

# 2.672 Experiment Design: Heat Sink Fin Configurations

by

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Submitted to the Department of Mechanical Engineering  
in partial fulfillment of the requirements for the degree of

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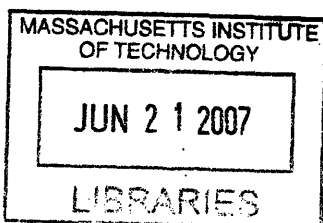
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### **Abstract**

2.672 is an undergraduate mechanical engineering laboratory course which requires students to solve real-world problems using both theoretical calculations and laboratory experiments. Many of the experiments currently in the laboratory have aged and their replacement presents an opportunity for the introduction of a new experiment. In this proposed experiment, students will optimize a heat sink for a certain type of rack-mount server. For a correct execution of the experiment, students will test the power dissipation of several different heat sinks against a model for how they should behave using principles of incompressible flow, extended surfaces and heat exchangers. An apparatus has been designed and constructed to simulate the air duct inside one possible server, and allow for measurements to be taken of power dissipation, temperature, and pressure in the duct. Seven different heat sink configurations were chosen to provide students with insight into how each parameter alters the effectiveness of the heat sink. Students are then asked to choose the parameters which give the optimal configuration.

Thesis Supervisor: Douglas P. Hart

Title: Professor of Mechanical Engineering

## Acknowledgments

I would first like to thank Professor Doug Hart for suggesting this project for undergraduate thesis work, and for providing well-needed guidance during the design process.

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# Chapter 1

## Introduction

### 1.1 2.672 Course Description

Project Laboratory in Mechanical Engineering, course number 2.672, is a laboratory subject for engineering juniors and seniors. Major emphasis is placed on the interplay between analytical and experimental methods in solution of research and development problems. Written and oral communication are strong components of the course[[2]

Over the course of the term, students complete three laboratory projects posed as engineering consulting problems. Students are instructed to design an analytical model and use the available apparatuses and measurement equipment to support the predictions of the model. In most cases, students are then requested to make recommendations regarding the design and implementation of new systems for (what is implied to be) a real-world application.

### 1.2 Purpose of New Experiments

The selection of experiments currently in place in the 2.672 laboratory provide interesting challenges for students in the class. However, it has been suggested by several faculty that it is time for new apparatuses or new experiments altogether. The field of mechanical engineering is ever-changing, with new technologies requiring a mechanical engineer's expertise as well as new tools and techniques at the engineer's disposal.

This is not to mention the wear and tear some of the set-ups show after years of use.

### 1.3 Proposed Project

As processors have been made with more and more processors and made to operate at higher and higher speeds, they have also produced more and more heat. Processors run more efficiently at lower temperatures, so fast computers depend on being able to remove heat quickly from the processor.

A popular method of removing heat from a processor is by using a heat sink. Heat sinks are finned surfaces which increase convective heat transfer by increasing the overall surface area over which such heat transfer can occur. Design of a good heat sink has evolved into something of an art form. Designers must have an understanding of air flow, extended surface approximations, and heat exchangers. Heat sinks are also subject to volume restrictions, since they must fit inside the case of a computer.

In this proposed experiment, students will be asked to find an optimal configuration for a fin array. The setup will be a simplified version of a standard rack-mount server (which contains a processor just like a PC does). Some simplifications will include air flow constrained to one direction, an air duct of simple, uniform cross-section, and a fin array with a limited number of parameters to change. Students will be expected to come up with a model to predict the overall thermal resistance for a heat sink as a function of certain parameters. They will be provided with a selection of several sample fin arrays to test on the apparatus which they can use to verify their model. Finally, they will be asked to find the optimal parameters for a fin array of this type, which should look something like what is shown in figure 1-1.

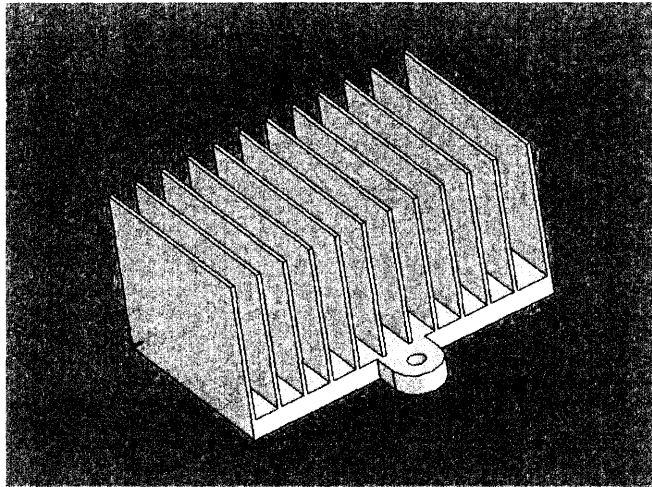


Figure 1-1: Solid model of a possible heat sink to be provided for students to test. The heat sinks will come with a variety of fin thicknesses, numbers of fins, and overall lengths.

# Chapter 2

## Theory

### 2.1 Fins

Extended surfaces in the form of fins can be used to increase the amount of convection heat transfer by increasing the surface area over which convection occurs. They are interesting configurations to model in part because convection occurs in a direction perpendicular to that of conduction. The part of the fin closer to the high-temperature base will, naturally, be hotter than the part at the tip of the fin. Thus, greater heat transfer due to convection occurs near the base than near the tip. For an in-depth analysis of the conduction and convection equations for a fin, see Dewitt[1]. For the assumption that the tip of the fin is adiabatic, that is, if  $d\theta/dx = 0$  at  $x = L$ , the temperature distribution  $\theta(x)$  is given by

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L - x)}{\cosh mL}, \quad (2.1)$$

where  $\theta$  is the difference between the temperature of the fins and temperature of the surroundings. Also,  $\theta_b$  is the value of  $\theta$  at the base of the fin,  $L$  is the length of the fin from base to tip, and  $m$  is given by

$$m \equiv \sqrt{\frac{hP}{kA_c}}. \quad (2.2)$$

The heat transfer rate is then given by (for a derivation see [1])

$$q_f = M \tanh mL, \quad (2.3)$$

where  $M$  is given by

$$M \equiv \sqrt{hPkA_c\theta_b}. \quad (2.4)$$

In equations 2.2 and 2.4,  $h$  is the convection heat transfer coefficient,  $P$  is the perimeter of the fin drawn in the plane parallel to the base,  $k$  is the thermal conductivity of the fin material, and  $A_c$  is the cross-sectional area of the fin ( $A_c$  is assumed to be constant for the entire fin).

It is desirable to have some measure for how well a fin is doing to increase heat transfer. There are two useful measures for this: fin effectiveness and fin efficiency. Fin effectiveness is defined as the ratio of heat transfer from the fin to the heat transfer that would have occurred had there been no fin present, and it is given by the formula

$$\epsilon_f \equiv \frac{q_f}{hA_{c,b}\theta_b}, \quad (2.5)$$

where  $A_{c,b}$  is the fin cross-sectional area at the base. Fin effectiveness should be greater than 1. Fin efficiency is defined as the ratio of heat transfer with the fin to the heat transfer that would occur if the entire fin were at the temperature of the base of the fin. It is given by the formula

$$\eta_f \equiv \frac{q_f}{hA_f\theta_b}. \quad (2.6)$$

## 2.2 Incompressible Flow

Air flow across the fin array will be driven by a fan at one end of the duct. The fan creates a pressure differential across the fin array which forces air to flow. While the fan curve (the relationship between volumetric air flow and pressure drop across the

fan, a characteristic the fan installed) is not linear, we model it as

$$Q = Q_{max} - \frac{Q_{max}}{\Delta P_{max}} \Delta P, \quad (2.7)$$

where  $Q_{max}$  is the airflow across the fan when there is zero pressure drop and  $\Delta P_{max}$  is the pressure drop across the fan when air flow is zero.

To find the actual pressure drop across the flow rate, an equation relating the air flow to pressure drop for the fin array is needed. For this equation, we assume that all pressure drop takes place across the fin array, and we look at the gaps between fins through which air is allowed to flow. Looking at just one gap, we find the hydraulic diameter,  $D_h$  for the gap as

$$D_h = \frac{4A_{gap}}{\mathcal{P}_{gap}}. \quad (2.8)$$

For viscous flow (flow through the small gaps between fins is certainly viscous, but we'll make sure by looking at the Reynolds number), the Darcy-Weisbach friction factor  $f$  can be used to determine the relationship between flow velocity and pressure drop[3].

$$f = \frac{\left(-\frac{dP}{dx}\right) D_h}{\frac{1}{2}\rho\vartheta^2} \quad (2.9)$$

In (2.9),  $dP/dx$  is the pressure drop per unit length past the fin and  $\vartheta$  is the velocity of air. For the gap width used for the heat sinks, it is appropriate to use the approximation that the gap is a long rectangular slit. This implies the friction factor is also given by

$$f = \frac{96}{Re} = \frac{96\mu}{\rho_{air}\vartheta D_h}, \quad (2.10)$$

where  $Re$  is the Reynolds number,  $\mu$  is the viscosity of air, and  $\rho_{air}$  is the density of air. From the friction factor equation and the fan curve, we can solve for the flow rate and pressure drop through the fin array. Noting that

$$Q = nA_{gap}\vartheta, \quad (2.11)$$

where  $A_{gap}$  is the gap width times the distance from the base of the fin to its tip and

$n$  is the number of gaps between fins, we solve the system of equations for  $\Delta P$  and  $Q$ .

