model yielded a 10% profitability improvement driven by both better pricing and better sourcing, resulting in, among other benefits, the elimination of all overtime. Both factors must be taken in context. The results with price paths are given in Appendix B; production is given in Appendix C. Note that the model sets the price at the lowest possible price when no demand signal exists, as seen in product 2 and, for a short time, in product 4.

It is unlikely that overtime can be completely eliminated, as it can often serve as a reward to diligent employees, and is dictated more by plant policy, not total production

demand. As a result, the full \$2 million cost savings presented in the model is not completely realistic. A large chunk of that savings results from simply making less product, a full 40% less. Less product has less need for overtime. The model dictates 160% price points for all products. Company X is a sales driven business, and the sales force is heavily incentivized to drive up volume at all costs. Company X would have to be pretty confident in the price elasticity function to drive all prices across the board up 60%. With the given base parameters, it is apparent that the model is still quite incomplete. Restricting the price range to only a maximum of 120% of base price may provide a more applicable policy, but it does not change the intuitions gleaned, as a high price policy is still optimal.

3.2 PARAMETER MANIPULATION AND SENSITIVITY ANALYSIS

3.2.1 Price Elasticity

Price elasticity easily represents the single most significant factor in determining the correct price path. To test the effect of elasticity in the model, I tested the base model with elasticity values of 0.3,0.6,0.9,1.19,1.5,2,3,4, and 6. A low elasticity number means that demand is relatively unaffected by price changes. As a result, higher prices should be charged, the extreme being perfectly inelastic demand, in which case an infinite price would prove optimal. Perfect inelasticity of demand can best be related to a monopoly of an essential product with unwavering demand. A high elasticity implies that customers will quickly jump from product to product when prices drop. This is generally indicative

of a very competitive environment, where the lowest priced competitor will see a significant increase in volume.

Below a price elasticity of 0.9, the optimum pricing strategy for all products was to charge the maximum possible price in all periods. This result naturally follows from the revenue formulation, in which any elasticity below 1 generates a revenue function that is nondecreasing with price. The dynamics of these conditions are obvious, raising prices increases revenue while decreasing costs through lowered demand, and so the maximum price is always the optimum price.

AVERAGE PRICE VS PRICE ELASTICITY

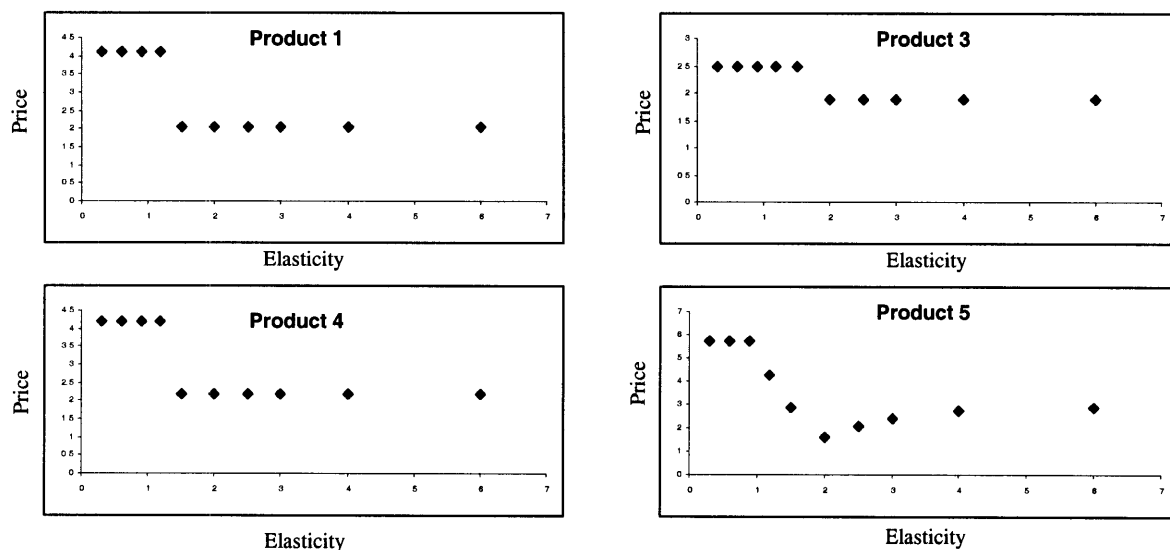


Exhibit 15

As shown in Exhibit 15, all prices tend to sit at the price break points (40%, 80%, 120%, and 160% of base price) in all elasticities. Products 1 and 4 jumped from 160% to 80% in synch between elasticities of 1.19 and 1.5. Product 3 jumped from 160% to 120% between elasticities of 1.5 and 2. The only interesting exception was product five. The drop and then increase in prices for product 5 was simply the result of

