THE EFFECT OF THE WIND VELOCITY GRADIENT ON AIRPLANE PERFORMANCE

by

James H. Doolittle

A.B. University of California
1922

M.S. Massachusetts Institute of Technology
1924

Submitted in Partial Fulfillment of the Requirement

For the Degree of
DOCTOR OF SCIENCE

From the

Massachusetts Institute of Technology
1925

Signature of Author

Certification by the Department of

TABLE OF CONTENTS

I.

INTRODUCTION.

There has long been an uncertainty in the minds of aviators regarding the effect of the wind on the flying qualities of an airplane.

Some pilots claim that it is much easier to turn into the wind than with it and that, at any altitude, they can tell the wind direction by the feel of the ship in a turn and this even though in a dense cloud which would preclude the possiblity of obtaining their relative motion from any stationary object.

Other pilots maintain that, regardless of the wind velocity or the proximity of the ground, there is no difference in the feel of the plane when turning into the wind and when turning with it. They claim that any apparent difference is due wholly to the psychological effect on the pilot, resulting from the difference in ground speed in the two cases, and if there is any difference in the ship's performance, from a time altitude standpoint, it is because the pilot handled the controls differently. In other words, if the pilot were blindfolded he could not tell the wind direction when turning and a turn made into the wind would be identical with a turn made with the wind. This is, of course, considering the turn in relation to the medium in which it is being executed and not in relation to the curves traced out in the ground.

There is a similar difference of opinion regarding the effect of a strong wind on the rate of climb. Experienced pilots are about evenly divided, half feeling that a plane climbs better into the wind, and the other half feeling that the wind makes absolutely no difference.

The following questions were asked some of America's leading pilots and their monosyllabic answers follow in tabulated form.

Assuming that the air is free from vertical currents, and the ground surface level:

1. *Is there any tendency to stall or "fall off" when turning with a strong steady wind near the ground?*

2.(a) *If there is a difference could a pilot feel it if he were blindfolded?*

(b) *Even though he felt no difference would his turns be different into and away from the wind?*

3. *Can a plane be climbed faster into the wind than it can when going with the wind?*

4. *If there is any effect, due to the wind, would it be apparent at higher altitudes, say 10,000'?*

Pilot	Question				
	1	2a	2b	3	4
Lt. J. A. Macready, U.S.A.	Yes	—	—	Yes	No
Lt. H. R. Harris, U.S.A.	No	No	No	No	No
Mr. Art Smith, Air. Mail	Yes	—	—	No	Yes
Capt. Lowell Smith, U.S.A.	No	No	No	No	No
Lt. Eric Nelson, U.S.A.	Yes	—	—	Yes	No
Lt. Leigh Wade, U.S.A.	Yes	No	No	Yes	No
Lt. Russell Maughan, U.S.A.	Yes	—	Yes	No	No

The answers given above are the results of actual observation in free flight and do not deal, in any way, with theoretical considerations. That is, there might actually be a difference in turning with and against the wind, but if this difference was not great enough to enable the pilot to detect it, either in the feel of the controls or the reaction of the ship, his answer would be that there was no difference.

Lt. Harris and Capt. Smith, whose observations are in agreement with each other but at variance with all the rest were the most positive in their assertions.

Lt. Macready, in his experience with the T2 when he and Lt. Kelly were setting their endurance record, observed that there appeared to be a difference in the feel of the ship when turning with and into the wind while the plane was heavily loaded.

Art Smith bases his opinions on experience with the early airplanes which, because of their low powered motors and poor aerodynamic characteristics, flew very poorly at best.

Lt. Nelson took his heavily loaded World Cruiser off the water at Hong Kong, China, into a 30 mile an hour wind. He turned to go with the wind and was unable to prevent the plane from settling until he came to the lee of an island where, in the comparatively quiet air, he was able to turn and as soon as the plane was nosed into the wind was able to climb again.

II.

THE WIND VELOCITY GRADIENT.

In the work which follows, the terms "Velocity Gradient" will be taken to mean the rate of change of wind velocity with respect to altitude and will be given in feet per second per foot.