## 2.3 Heat Exchange

To calculate the overall resistance to heat transfer, we must first calculate  $UA$ , which is given by

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o} \quad (2.12)$$

This equation assumes that the fouling factor of the heat exchanger is zero, that is, all surfaces inside the heat exchanger are clean. We can use the equation

$$NTU = \frac{UA}{C_{min}}, \quad (2.13)$$

where  $NTU$  is the number of thermal units, an engineering characteristic used to calculate the amount of heat transfer which occurs, and  $C_{min}$  is the smaller of the two heat capacities of the two working fluids. In our case,  $C_{min}$  will just be the specific heat capacity of air times the air flow rate, because the hot side of the heat exchanger (the aluminum) doesn't have an actual flow rate associated with it. To calculate resistance we first must use the equation

$$\epsilon = 1 - e^{-NTU}, \quad (2.14)$$

where  $\epsilon$  is the fraction of actual heat transfer to the theoretical maximum calculated from the flow rate and  $C_{min}$ . To calculate resistance, we can simply use

$$R_{overall} = \frac{1}{\epsilon C_{min}}, \quad (2.15)$$

where  $R_{overall}$  is the resistance to heat transfer for the heat exchanger for a given temperature difference.



## 2.4 Optimization

Optimization of a function over four variable can be done in a simple way as long as the function is well-behaved. First, three of the parameters are chosen from ballpark ideas of what they should be, and the function is minimized over the remaining free variable. Then, the function is minimized over the second variable with the first variable chosen from before. The process is repeated for the other two variables, and then several more iterations are performed, each leading to a better value for the function. The process ends when the value for the function is unchanged (or very close to unchanged) after a whole iteration.

# Chapter 3

## Design of Apparatus

One intent in the design of the apparatus is to convey the impression that the students are experimenting with a rack-mount server. The length and width of the box are approximately in line with a server of this type. However, the apparatus is much thicker due to the sizes of some of the electronic parts which are contained within. The height of the duct, on which the students focus most of their efforts, however, is comparable to the height of a typical rack-mount server.

### 3.1 Choice of Components

#### 3.1.1 Materials Selection

The case of the server is made of Lexan, a strong, non-conducting polymer. Making the case out of Lexan, particularly the part of the case near the fin array, prevents heat transfer from the heat source to the environment via the case. In other words, having a non-metallic case makes it easy to assume that all heat transfer from the heat source to the environment occurs via the fin array.

The square plate meant to represent the processor is made of copper. In a real server, the heat is generated by this plate itself (because the processor runs) and not by some other heat source. The choice of copper for the plate was necessary to simulate the processor as a source of heat as accurately as possible. Copper has a

Material	Thermal Conductivity (W/mK)	fin width	gap width	fin array length
Al 6061	237	.295 mm	.79 mm	59 mm
Cu 110	388	.25 mm	.83 mm	59 mm
Brass 360	115	.319 mm	.72 mm	57 mm
Steel 1018	51.9	.29 mm	.61 mm	55 mm
Stainless 316	16.3	.2 mm	.44 mm	51 mm

Table 3.1: Comparison of optimal geometric configurations for different heat sink materials.

high thermal conductivity which will encourage the temperature across the surface of the plate to be as uniform as possible.

### 3.1.2 Fin Array Material Selection

The thermal conductivity of the material that the fin array is made of has a significant effect on how well the heat sink works. We can optimize the parameters for a heat sink just by plugging in different thermal conductivities. The results are summarized in table 3.1. This analysis encourages the use of brass as a heat sink material because it yields the thickest fins in the optimal case. Since the heat sinks will be cut on the wire EDM, it is preferable to 6061 aluminum because of the absence of non-metallic elements.

### 3.1.3 Heat Generation and Control Components

For the power resistor we chose the SOT227 package power resistor which will be operating at a maximum heat dissipation rate of 144W. The power resistor is attached to the copper plate by screws, and thermal grease is applied at the interface between the two components to provide as low a thermal resistance as possible. The copper plate is machined with as good a finish as possible to keep the small gaps between it and the power resistor small.

Our temperature controller depends on an accurate measurement of the temperature of the power resistor. For this purpose we use an LM35 temperature sensor. However, it is not feasible to mount the sensor directly to the power resistor, so we do the next best thing. As shown in figure 3-1, the LM35 is mounted on the under



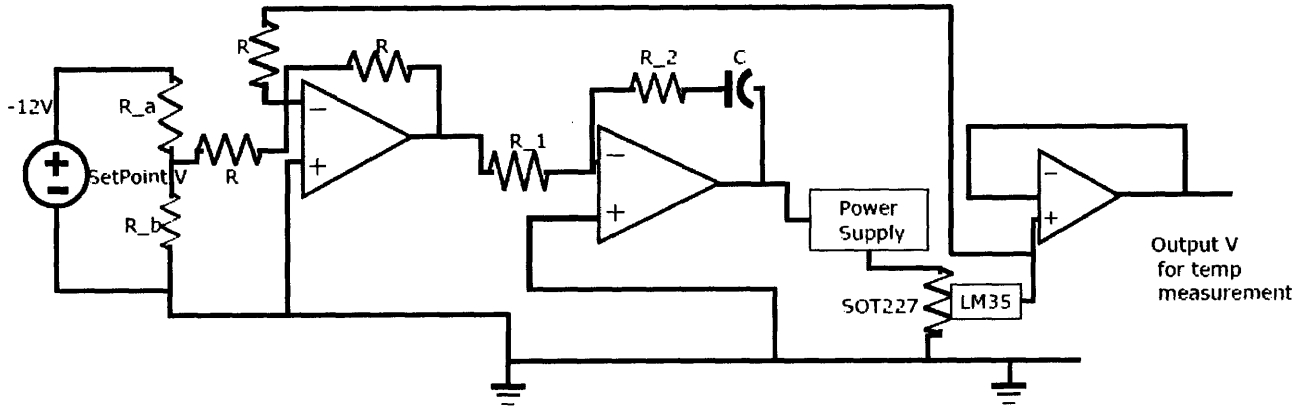


Figure 3-2: Circuit diagram for the temperature controller. The input voltages for the op-amps and LM35 are not shown. The power supply accepts voltages between 0V and 12V. The setpoint voltage could also be set using a variable resistor (effectively providing an  $R_a$  and  $R_b$ ).

without a heat sink attached, the time constant of the system is small and making the temperature hard to control. In order to design an appropriate temperature controller, a good thermal model of the system is needed. The controller will be designed to work in a worst-case scenario: with the fan off, no heat sink attached, and the cover of the duct closed. It will then be tested to make sure it works well under normal operating conditions.

### 3.2.1 Thermal Model

Heat will move by conduction from the power resistor to the copper plate to the fin array, and finally to the air (or straight from the copper plate to the air in the case without the fin array attached). Temperature will be measured at the copper plate, so the measured temperature will be lower than the temperature of the power resistor.

The thermal model is simulated using Simulink. Graphical representations of the models, as well as the input parameters used, can be found in the Appendices. A proportional-plus-integral controller is then inserted into the Simulink model for testing. Results of the simulated tests with the controller included are shown in figures 3-3, 3-4, and 3-5.

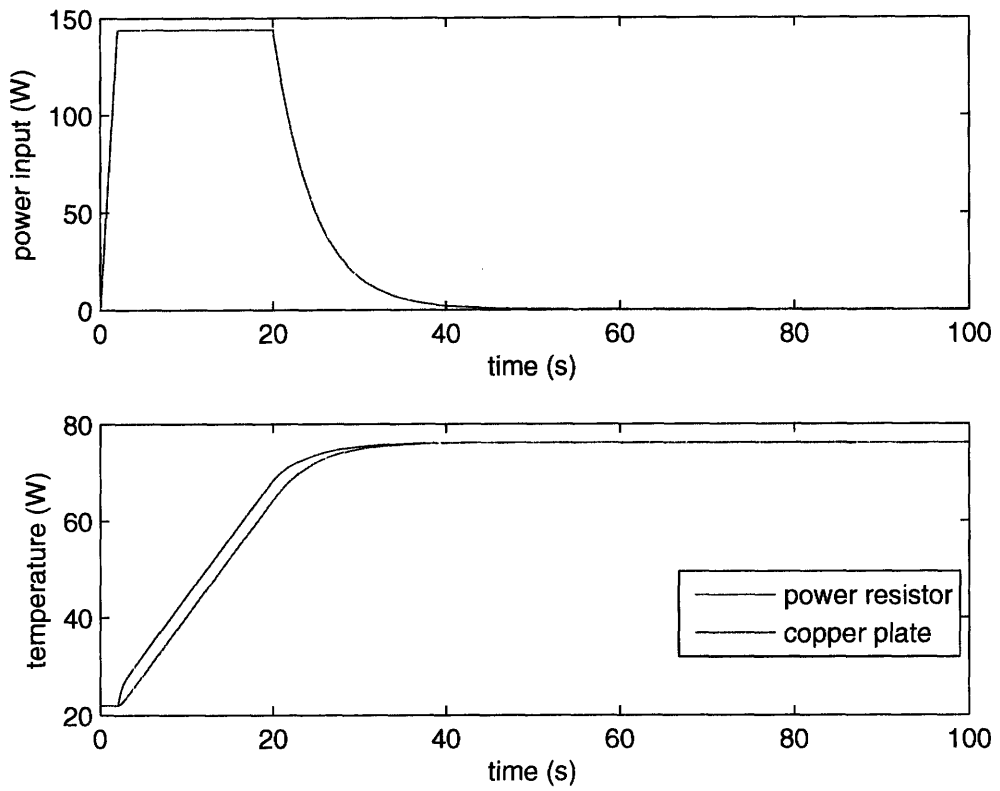


Figure 3-3: Plot showing simulation of case when the fan is off and the fin array is unattached. Once the copper plate reaches the set point, the power supply shuts off to allow the plate to cool. Overshoot is less than 2 degrees C.