the solver picking product 5 to manage capacity. With high elasticities, total production capacity quickly becomes the constraint, as more profit can be yielded through ever higher production volumes. The solver engine in this case used product 5 as to manage the capacity, although the exact dynamics dictating that choice are unclear.

The results with high elasticities of over 1.5-2 may be considerably inaccurate as evidenced by the rather large error bounds as shown in Exhibit 16.

Price Factor Error Bounds				Revenue Factor Error Bounds			
Elasticity	Factor			Elasticity	Factor		
	1	2	3		1	2	3
0.3	17.9%	7.8%	3.9%	0.3	1.3%	0.4%	0.2%
0.6	2.2%	1.0%	0.0%	0.6	1.4%	0.5%	0.2%
0.9	-9.2%	-3.3%	-1.7%	0.9	0.5%	0.2%	0.1%
1.19	-12.0%	-4.0%	-1.7%	1.19	-1.3%	-0.5%	-0.2%
1.5	-5.5%	-0.6%	-0.1%	1.5	-4.4%	-1.5%	-0.8%
2	32.8%	14.3%	4.5%	2	-11.1%	-4.0%	-2.0%
3	461.3%	97.5%	16.0%	3	-29.2%	-11.4%	-6.0%
4	5877.0%	356.5%	23.0%	4	-48.6%	-21.3%	-11.5%
6	9072675.0%	4876.2%	4.3%	6	-78.1%	-43.9%	-25.9%

Exhibit 16

The total profit gains yielded against the base case were most pronounced with low elasticities because of the obvious improvement of charging the maximum rate. For all elasticities above 2, the gains were between 1-5% and highly variable most likely as a result of the high error bounds.

One significant take away is that the optimum price path was most variable right around the elasticities closest to the baseline indicating how critical an accurate elasticity number is.

3.2.2 Changing Demand Scenarios

Three demand scenarios were tested. The unadjusted demands are given in Appendix A. With the given demands, there does not seem to be any correlation between variability of demand and improvement over the base case. Intuitively this makes sense, as the model assumes a deterministic demand, so the only demand characteristic that should be critical is the frequency and size of peaks over capacity. This only becomes constraining in the base parameter set if the max price strategy does not reduce prices below capacity, necessitating build-up volume. In all the demand cases given, the max price strategy always proved optimal, so demand peaks never brushed up against demand.

I also tested the model with an averaged demand signal used for all periods. This was used to represent an approximated forecast. Using an averaged demand signal as a simple forecast, I tested to see if running the model against forecasted data would detract from the benefit of the solution. With the simple high price strategy of the base parameters, working from a steady demand signal had no effect on the selected price path.

3.2.3 No Overtime

The results without overtime are predictable. The price path is the same, yielding the same demand pattern, but costs are greater. The no overtime results are probably more realistic in that simply lowering demand will not eliminate overtime completely. The net improvement in profit drops from 10% to 3.4%. Eliminating overtime from the formulation implies that overtime will be a consistent percent of total labor costs.

3.2.4 Discount Rate

Discount rate has no effect on the selection of price path, indicating that the myopically optimal policy would seem to always be the optimal policy. This seems a natural result of a model with no intertemporal effects of price, where the time focus shifting effects of discount rate will not yield much benefit in demand and elasticity formulations that never require more than a weeks worth of inventory. To test discount rates, I tried rates of 0%,5%,10%, 20%,40%, and 1000%. I tested the discount rate at various elasticities to see if it had any effect when capacity was constrained or when a simple max price strategy was not optimal. There was no effect. Price paths remained the same regardless.