The main factors affecting the velocity gradient at any given point are:

(1) Convection currents resulting from temperature differences.

(2) Turbulence due to large distant surface irregularities.

(3) The nature of the ground surface in the immediate vicinity.

(4) The wind velocity.

The first two affect, primarily, the gradient at altitude and the last two affect it near the ground.

Experiments made in England and published in "Reports and Memoranda" #296 and #531, Advisory Committee for Aeronautics, indicate that very near the ground, if the surface is level and fairly smooth, like the average Government Flying Field, the velocity gradient varies directly as the velocity and inversely as the altitude, following, very closely, the approximation formula

$$\text{Velocity Gradient} = \frac{dV}{dh} = .2\,\frac{V_{40}}{h} \qquad (1)$$

where V40 is the wind velocity at forty feet and h is the height at which we desire to determine the gradient.

If the surface is absolutely smooth and for altitudes greater than forty feet a closer approximation is obtained if we use the formula

$$\frac{dV}{dh} = .2\,\frac{V}{h} \qquad (2)$$

where V is the wind velocity at the altitude h.

Both of these formulas were obtained by plotting velocity curves from the data cited, drawing tangents, determining the slope of the tangents, drawing curves of slope against altitude at various velocities and deriving the formulas which most nearly fitted these curves. They may be used interchangeably in the vicinity of 40' as the difference in the results obtained would be no greater than the errors in experimental measurements or the variations due to instantaneous velocity fluctuations, being of the order of 9% at 25' and 16% at 100'.

Hellman gives the following formula, which is generally accepted by meteorologists, for the average wind velocity at any altitude:

$$V = 7.0\,h^{\frac{1}{5}} \qquad (3)$$

differentiating

$$\frac{dV}{dh} = 1.4\,h^{-\frac{4}{5}} \qquad (4)$$

This becomes identical with (2) when the value of V, as given in (3), is substituted.

The velocity at any altitude up to forty feet may be obtained by integrating (1).

This gives

$$V = V_{40}\left(1 + .2\ln\frac{h}{40}\right) \qquad (5)$$

It may be seen that this formula does not hold for $h = 0$, giving a negative value for the velocity where as we know it to be zero. The zero value of the velocity occurs at $h = .27'$. This does not affect the accuracy of our work, as the lowest lifting surface of an airplane, the wing sometimes placed on the landing gear spreader bar, is never closer to the ground than one foot and the main lifting surfaces are rarely closer than three feet. If we desired to work under $h = .5'$, actual conditions would be more closely approximated if, in the solid curve on page 11, a tangent were drawn from the origin to the curve. This tangent would meet the curve at $h = .5$.