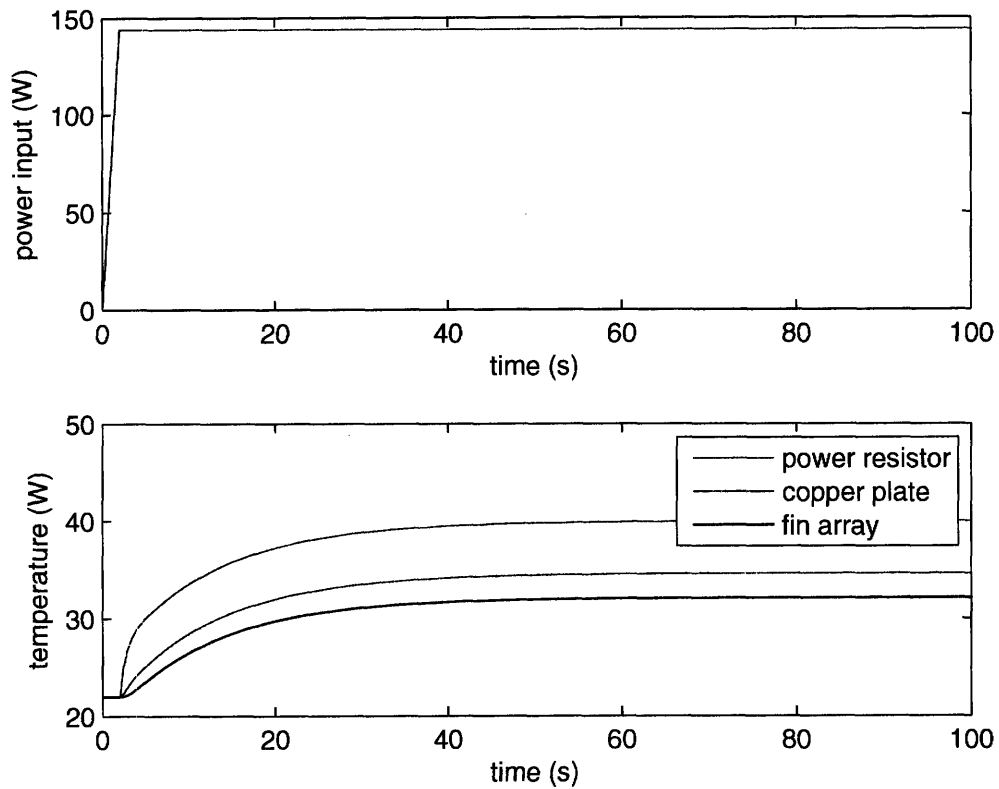


Figure 3-4: Plot showing simulation of case when the fan is on and a fin array with low resistance value is attached. The power supply cannot provide enough power to reach the setpoint temperature, so the power stays on at its maximum value and the copper plate reaches a steady state temperature lower than the set point. The temperature differences between the power resistor, copper plate, and fin array are due to the thermal resistance between them.

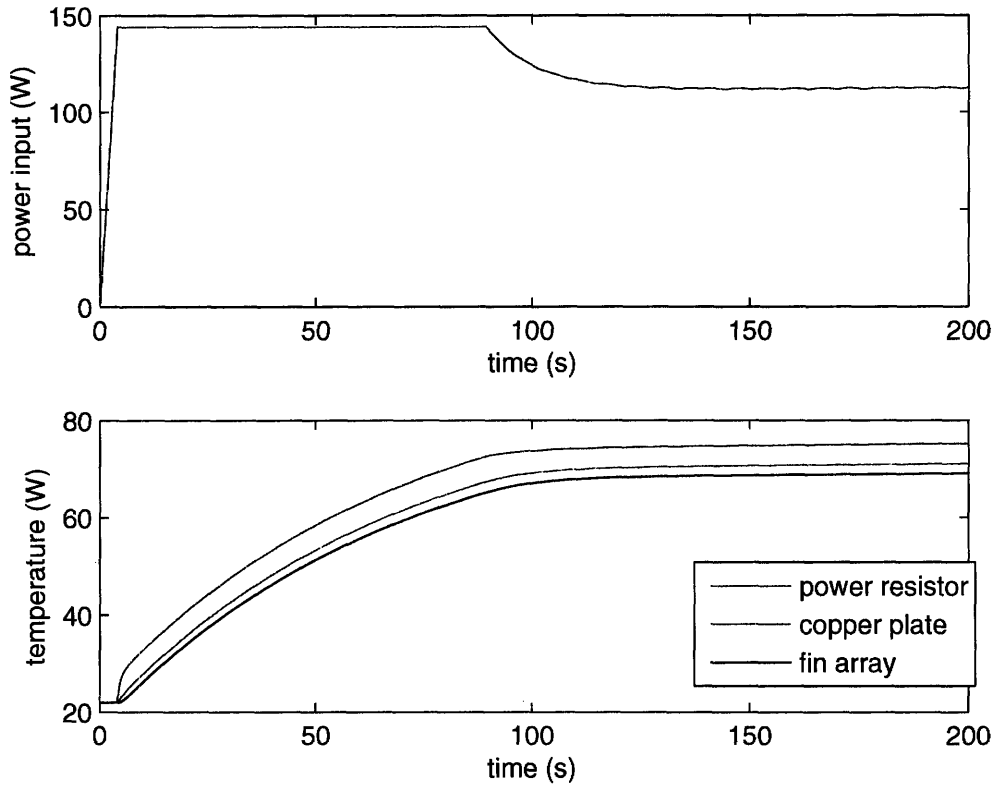


Figure 3-5: Plot showing simulation of case when the fan is on and a fin array with fairly high resistance value is attached. The power supply runs at maximum and then settles to a lower steady state once the copper plate reaches the setpoint temperature. There is no overshoot in this case.

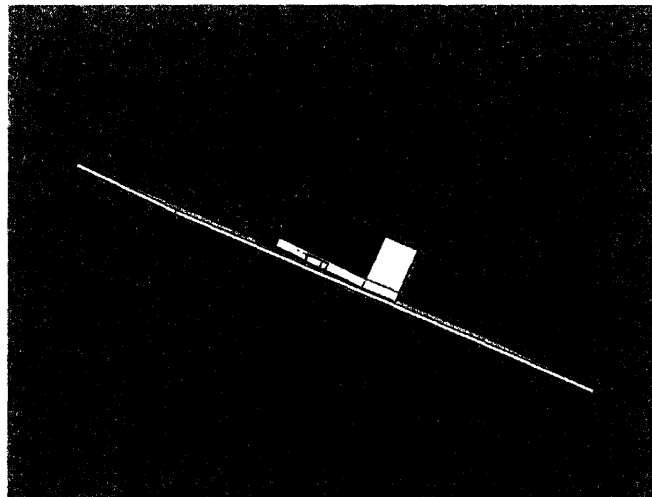


Figure 3-6: Solid model showing the fin array mounted atop the copper plate, as it will be in the duct.



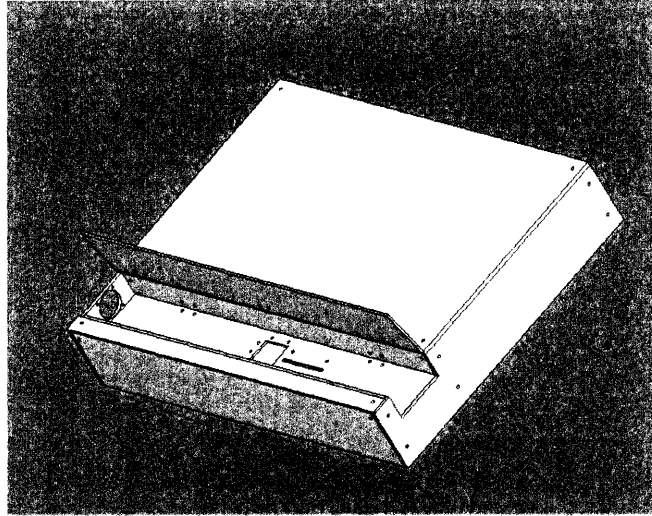


Figure 3-7: Solid model of the overall box for the apparatus.

### 3.3 Overall Apparatus Design

The experiment is to be presented to students in the form of a box. This box is supposed to be of comparative size of a typical rack-mount server, since the goal of the experiment is to optimize a heat sink for such a server. The server box will contain a heat source meant to be of similar size, shape, and temperature of a the kind of processor which would run inside a server. The heat source will be simulated by a square copper plate 50mm on each side with a power resistor on the bottom, whose power output will be controlled by the temperature controller described in section 3.2. Like a typical server box, a fan will provide air flow over the heat sink. However, there are several key features not present in a server.

One feature is the presence of a hinged door on the top of the box. This door allows access to the heat sink so that heat sinks can be easily interchanged. Other features include devices to measure temperature, pressure, and power. The case of the “server” is clear to allow the students to see the electronics on the inside.

#### 3.3.1 Sealing the Duct

When the door is opened, the air duct through which air forced by the fan passes through the heat sink is opened to the atmosphere. If the fan is run in this configura-

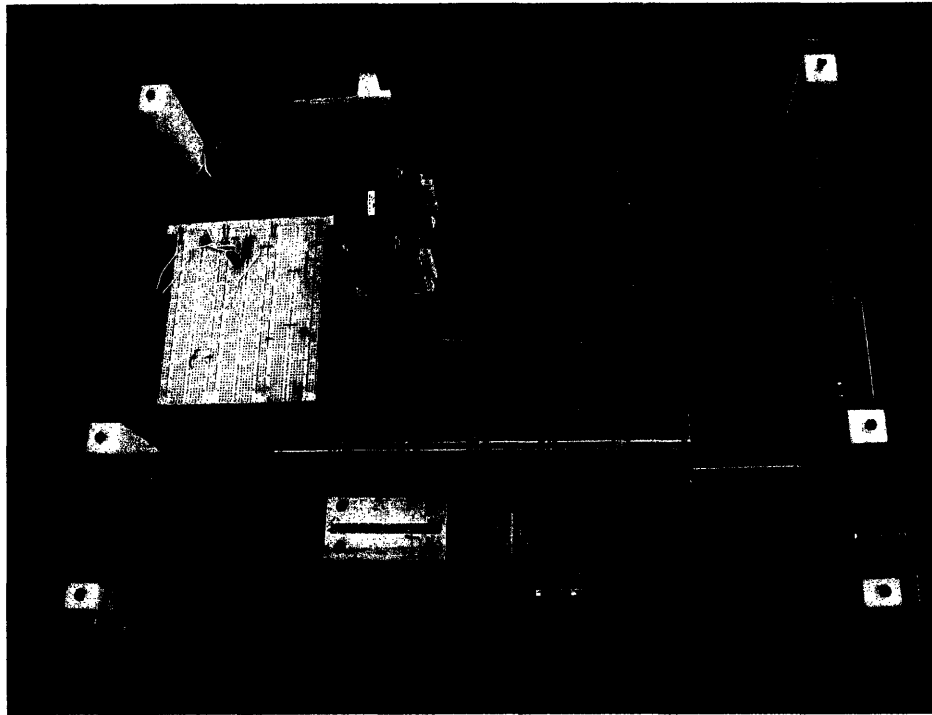


Figure 3-8: Overhead picture of the partially-assembled, unwired apparatus.

tion, very little air will pass through the fin array because the pressure drop required to force air through is high compared to forcing air over and around the array. Since the fan causes a pressure increase upwind of the fin array, that part of the duct must be sealed to the atmosphere to insure that all of the pressure from the fan is used to push air through the array, and not to push air out of the duct through other exits.