CHAPTER 4:

CONCLUSION

Dynamic Pricing has proven broadly applicable in a range of businesses and applications, but implementation is heavily dependent on accurate input data. Company X operates in a relative monopoly and the low price elasticity seems to reflect that. The model indicates that the low price elasticity should allow Company X to charge higher prices to yield a greater profit. The model does not capture the negative intertemporal effects that high prices might have through brand damage and poor customer relations. The elasticity may only be applicable in a very narrow range of prices and demand. Outside of the limitations of the base parameters, the model could easily be increased in complexity and further research could be done to increase the descriptive accuracy of the formulation

4.1 MODEL LIMITATIONS

The Dynamic Pricing model as described could not be directly implemented as a price path generating tool at Company X. That is not the intention of the research. The tool is intended to generate insights and highlight areas needing further investigation. Some of the limitations to implementation are in the data available, but a stronger limitation is the complexity of the actual pricing decision making process not captured by the model.

The data is most significantly limited in the formulation of the price elasticity function. Demand and cost data was also limited, but is less critical to the selection of the

optimum pricing path. Assigning a single number for price elasticity and eliminating intertemporal effects is a gross oversimplification. The difficulty in final implementation is that the complexity of the actual price elasticity might preclude development of a useful model for generating accurate pricing paths. I would propose that this is not critical to the general understanding that the pricing model can generate, but eliminates the chance of the model giving a single optimum price path or even period price choice. Price elasticity depends on many unpredictable external market factors such as the entrance of new competitors, consumer purchasing patterns, and even weather and catastrophic events.

Elasticity is also dependent on internal factors that are difficult to accurately correlate to price elasticity, such as promotional activity and inter product sales cannibalization effects from new product introductions. An accurate representation of price elasticity should include several stepped trigger prices to represent substitution effects or mental price break points, such as \$100 or \$200. Easily substitutable products will experience significantly varying elasticities as price paths cross.

Price elasticity represents the single most significant data limitation to applicability. Other limitations in the current data available at Company X could be more easily rectified. The cost formulation could be improved with an activity study of materials and labor inputs into products. Company X is unlikely to switch to an activity based cost accounting system, but a simple study of production and labor schedules coupled with a plant and warehouse floor activity study could easily yield more accurate allocations. Within transportation, a more thorough accounting of costs would require lane shipment histories, destination demand histories, and lane rates per cubic feet. From

this data, a network model could be constructed matching origin and destination pairs to linear cost factors including production costs and the costs on each individual lane.

The model did not show great sensitivity to the cost formulation, so this would only be a step to gain more accuracy in the numbers generated, rather than gaining more insight into the optimality of the pricing decisions.

4.2 DECISION PROCESS

Company X currently incentivizes its sales force to drive additional volume with low price promotions. The model indicates that promotions and unbounded sales growth may actually be less profitable than charging slightly higher prices to the detriment of total volume and potentially, revenue. This is not an immediately obvious conclusion, and may in fact be undesirable. The ability to charge high prices correlates with low elasticity and is driven by the oligopolic and, in some regions, monopolic structure of competition. Reducing total sales volume could increase competition and competitor market share, thereby threatening Company X's dominance and raising the elasticity of prices. Such market dynamics are not captured by the given formulation.

The most interesting question for future study then is: what is the balance point between driving sales growth and maximizing profits? How do you balance and prioritize the two? By increasing market dominance, price elasticities are potentially lowered thereby allowing price increases. This of course could be negated if consumer choice models are heavily guided towards magnet price points, like \$100 and \$200, If a product

is then priced at \$95 with an indicated low elasticity at that price point, the elasticity might increase drastically in the range right above \$100 thereby eliminating many potential benefits of incremental price increase past \$100. Within company X, the priority is sales growth, but the model seems to indicate that at some level of market penetration, perhaps the current level, Company X should change its pricing strategy to maximize profits. Intuitively, such a solution may not support long term sales growth, but it speaks strongly for the elimination of short term promotional activity.