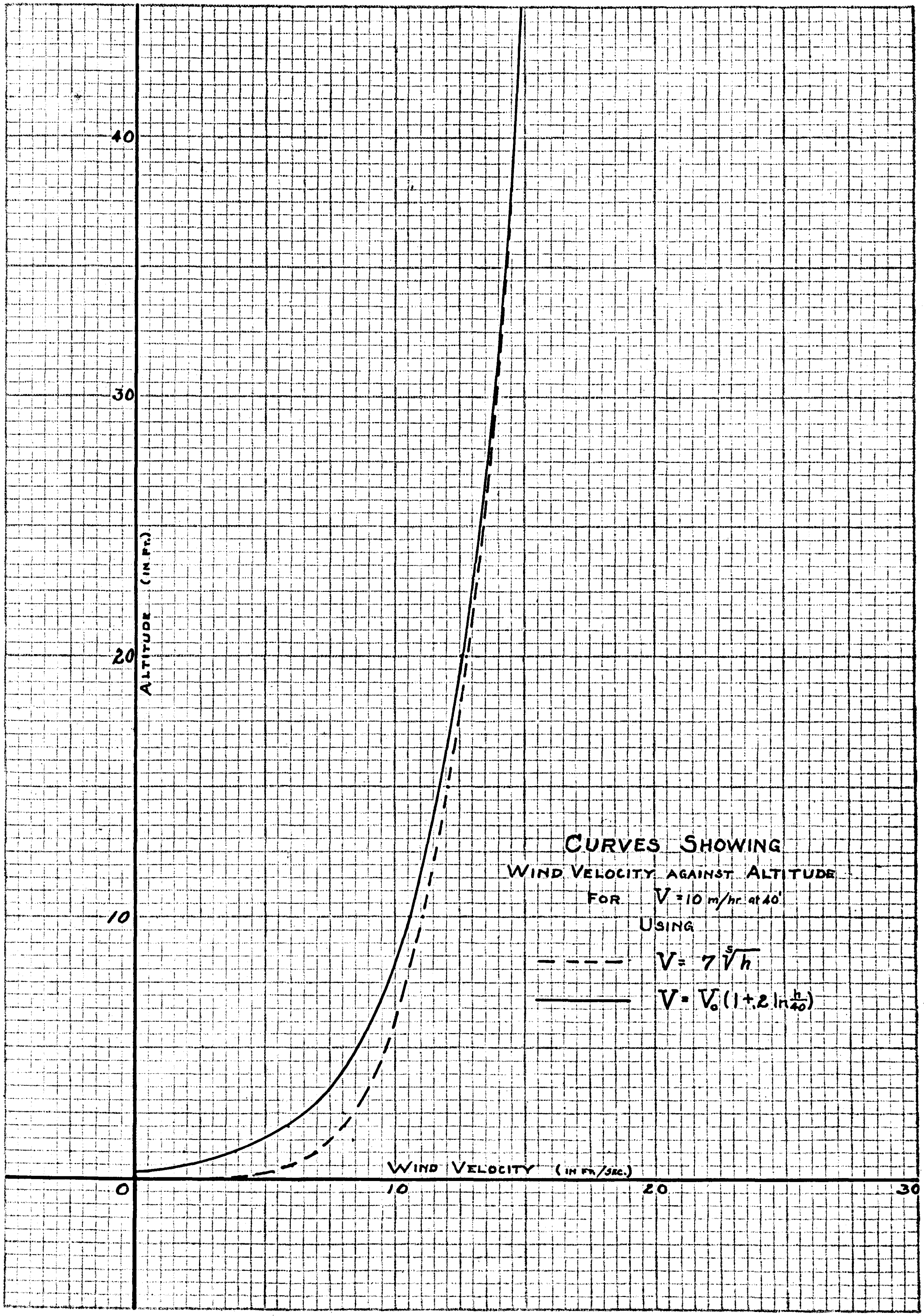
Formulas (1) and (2) give the <u>general</u> conditions near the ground but only <u>average</u> conditions at altitude. This is due to conditions being fairly steady in the immediate vicinity of the earth's surface but decidedly erratic above a few hundred feet. The degree of this erraticalness is indicated by the fact that a velocity gradient inversion often occurs between one and five thousand feet. Above six thousand feet the flow generally becomes fairly steady again, following formula (2) to an altitude of about 30,000' when another gradient inversion often occurs. The difficulty encountered in predicting the gradient, even in this second region of comparatively steady flow, becomes apparent when we observe that at 20,000' the formulas give