There are several exits which must be sealed. One is the space between the top hinged door and the side of the duct. This exit is sealed with a foam rubber strip which, when compressed, provides a perfect seal. To keep the foam rubber compressed, we include clasps to hold the door tightly shut. Another exit is through the bottom of the duct. The heat sink is held in place by screws into the bottom, which are placed in holes in the bottom of the duct. To prevent air from escaping there, aluminum blocks with foam rubber on the edges are placed underneath the duct to provide sinks for the screws.

Finally, for all other gaps in the walls in the duct, we apply a clear silicone glue/sealant to ensure that the duct is air-tight.



Figure 3-9: This picture shows the duct exit. The silicone glue used to seal the duct is visible.



Figure 3-10: This picture shows the fan attached at the entrance of the duct.



Figure 3-11: Picture showing where the heat sink will be attached. The slot on the left allows for attaching heat sinks of different length.

### **3.3.2 Heat Sink Attachment**

An important feature of the design involves how students will go about attaching different heat sinks to the copper plate. They must be able to ensure a solid thermal connection with the copper plate. This is done by screwing the fin array directly into the bottom of the duct.

However, knowing where to drill the holes in the duct is trickier, since the heat sinks will have different lengths. Instead of many different holes, a t-slot is available to screw one end of the heat sink into. Inside the t-slot is a square nut which catches the screw. This nut is free to slide back and forth in the slot. Figure 3-11 shows the result.

# Chapter 4

## Analytical Model

### 4.1 Choice of Fin Configurations for Experiment

The fin configurations were chosen so that students would have at least two heat sinks to compare with three of the variables fixed and only one varied. They were chosen by looking at graphs of the resistance as a function of one variable. Figure 4-1 shows an example of this method.

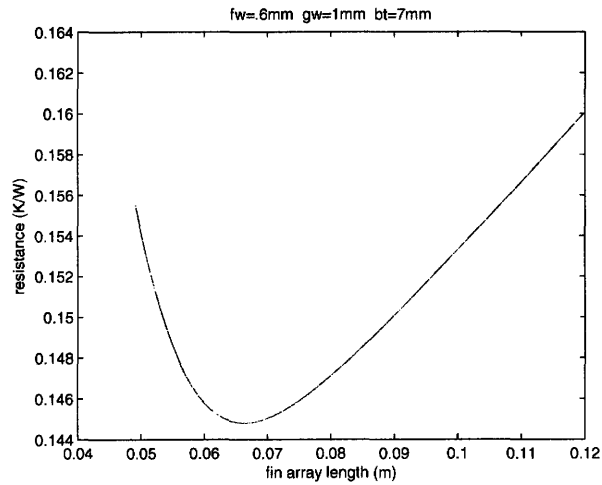


Figure 4-1: Graph showing resistance as a function of fin array length. In this case, base thickness is 7mm, fin thickness is .6mm, and gap thickness is 1mm. From the graph, we chose fin lengths of 50mm, 66mm, and 90mm so that students could see that the fin array length should have some overhang beyond the heat source, but not too much overhang

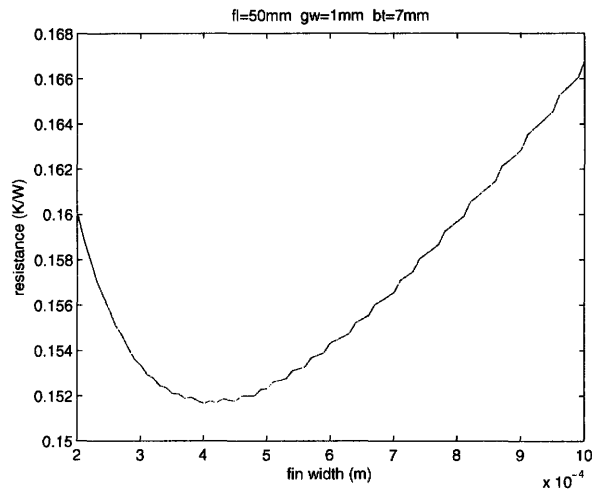


Figure 4-2: Graph showing resistance as a function of fin width. In this case, base thickness is 7mm, fin array length 50mm, and gap thickness is 1mm. From the graph, we chose fin widths of .4mm and .6. A fin width smaller than .4mm would have been chosen except the fins become too fragile at that thickness.

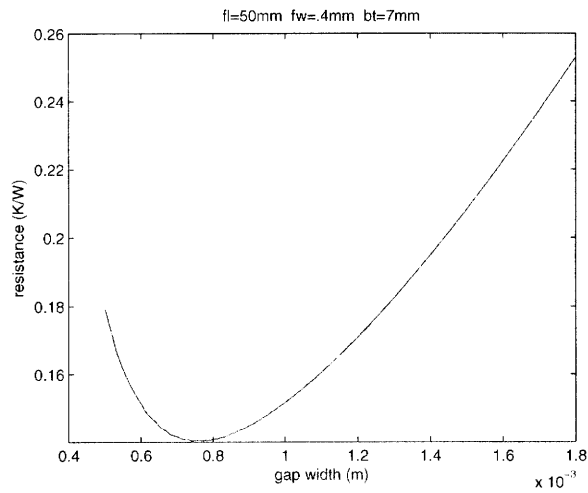


Figure 4-3: Graph showing resistance as a function of gap width. In this case, base thickness is 7mm, fin array length 50mm, and fin thickness is .4mm. From the graph, we chose gap widths of 1mm and 1.5mm.

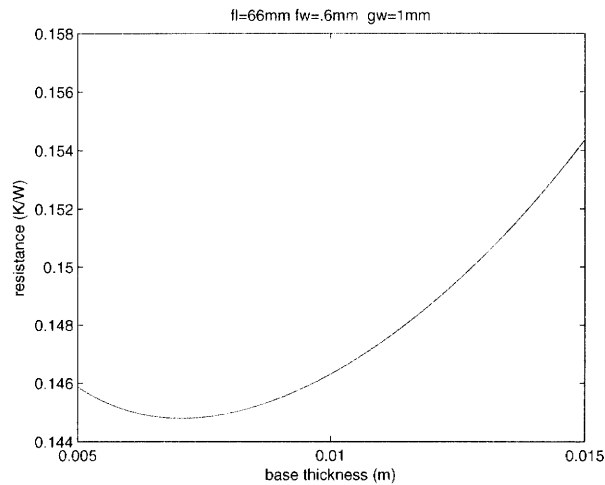


Figure 4-4: Graph showing resistance as a function of base thickness. In this case, fin array length 66mm, fin thickness is .6mm, and gap width is 1mm. From the graph, we chose base thicknesses of 5.5mm and 10mm to go with the configuration that already was 7mm. This will help students get an idea of the optimal base thickness for the other variables given.

# Chapter 5

## Simulated Results

The following results were simulated in MATLAB by taking the predicted results and adding a randomly distributed error term. For the resistance measurements, an addition constant term was added to each resistance because of resistance from the copper plate to the fin array. In the actual experiment, the theoretical resistance will be subtracted from the experimental resistance to calculate this extra resistance from the thermal grease. This could also be calculated from the properties of the thermal grease, but the thickness of the grease between the surfaces is unknown due to variation in how well the students apply it and how good the surface finish is. Also note that the predictions were based on Aluminum fins, the material of the first fin array, but the fins used in the experiment will probably be brass but could be some other material.

### 5.1 Pressure Drop

The pressure readings provide a quick check of the incompressible flow aspect of the model. Since, theoretically, the pressure drop is just a function of the fan curve and the duct geometry (primarily the fin gap geometry), confirming the pressure model correct will give a fairly accurate volumetric flow rate.

The pressure data, compared to the predicted values, is printed in table 5.1. Some of the measurements have significant error, with the majority of the measured



Heat Sink Number	Measured Pressure (Pa)	Predicted Pressure (Pa)	Percent Diff.
1	53.6	54.6	1.8
2	25.8	25.8	-0.03
3	63.0	60.2	-4.6
4	63.7	73.8	14
5	85.3	90.9	6.1
6	58.3	71.4	18
7	73.9	79.2	6.7

Table 5.1: Comparison of measured to predicted pressure drops for each heat sink.

Heat Sink Number	Measured Resistance (K/W)	Predicted Resistance	Percent Diff.
1	0.206	0.152	40.8
2	0.263	0.208	28.7
3	0.209	0.154	41.3
4	0.200	0.145	41.4
5	0.205	0.150	41.4
6	0.200	0.145	40.7
7	0.201	0.146	39.6

Table 5.2: Comparison of measured resistance to predicted resistance values for the different heat sinks. The measured resistance was calculated by dividing steady state temperature by steady state power.

pressures lower than the predicted values. A possible explanation for this is that the fins in the fin arrays are not perfectly straight, so some of the spaces between fins are larger than they should be, letting more air pass through. It is unclear whether the fins are bent because of damage to them or because of the way they were manufactured.

## 5.2 Resistance

Resistances are not measured directly. Instead power dissipation and temperature is measured. Resistance is calculated using the difference in temperature between the copper plate and room temperature. Tables 5.2 and 5.3 summarize the differences between measured and predicted values.