4.3 EXTENSIONS

The choice of price elasticity model used in this thesis was motivated primarily by both a desire to conform to Company X's demand planning models and to constrain the solution space of the mixed integer program. The effect of price on demand could be more accurately modeled by considering the initial effect on retailer demand, the possible effect on retail prices, and the eventual time displaced effect on consumer demand. Such a price model would prove useful to most manufacturing companies who can only leverage pricing changes through the retailer by both changing wholesale prices and suggesting retail price changes. Retail demand could be modeled more accurately by considering the gaming effects of the retailers and accounting for stochastic demand and price elasticity.

Working with real data, it is very difficult to ever construct a price elasticity that captures all possible real effects and agent interactions, but there are several ways in which research could produce a more accurately descriptive model and hence more

precise solutions. This model assumed instantaneous price elasticity with no intertemporal effects. No consumer choice model or assumptions about rational consumers were made. In the actual data there is evidence of gaming and delayed purchasing decisions so as to take advantage of price drops. In this case, the buyers are retail stores with a vested interest in optimum purchase timing. With highly perishable products such as those studied in this case, the effects are somewhat diluted. Buyers can at most build two to three weeks of inventory.

Demand and price promotions are seasonal, so a more accurate accounting of costs should take into account the seasonality of the energy costs and the costs of materials. A stochastic demand formulation may also provide further insight and would be critical to the proper implementation of a pricing model. Another key supply chain consideration is the potential for fixed cost reduction as a percent of revenue with stabilized demand, with greater demand stability, although in the base case, the demand was only slightly more stable. Stable prices could also potentially increase forecast accuracy, as the current elasticity model is certainly not capturing all of the price dynamics.

4.4 SUMMARY

There is significant opportunity for either profit improvement or sales growth through the use of Dynamic Pricing strategies. Initial results indicate that there is a trade-off between the two in the short term, although strategically it makes sense to assume that long term profit growth is dependent on driving up sales volume. This may be better driven by

stabilizing the pricing path at a lower base, so as to drive volume, or at a slightly higher base, so as to drive profitability. The key factor that will actually drive the decision process is the accuracy of the price elasticity model. If the current elasticity model holds, then temporary price promotions yield absolutely no long term benefit, and prices should be pushed upwards or stabilized without upsetting market dynamics. If greater intertemporal effects are observed, then this could potentially have a downward effect on prices, as a model in which price changes had long term effect could indicate that lower prices are the key to long term growth prospects.

Pricing decisions are strategic to any company, and any model is at best an approximation of reality. Final validation depends on whether the model results align with company policy. In this case, the model serves to bring the tradeoffs of current policy into focus, allowing more informed decision making and better strategic guidance.

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APPENDIX A

Base	Plant 1					Plant 2				
	1	2	3	4	5	1	2	3	4	5
1	43291.88	0	270111.1	2034	31446	15123.28	0	146893.6	0	20428.75
2	65391.56	0	358692	0	58491	15695.63	0	162423.5	0	16653.75
3	114412	0	407999.2	0	59568	25513.13	0	212365.2	0	25376.25
4	93524.06	0	321697.1	1194	57921.75	9796.875	0	167051	0	23973.75
5	86176.41	0	304370.3	1128	33447	14432.34	0	188204.4	0	27836.25
6	83649.84	0	267120.6	1080	35452.5	15442.97	0	158389.4	0	24746.25
7	81143.91	0	327692.3	1116	27105	23930.16	0	209067.7	0	34147.5
8	79937.34	0	267472	1512	58736.25	11539.69	0	171515.3	0	25683.75
9	83752.97	0	317638.7	1122	57442.5	20501.25	0	219391.4	0	36266.25
10	127405.8	0	419059	648	76305	17887.03	0	245652.3	0	35167.5
11	50412.66	0	292387.9	918	38013.75	23888.91	0	223306.8	0	34357.5
12	101222.3	0	327810.2	648	29010	15112.97	0	294361.4	0	37042.5