H.S. Number	Measured Resistance (K/W)	Predicted Res (adjusted)	Percent Diff.
1	0.206	0.207	3.41
2	0.263	0.263	1.89
3	0.209	0.209	4.16
4	0.200	0.200	2.51
5	0.205	0.205	3.55
6	0.200	0.200	2.14
7	0.201	0.201	1.52

Table 5.3: Comparison of measured resistance to predicted resistance values for the different heat sinks. In this case, the predicted values have been adjusted to include resistance between the power resistor and the fin array. All the predicted resistances simply have a constant value of  $0.055 K/W$  added to them.

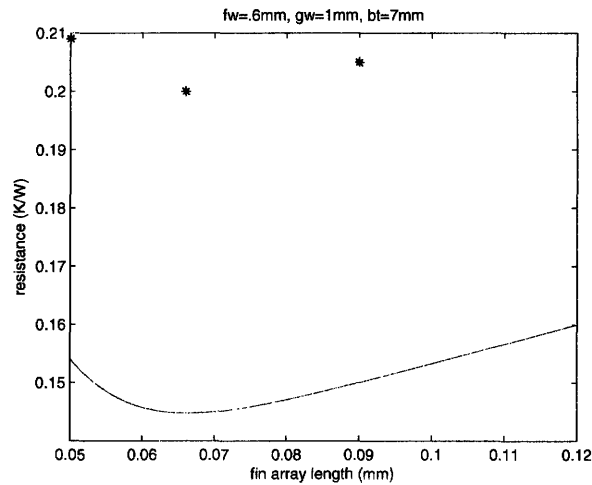


Figure 5-1: Graph showing theoretical resistance as a function of fin array length. In this case, base thickness is 7mm, fin thickness is .6mm, and gap thickness is 1mm. The experimental data (red stars) overlays the theoretical prediction.

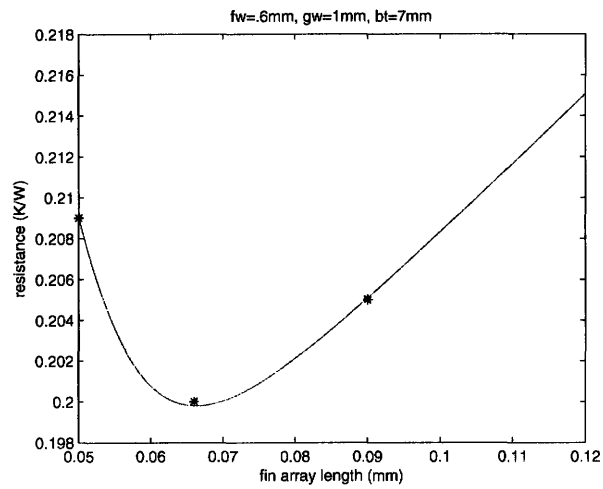


Figure 5-2: Graph showing adjusted theoretical resistance as a function of fin array length. In this case, base thickness is 7mm, fin thickness is .6mm, and gap thickness is 1mm. The adjustment is made by adding a resistance value for the resistance between the power resistor and the fin array, which is determined to be approximately .0055 K/W. After the adjustment, the measured data is closer to the prediction.

# Chapter 6

## Conclusions

### 6.1 Direction for Further Work

Since the heat sinks have not been tested, it is unclear whether or not they give good enough results in reality that will allow student to understand how varying different parameters affects the resistance. If the reality doesn't match up with the model closely enough, it might be advisable to make a new batch of heat sinks which conveys the best information to the students.

### 6.2 Suggestions for Improvement of Apparatus

The case suffers from imprecise construction, and has some aesthetic and structural problems which could be fixed by a rebuild. For example, the outer walls of the box don't line up properly, and the duct is held together more by glue than by the L-shaped braces that attach the bottom of the duct to the sides.

The temperature sensors are not good enough to measure the bulk temperature of exiting air because the of the temperature variation across the profile of the air flow. Maybe more sensors could be added and their outputs averaged.

The temperature controller is currently wired on a breadboard. It should be eventually done on a printed circuit board for a more permanent solution.

The hinge on the top flap is much too large.

The screws that hold the copper plate to the bottom of the duct should be countersunk so that dimples don't need to be drilled in the bottoms of heat sinks.

# Appendix A

## Sample 2.672 Lab Report

This appendix presents a demonstration lab writeup which might be written by a 2.672 student assigned to this experiment. Because it uses simulated data for heat sinks made of aluminum instead of brass and for parameters not actually used in their construction, it should not be used as a model lab report.

### A.1 Abstract

Fans and heat sinks are a popular method of keeping computer processors cool. In this report, certain parameters of a heat sink fin configuration are optimized for a particular rack-mount server. A model is developed to analyze how fin thickness, fin spacing, base thickness, and heat sink length affect the resistance to heat transfer of the fin array. Several different heat sinks are used to verify the model, and the optimum configuration is found using numerical optimization techniques. For the given fan specifications and duct dimensions, the optimum configuration is given by an array length of 59 mm, a fin thickness of .30 mm, a fin spacing of .79 mm, and a base thickness of 6.5 mm. The model predicted the experimental results with less than 15 percent error (in the resistance measurement).

## A.2 Introduction

Computer processors generate heat during operation, but their performance suffers when they operate at too high temperatures. They can even be destroyed if they overheat too much. In order for processors to run at high speeds, they need to be kept at low temperature by some sort of cooling system. One particular kind of cooling system, the kind investigated in this report, uses a heat sink with forced convection.

Heat sinks are finned surfaces which increase convective heat transfer by increasing the overall surface area over which such heat transfer can occur. Design of a good heat sink has evolved into something of an art form. Designers must have an understanding of air flow, extended surface approximations, and heat exchangers. Heat sinks are also subject to volume restrictions, since they must fit inside the case of a computer.

The purpose of this lab is to design an optimal heat sink for a given processor size, air duct, and fan characteristic. The material of the heat sink is restricted to aluminum (6061 alloy), but there are four geometric parameters which

## A.3 Apparatus and Procedure

The apparatus is made of clear Lexan and contains a heating element that must be kept cool. The system to be cooled consists of a processor of square shape, 48 cm on a side, situated in an air duct of a rack-mount server. On one end of the duct is a fan with fan curve shown in figure A.3.1; the other end of the duct is open to the atmosphere inside the server room. The duct is about 60 cm long, and its cross-section is approximately 8 cm by 4 cm (width and height, respectively). In order to investigate the system, a model of the server box is available, with a power resistor mounted to a copper plate to simulate the processor's heat generation.

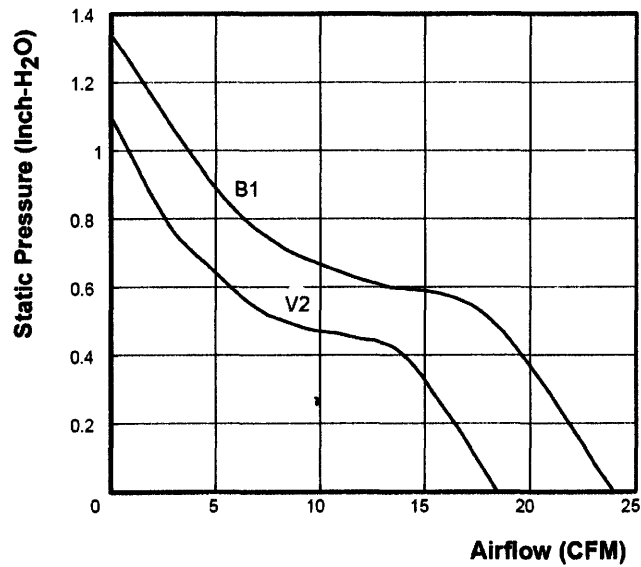


Figure A-1: Fan curve for the fan used in lab. The curve B1 is the correct one. The fan curve shows static pressure developed by the fan as a function of flow rate in cubic feet per minute.

### A.3.1 Sensors and Measurement

Power provided to the resistor and the temperature of the copper plate have analog outputs provided. Temperature is measured by an LM35 temperature sensor, which is already calibrated to give .01 Volts per degree Celsius. Pressure drop across the fin array is measured by a PC Board Mount pressure sensor with a range of -5 to 5 inches of  $H_2O$ .

### A.3.2 Procedure

Data was collected by attaching a heat sink to the duct and copper plate, closing the top flap, turning on the fan, and turning on the power to the resistor. After about two minutes, the temperature and power levels out to a steady state, and the values for power, temperature, and pressure drop are recorded. The power to the resistor is turned off and the apparatus is allowed to cool before removing the heat sink. A different heat sink is attached and the procedure is repeated.



## A.4 Theoretical Analysis

### A.4.1 Fins

Extended surfaces in the form of fins can be used to increase the amount of convection heat transfer by increasing the surface area over which convection occurs. They are interesting configurations to model in part because convection occurs in a direction perpendicular to that of conduction. The part of the fin closer to the high-temperature base will, naturally, be hotter than the part at the tip of the fin. Thus, greater heat transfer due to convection occurs near the base than near the tip. For an in-depth analysis of the conduction and convection equations for a fin, see Dewitt[1]. For the assumption that the tip of the fin is adiabatic, that is, if  $d\theta/dx = 0$  at  $x = L$ , the temperature distribution  $\theta(x)$  is given by

$$\theta/\theta_b = \frac{\cosh m(L - x)}{\cosh mL}, \quad (\text{A.1})$$

where  $\theta$  is the difference between the temperature of the fins and temperature of the surroundings. Also,  $\theta_b$  is the value of  $\theta$  at the base of the fin,  $L$  is the length of the fin from base to tip, and  $m$  is given by

$$m \equiv \sqrt{hP/kA_c}. \quad (\text{A.2})$$

The heat transfer rate is then given by (for a derivation see [1])

$$q_f = M \tanh mL, \quad (\text{A.3})$$

where  $M$  is given by

$$M \equiv \sqrt{hPkA_c\theta_b}. \quad (\text{A.4})$$

In equations A.2 and A.4,  $h$  is the convection heat transfer coefficient,  $P$  is the perimeter of the fin drawn in the plane parallel to the base,  $k$  is the thermal conductivity of the fin material, and  $A_c$  is the cross-sectional area of the fin ( $A_c$  is assumed to be

constant for the entire fin).

It is desirable to have some measure for how well a fin is doing to increase heat transfer. There are two useful measures for this: fin effectiveness and fin efficiency. Fin effectiveness is defined as the ratio of heat transfer from the fin to the heat transfer that would have occurred had there been no fin present, and it is given by the formula

$$\epsilon_f \equiv \frac{q_f}{hA_{c,b}\theta_b}, \quad (\text{A.5})$$

where  $A_{c,b}$  is the fin cross-sectional area at the base. Fin effectiveness should be greater than 1. Fin efficiency is defined as the ratio of heat transfer with the fin to the heat transfer that would occur if the entire fin were at the temperature of the base of the fin. It is given by the formula

$$\eta_f \equiv \frac{q_f}{hA_f\theta_b}. \quad (\text{A.6})$$

#### A.4.2 Incompressible Flow

Air flow across the fin array will be driven by a fan at one end of the duct. The fan creates a pressure differential across the fin array which forces air to flow. While the fan curve (the relationship between volumetric air flow and pressure drop across the fan, a characteristic the fan installed) is not linear, we model it as

$$Q = Q_{max} - \frac{Q_{max}}{\Delta P_{max}} \Delta P, \quad (\text{A.7})$$

where  $Q_{max}$  is the airflow across the fan when there is zero pressure drop and  $\Delta P_{max}$  is the pressure drop across the fan when air flow is zero.

To find the actual pressure drop across the flow rate, an equation relating the air flow to pressure drop for the fin array is needed. For this equation, we assume that all pressure drop takes place across the fin array, and we look at the gaps between fins through which air is allowed to flow. Looking at just one gap, we find the hydraulic

diameter,  $D_h$  for the gap as

$$D_h = \frac{4A_{gap}}{\mathcal{P}_{gap}}. \quad (\text{A.8})$$

For viscous flow (flow through the small gaps between fins is certainly viscous, but we'll make sure by looking at the Reynolds number), the Darcy-Weisbach friction factor  $f$  can be used to determine the relationship between flow velocity and pressure drop.

$$f = \frac{\left(-\frac{dP}{dx}\right) D_h}{\frac{1}{2}\rho\vartheta^2} \quad (\text{A.9})$$

In (A.9),  $dP/dx$  is the pressure drop per unit length past the fin and  $\vartheta$  is the velocity of air. For the gap width used for the heat sinks, it is appropriate to use the approximation that the gap is a long rectangular slit. This implies the friction factor is also given by

$$f = \frac{96}{Re} = \frac{96\mu}{\rho_{air}\vartheta D_h}, \quad (\text{A.10})$$

where  $Re$  is the Reynolds number,  $\mu$  is the viscosity of air, and  $\rho_{air}$  is the density of air. From the friction factor equation and the fan curve, we can solve for the flow rate and pressure drop through the fin array. Noting that

$$Q = nA_{gap}\vartheta, \quad (\text{A.11})$$

where  $A_{gap}$  is the gap width times the distance from the base of the fin to its tip and  $n$  is the number of gaps between fins, we solve the system of equations for  $\Delta P$  and  $Q$ .

### A.4.3 Heat Exchange

To calculate the overall resistance to heat transfer, we must first calculate  $UA$ , which is given by

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o} \quad (\text{A.12})$$

This equation assumes that the fouling factor of the heat exchanger is zero, that is, all surfaces inside the heat exchanger are clean. We can use the equation

$$NTU = \frac{UA}{C_{min}}, \quad (\text{A.13})$$

where  $NTU$  is the number of thermal units, an engineering characteristic used to calculate the amount of heat transfer which occurs, and  $C_{min}$  is the smaller of the two heat capacities of the two working fluids. In our case,  $C_{min}$  will just be the specific heat capacity of air times the air flow rate, because the hot side of the heat exchanger (the aluminum) doesn't have an actual flow rate associated with it. To calculate resistance we first must use the equation

$$\epsilon = 1 - e^{-NTU}, \quad (\text{A.14})$$

where  $\epsilon$  is the fraction of actual heat transfer to the theoretical maximum calculated from the flow rate and  $C_{min}$ . To calculate resistance, we can simply use

$$R_{overall} = \frac{1}{\epsilon C_{min}}, \quad (\text{A.15})$$

where  $R_{overall}$  is the resistance to heat transfer for the heat exchanger for a given temperature difference.

#### A.4.4 Theoretical Predictions

Predictions for the resistance of each heat sink can be found by using the three models discussed above. The flow rate is calculated by finding the hydraulic diameter for the gaps between the fins, which is given approximately by  $D_h = 2 * gw$ , where  $gw$  is the fin spacing. The hydraulic diameter and fin array length gives a friction factor, which can then be used to construct a pressure vs. flow rate curve. The intersection of this curve with the fan curve yields the actual flow rate and pressure drop.

Similarly, the convection heat transfer coefficient  $h$  can be calculated from the geometry of the fins. The fin equations are used to find an effective heat transfer

Heat Sink Number	Measured Pressure (Pa)	Predicted Pressure (Pa)	Percent Diff.
1	53.6	54.6	1.8
2	25.8	25.8	-0.03
3	63.0	60.2	-4.6
4	63.7	73.8	14
5	85.3	90.9	6.1
6	58.3	71.4	18
7	73.9	79.2	6.7

Table A.1: Comparison of measured to predicted pressure drops for each heat sink.

coefficient and fin efficiency, and then the array overhang is modeled as a fin with the same thickness as the base thickness to further refine out efficiency calculation. Finally, the heat exchanger equations are used to calculate the overall resistance for the fin array.

## A.5 Results and Discussion

### A.5.1 Pressure Drop

The pressure readings provide a quick check of the incompressible flow aspect of the model. Since, theoretically, the pressure drop is just a function of the fan curve and the duct geometry (primarily the fin gap geometry), confirming the pressure model correct will give a fairly accurate volumetric flow rate.

The pressure data, compared to the predicted values, is printed in table A.1. Some of the measurements have significant error, with the majority of the measured pressures lower than the predicted values. A possible explanation for this is that the fins in the fin arrays are not perfectly straight, so some of the spaces between fins are larger than they should be, letting more air pass through. It is unclear whether the fins are bent because of damage to them or because of the way they were manufactured.

Heat Sink Number	Measured Resistance (K/W)	Predicted Resistance	Percent Diff.
1	0.206	0.152	40.8
2	0.263	0.208	28.7
3	0.209	0.154	41.3
4	0.200	0.145	41.4
5	0.205	0.150	41.4
6	0.200	0.145	40.7
7	0.201	0.146	39.6

Table A.2: Comparison of measured resistance to predicted resistance values for the different heat sinks. The measured resistance was calculated by dividing steady state temperature by steady state power.

H.S. Number	Measured Resistance (K/W)	Predicted Res (adjusted)	Percent Diff.
1	0.206	0.207	3.41
2	0.263	0.263	1.89
3	0.209	0.209	4.16
4	0.200	0.200	2.51
5	0.205	0.205	3.55
6	0.200	0.200	2.14
7	0.201	0.201	1.52

Table A.3: Comparison of measured resistance to predicted resistance values for the different heat sinks. In this case, the predicted values have been adjusted to include resistance between the power resistor and the fin array. All the predicted resistances simply have a constant value of  $0.055K/W$  added to them.

## A.5.2 Resistance

Resistances are not measured directly. Instead power dissipation and temperature is measured. Resistance is calculated using the difference in temperature between the copper plate and room temperature. Tables A.2 and A.3 summarize the differences between measured and predicted values.

## A.6 Conclusions

It is not possible to get an exact value of the resistance of the fin array due to unknown resistances between the power resistor and the heat sink. However, this does not affect the possibility of minimizing the resistance, since each measured resistance is off by a constant value (assuming the thermal grease was applied correctly). The optimal

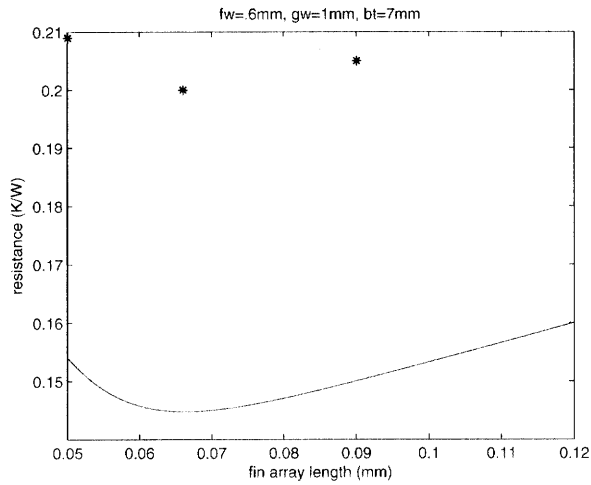


Figure A-2: Graph showing theoretical resistance as a function of fin array length. In this case, base thickness is 7mm, fin thickness is .6mm, and gap thickness is 1mm. The experimental data (red stars) overlays the theoretical prediction.

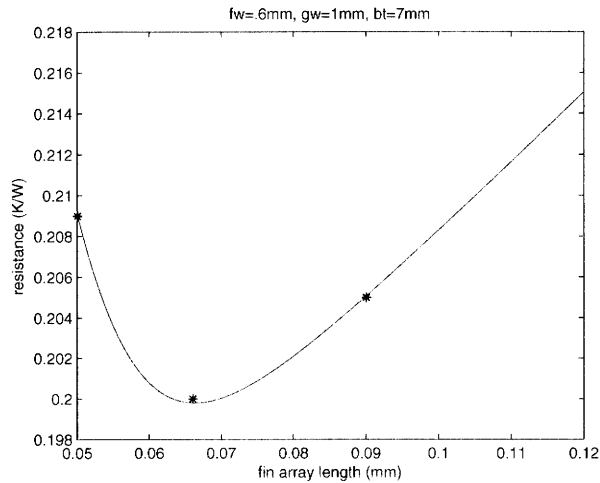


Figure A-3: Graph showing adjusted theoretical resistance as a function of fin array length. In this case, base thickness is 7mm, fin thickness is .6mm, and gap thickness is 1mm. The adjustment is made by adding a resistance value for the resistance between the power resistor and the fin array, which is determined to be approximately .0055 K/W. After the adjustment, the measured data is closer to the prediction.

resistance is given by and fin array length of 58.7 mm, a fin width of .295 mm, a gap width of .786 mm, and a base thickness of 6.53 mm. The resistance of such an array would be .1370 K/W, good enough to dissipate 144 W at a temperature of only 42 degrees Celsius.