Demand 2	Plant 1					Plant 2				
	1	2	3	4	5	1	2	3	4	5
1	58446.09	0	271504.8	864	35831.25	13535.16	0	252440.3	0	20212.5
2	79876.56	0	356222.9	864	25835.5	13581.56	0	271784.6	0	21626.25
3	126378.6	0	387078.5	1242	56731.25	14298.28	0	246262.5	0	19833.75
4	86975.63	0	351675.1	648	29006.25	11250.94	0	199840	0	24123.75
5	86068.13	0	350891.7	1296	35787	13519.69	0	221475.4	0	17587.5
6	83500.31	0	342784.3	804	42952.5	13236.09	0	173424.1	0	21652.5
7	125394.8	0	399022.5	1362	48138.75	20532.19	0	205293.1	0	24690
8	51232.5	0	202930.6	0	42555	9147.188	0	211775.2	0	17943.75
9	84701.72	0	388666.5	0	60648	19753.59	0	266532.6	0	40923.75
10	100608.8	0	438912	1296	55996.5	12271.88	0	251285.9	0	23283.75
11	102898.1	0	403220.8	708	62160.75	16742.34	0	255356.7	0	30251.25
12	73791.09	0	406985.2	0	63682.5	7450.781	0	168481	0	27585

Demand 3	Plant 1					Plant 2				
	1	2	3	4	5	1	2	3	4	5
1	78039.84	85642.45	371433.3	0	21607.5	12142.97	0	150633.7	0	12667.5
2	65391.56	65781.86	312340.2	1296	44820	13963.13	0	223165.9	0	21345
3	114412	102509.3	317347.5	432	19503.75	17196.09	0	258606.7	0	26823.75
4	93524.06	48968.17	312197.3	2592	23160	13999.22	0	210378.4	0	26028.75
5	86176.41	3493.548	489068.4	0	69386.25	22460.63	0	235526.2	0	42123.75
6	83649.84	0	465508.8	1866	71501.25	17402.34	0	320453.9	0	36352.5
7	81143.91	0	568801.8	1176	86490	10565.16	0	328090.2	0	39075
8	79937.34	0	176515.8	648	25998.75	15123.28	0	154968.6	0	22567.5
9	83752.97	44799.31	252992.8	0	28417.5	15360.47	0	181587.9	0	17175
10	127405.8	0	336754.5	2328	10245	14550.94	0	226511.7	0	20418.75
11	50412.66	0	409475	0	35208.75	14550.94	0	225004.9	0	20782.5
12	101222.3	0	389783.8	1572	36993.75	15566.72	0	216001.2	0	23111.25

APPENDIX B

	Original Data
Revenue	15,667,932.79
Costs	5,654,327.22
Profit	10,013,605.57
Aggregate Period to Period Demand Std Dev as % of Mean	19.29%
Total Production	8,395,447.33

	Base Case
Revenue	14,448,623.23
Costs	3,457,354.91
Profit	10,991,268.32
Improvement	9.76%
Aggregate Period to Period Demand Std Dev as % of Mean	19.19%
Total Production	4,959,318.03

	Original Data			Prices	
	1	2	3	4	5
1	2.603616	2.857143	1.564247	3	3.644163
2	2.612899	2.857143	1.667828	3	3.670662
3	2.671898	2.857143	1.49166	3	3.634712
4	2.199354	2.857143	1.381595	3	3.658895
5	2.587696	2.857143	1.720064	3	3.699341
6	2.551204	2.857143	1.723613	3	3.699341
7	2.58494	2.857143	1.777496	3	3.699341
8	2.599708	2.857143	1.718707	3	3.699341
9	2.597604	2.857143	1.540739	3	3.699341
10	2.620028	2.857143	1.56798	3	3.706029
11	2.701232	2.857143	1.512375	3	3.22678
12	2.608994	2.857143	1.497362	3	3.185544

t	Base Case			Prices	
	1	2	3	4	5
1	4.121549	1.142857	2.496701	4.8	4.289807
2	4.121549	1.142857	2.496701	1.2	4.289807
3	4.121549	1.142857	2.496701	1.2	4.289807
4	4.121549	1.142857	2.496701	4.8	4.289807
5	4.121549	1.142857	2.496701	4.8	4.289807
6	4.121549	1.142857	2.496701	4.8	4.289807
7	4.121549	1.142857	2.496701	4.8	4.289807
8	4.121549	1.142857	2.496701	4.8	4.289807
9	4.121549	1.142857	2.496701	4.8	4.289807
10	4.121549	1.142857	2.496701	4.8	4.289807
11	4.121549	1.142857	2.496701	4.8	4.289807
12	4.121549	1.142857	2.496701	4.8	4.289807