# Appendix B

## Simulink Models

The Simulink program is a part of the MATLAB package, which is available to MIT students who are connected to the internet through the MIT network. Simulink provides a stateflow approach to modeling. Objects have inputs and outputs, and lines connect the output from one object to the input of another.

The Simulink models shown herein model the controlled temperature and power dissipation of the resistor, plate, and fin array system. The model in figure B is for the case when the heat sink is in place, the duct is closed, and the fan is running.

The large blocks in B refer to smaller systems. One such system, the one describing heat transfer through the copper plate, is shown in figure B.

Figure B shows the Simulink model for our worst-case scenario in terms of controlling the temperature: the duct closed, the fan off, and no heat sink in place. Note that it doesn't have the block to model the heat sink that figure B has.

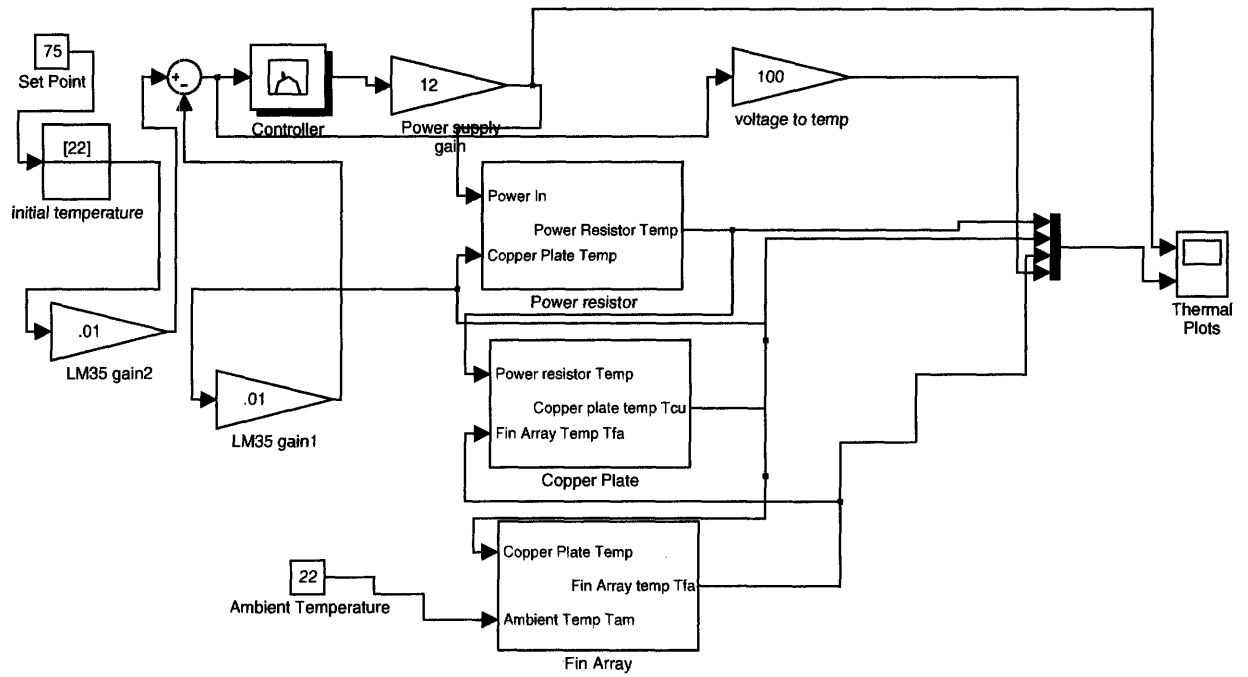


Figure B-1: Simulink model showing the temperature controller operating on the system under normal operating conditions.

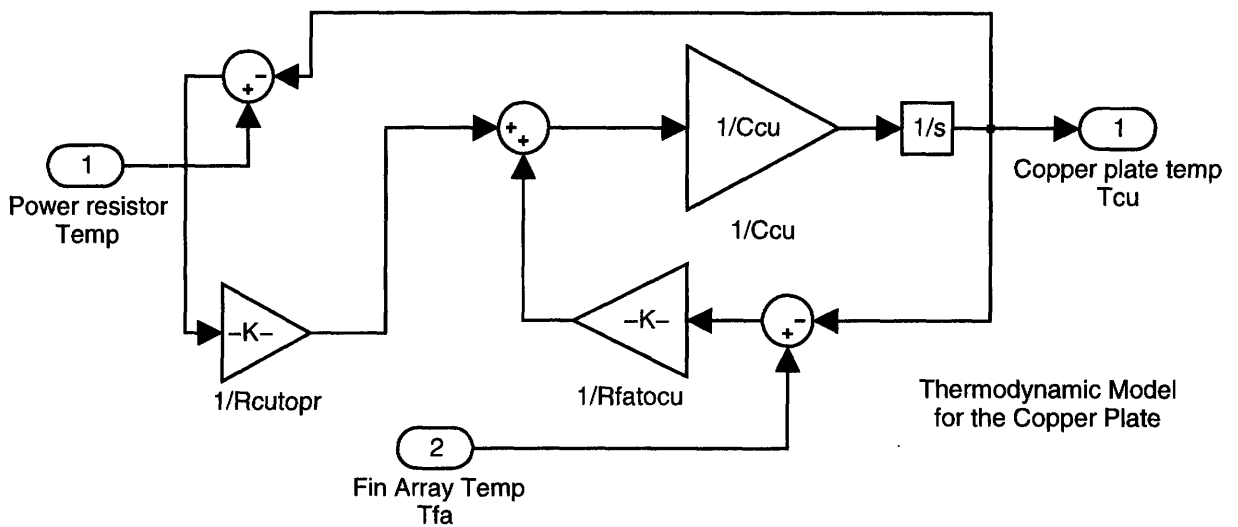


Figure B-2: Simulink model of heat transfer through the copper plate.

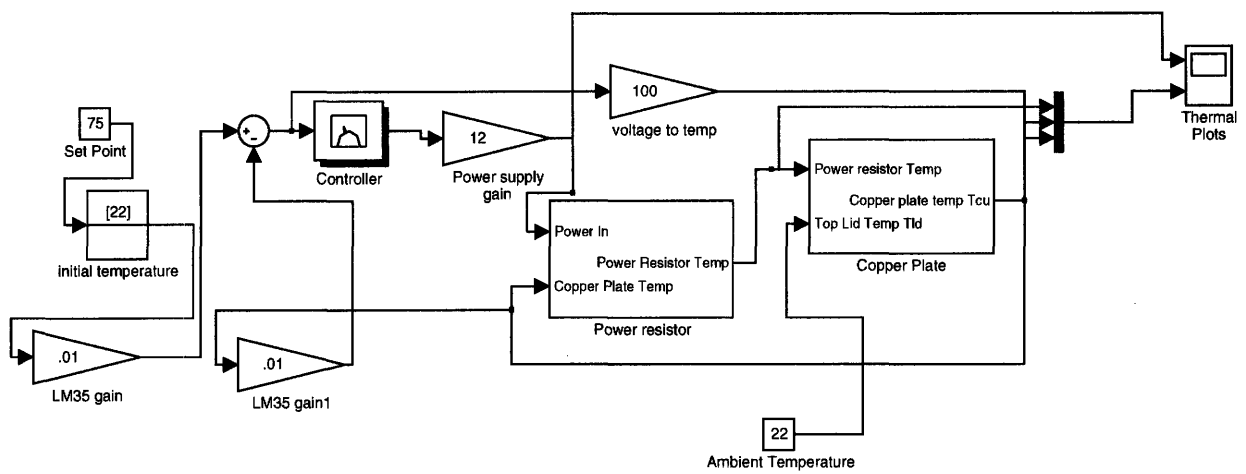


Figure B-3: Simulink model for the system operating under worst-case conditions.

# Appendix C

## MATLAB Code

```
function resistance = heat_sink_resistance(fin_length, ...
    fin_width, ...
    gap_width, ...
    base_thickness)

    %apparatus variables
    heat_source_length=.04572; %m for AMD athlon processor
    heat_source_width=.04572;
    %http://www.amd.com/us-en/assets/content_type/white_papers_and_tech_docs/23792.pdf
    duct_height=.04; %m
    duct_width=.08; %m

    max_pressure_drop=250; %Pa (modeled fan curve pressure drop)
    max_flow_rate=.01165; %m3/s (modeled fan curve volumetric flow rate)
    air_temp=273+22; %K

    %material constants
    k=237; % W (m*K) from http://en.wikipedia.org/wiki/Aluminium
    rho=1.17; %kgm-3
```

$\mu=1.8e-5$ ; %Pa\*s 20

$c_{p\_air}=1012$ ; %J (kg\*K) specific heat of air

$k_{air}=0.0257$ ; %W (m\*K)

*%calculation of h*

$a=gap\_width$ ;

$b=duct\_height-base\_thickness$ ;

$b.a$ ;

$Dh=4.(a.*b).(2.*a+2.*b)$ ;

*%Dh=a<sup>2</sup>; %true when  $b \gg a$ , but we don't need to make this assumption*

30

$Nu=7.54$ ; % $Nu=hDk$ , assuming constant surface temp and  $b \gg a$

$f_{ReDh}=96$ ; %assuming  $b \gg a$

$h=Nu.*k_{air}.Dh$ ;

*%ceiling because we can often let close the fin array with a thinner fin*

$num\_fins=ceil(duct\_width.(fin\_width+gap\_width))$ ;

*%calculation of pressure drop and flow rate from fan curve and friction*

*%factor*

40

$flow\_rate=max\_pressure\_drop.((f_{ReDh}.2.*\mu.*fin\_length).(Dh.^2.*a.*b.*num\_fins))+$   
 $max\_pressure\_drop.(max\_flow\_rate))$ ;

$pressure\_drop=f_{ReDh}.2.*\mu.*fin\_length.(Dh.^2.*a.*b.*num\_fins).*flow\_rate$ ;

$vel\_air=flow\_rate.(num\_fins.*b.*gap\_width)$ ;

$Re=rho.*vel\_air.*Dh.\mu$ ;

*%calculation of fin efficiency*

$P=2.*fin\_length+2.*fin\_width$ ;

```

Ac=fin_length.*fin_width;
fin_effectiveness=sqrt(k.*P.(h.*Ac));
m=sqrt(h.*P.(k.*Ac));
L=b;
fin_efficiency=tanh(m.*L).(m.*L);

```

*%calculation of overall heat transfer coefficient*

```

Af=2.*(fin_length.*b).*num_fins; %total fin surface area
Aeb=(fin_length.*gap_width).(num_fins-1); %surface area of exposed base
%equivalent to Aeb/(Af+Aeb)+fin_efficiency.*Af/(Af+Aeb)
overall_fin_efficiency=1-Af.(Af+Aeb).(1-fin_efficiency);

```

*%calculate the efficiency of base plate as if it were a fin of thickness*

*%base\_thickness and it had an h=h\_eff*

```

h_eff=(h.*gap_width+(fin_efficiency.*h.*(2.*fin_length.*b).(fin_width.*fin_length)).*
fin_width).(fin_width+gap_width);
m_base_plate_l=sqrt(h_eff.*(2.*(heat_source_width)+
2.*base_thickness).(k.*((heat_source_width).*base_thickness)));
m_base_plate_w=sqrt(h_eff.*(2.*(heat_source_length)+
2.*base_thickness).(k.*((heat_source_length).*base_thickness)));
L_base_plate_l=(abs(fin_length-heat_source_length)+(fin_length-heat_source_length))2;
L_base_plate_w=(abs(duct_width-heat_source_width)+(duct_width-heat_source_width))4;

base_plate_efficiency_l=tanh(m_base_plate_l.*L_base_plate_l).(m_base_plate_l.*L_base_plate_l);
base_plate_efficiency_w=tanh(m_base_plate_w.*L_base_plate_w).(m_base_plate_w.*
L_base_plate_w);

```

*%calculate UA*

80

$UA = h_{\text{eff}} \cdot (\text{heat\_source\_width} \cdot \text{heat\_source\_length});$

$UA = UA + h_{\text{eff}} \cdot \text{base\_plate\_efficiency\_w} \cdot (\text{duct\_width} - \text{heat\_source\_width}) \cdot \text{heat\_source\_length};$

$UA = UA + h_{\text{eff}} \cdot \text{base\_plate\_efficiency\_l} \cdot (\text{fin\_length} - \text{heat\_source\_length}) \cdot \text{heat\_source\_width};$

$UA = UA + h_{\text{eff}} \cdot \text{base\_plate\_efficiency\_l} \cdot \text{base\_plate\_efficiency\_w} \cdot (\text{fin\_length} - \text{heat\_source\_length}) \cdot (\text{duct\_width} - \text{heat\_source\_width});$

*%because the temperature of the aluminum doesn't change much over the  
%length of the fin*

$C_{\text{min}} = c_{\text{p\_air}} \cdot \text{flow\_rate} \cdot \rho;$

$NTU = UA \cdot C_{\text{min}};$

90

$\epsilon = 1 - \exp(-NTU);$  *%from Incropera Dewitt p689, assuming  $Cr=0$ ;*

$\text{resistance} = 1.(\epsilon \cdot C_{\text{min}});$

```
%optimization.m
```

```
%run this script to find the optimal parameters for a heat sink.
```

```
%it can be used for different materials by commenting/uncommenting
```

```
%the appropriate lines.
```

```
fl=50e-3; fw=.63e-3; gw=1e-3; bt=7e-3;
```

```
%resist=heat_sink_resistance2copper(fl,fw,gw,bt);
```

```
%resist=heat_sink_resistance2brass(fl,fw,gw,bt)
```

```
% resist=heat_sink_resistance2steel(fl,fw,gw,bt)
```

```
resist=heat_sink_resistance2stainless(fl,fw,gw,bt)
```

10

```
oldres=10;
```

```
while oldres-resist>1e-6
```

```
    oldres=resist;
```

```
    gws=1e-5:1e-6:2*gw;
```

```
% res=heat_sink_resistance2copper(fl,fw,gws,bt);
```

```
% res=heat_sink_resistance2brass(fl,fw,gws,bt);
```

```
% res=heat_sink_resistance2steel(fl,fw,gws,bt);
```

```
res=heat_sink_resistance2stainless(fl,fw,gws,bt);
```

```
index=find(res==min(res));
```

20

```
gw=gws(index)
```

```
fls=49e-3:1e-5:fl+10e-3;
```

```
% res=heat_sink_resistance2copper(fl,fw,gw,bt);
```

```
% res=heat_sink_resistance2brass(fl,fw,gw,bt);
```

```
% res=heat_sink_resistance2steel(fl,fw,gw,bt);
```

```
res=heat_sink_resistance2stainless(fl,fw,gw,bt);
```

```
index=find(res==min(res));
```

```
plot(fls,res)
```

```
fl=fls(index)
```

30



```

fws=1e-5:1e-6:2*fl;
%   res=heat_sink_resistance2copper(fl,fws,gw,bt);
%   res=heat_sink_resistance2brass(fl,fws,gw,bt);
%   res=heat_sink_resistance2steel(fl,fws,gw,bt);
res=heat_sink_resistance2stainless(fl,fws,gw,bt);
index=find(res==min(res));
fw=fws(index)

```

```

bts=1e-5:1e-6:2*bt;
%   res=heat_sink_resistance2copper(fl,fw,gw,bts);
%   res=heat_sink_resistance2brass(fl,fw,gw,bts);
%   res=heat_sink_resistance2steel(fl,fw,gw,bts);
res=heat_sink_resistance2stainless(fl,fw,gw,bts);
index=find(res==min(res));
bt=bts(index)

```

40

```

%   resist=heat_sink_resistance2copper(fl,fw,gw,bt)
%   resist=heat_sink_resistance2brass(fl,fw,gw,bt)
%   resist=heat_sink_resistance2steel(fl,fw,gw,bt)
resist=heat_sink_resistance2stainless(fl,fw,gw,bt)

```

50

**end**

*%modelparams.m*

*%script that initializes variables for the Simulink models*

*%material constants from Wikipedia*

rho\_cu=8.96e3;

rho\_al=2.7e3;

c\_al=897; %J/kgK

c\_cu=385; %J/kgK

*%thermal grease properties*

10

k\_tg=.7; %W/mK

grease\_thickness=.001\*.0254; %meters

*%power resistor properties*

Cpr=13.5; %J/K

L\_pr=1.5\*.0254;

w\_pr=1\*.0254;

*%resistance pr to cu plate*

CA\_cupr=l\_pr\*w\_pr; %contact area

20

Rcutopr=grease\_thickness/(k\_tg\*CA\_cupr);

*%copper plate properties*

l1\_cu=1.8\*.0254;

w1\_cu=l1\_cu;

h1\_cu=.16\*.0254; %length, width, and height for contact area only

l2\_cu=2.8\*.0254;

w2\_cu=w1\_cu;

h2\_cu=.06\*.0254;

V\_cu=l1\_cu\*w1\_cu\*h1\_cu+l2\_cu\*w2\_cu\*h2\_cu;

30

```
Ccu=V_cu*rho_cu*c_cu; %J/K
```

```
%resistance cu plate to fin array
```

```
CA_facu=l1_cu*w1_cu; %contact area
```

```
Rfatocu=grease_thickness/(k_tg*CA_facu);
```

```
%fin array properties
```

```
fin_length=.05;
```

```
fin_width=.0004;
```

```
gap_width=.001;
```

40

```
base_thickness=.007;
```

```
duct_width=.08;
```

```
duct_height=.04;
```

```
num_fins=floor(duct_width./(fin_width+gap_width));
```

```
V_fa=duct_width*fin_length*base_thickness+
```

```
num_fins*fin_length*(duct_height-base_thickness)*fin_width;
```

```
Cfa=V_fa*rho_al*c_al;
```

```
Rfatoam = heat_sink_resistance2(fin_length, fin_width, gap_width, base_thickness) 50
```

```
%for case when fin array is not there
```

```
l1_cu;
```

```
w1_cu;
```

```
CA_facu;
```

```
c_p_air=1012; %J/(kg*K) specific heat of air
```

```
k_air=0.0257;
```

```
mu_air=1.8e-5;
```

60

```

rho_air=1.17;
nu_air=mu_air/rho_air;
alpha_air=k_air/(rho_air*c_p_air);
g=9.81;
Ts=50+273;
Tinf=22+273;
beta=2/(Ts+Tinf);
H=.05; L=.04;

```

```
Pr=mu_air*c_p_air/k_air;
```

70

```
Ra=g*beta*(Ts-Tinf)*L^3/(nu_air*alpha_air);
```

```
Nu=.18*(Pr/(.2+Pr)*Ra)^.29;
```

```
h=Nu.*k_air./L;
```

```
Rcutold=1/(h*H^2);%resistance from copper plate to the top lid of the box
```

```
TinIC=22;
```

80

```
%controller parameters found by trial and error
```

```
Kp=100;
```

```
Ki=.2;
```

```
Kd=0;
```

# Bibliography

- [1] Frank P. Incropera and David P. Dewitt. *Fundamentals of Heat and Mass Transfer*. Wiley USA, 2002.
- [2] Mit course catalogue '06-'07.
- [3] Frank M. White. *Fluid Mechanics*. McGraw-Hill Boston, 2003.