$$\frac{dV}{dh} = .0005$$

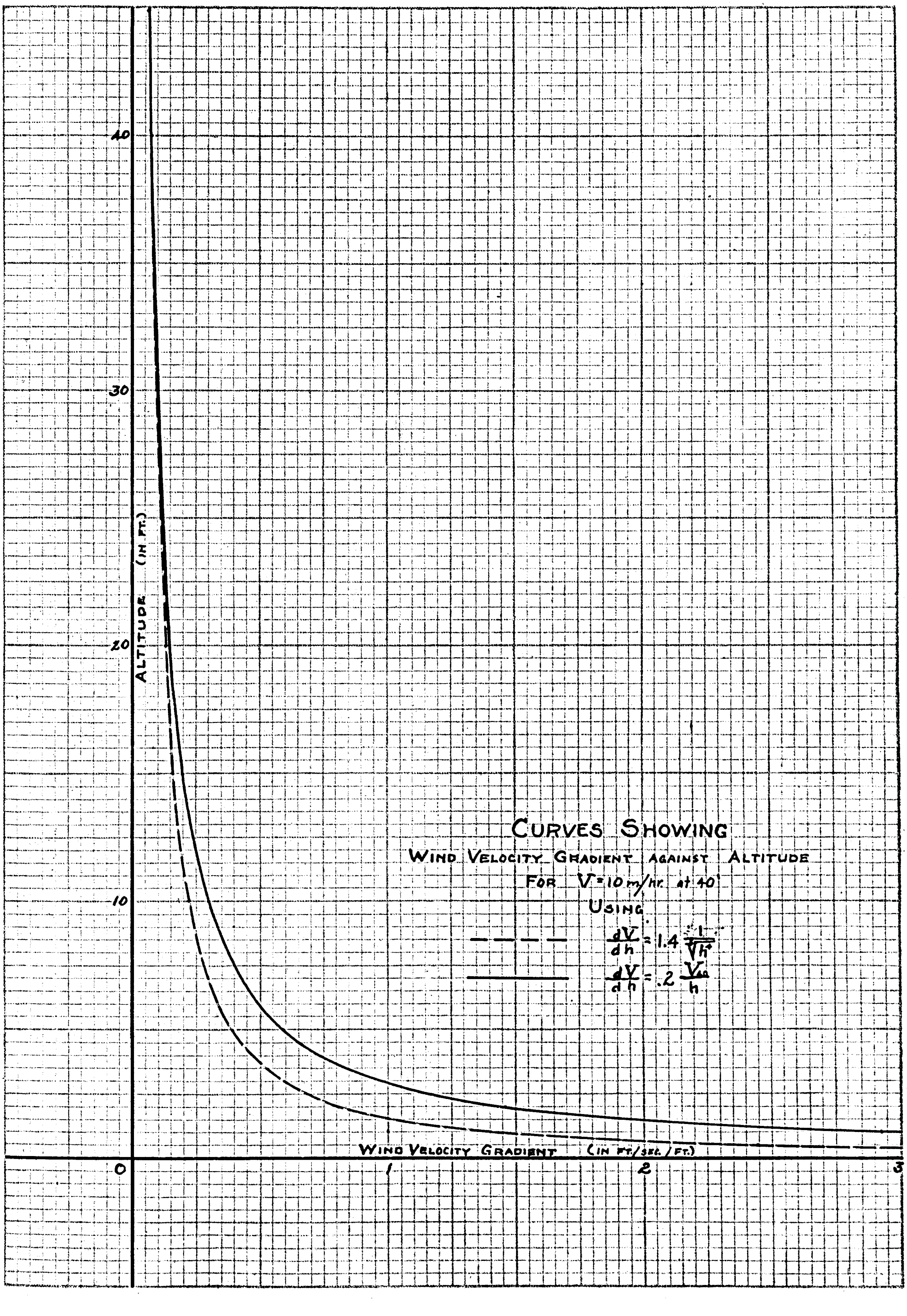
and data contained in "Aerology", a pamphlet published by the U.S. Naval Aerological Observatory at Pensacola, Florida, indicates that a value of .003 is not uncommon for this altitude.

If there are hills, tall trees, high buildings, or other obstructions nearby, the flow is disturbed and the variations from mean conditions become so great that it is impossible to predict the flow.

Obviously then, if quantitative results are desired in any tests depending upon the velocity gradient, they must be made over a large flat area, free from obstructions, and preferably on a day when the sky is completely overcast or at night, in order to eliminate local convection currents. Even when these precautions are taken, the fact that in our present state of knowledge we are unable to predict other meteorological phenomena with unfailing accuracy indicates that the previously cited formulas are not rigorously correct; but the consistency with which they check experimentally obtained data justifies their use, as a first approximation, at low altitudes.



CURVES SHOWING
WIND VELOCITY AGAINST ALTITUDE
FOR V = 10 m/hr at 40'
USING
V = 7 ∜h
V = V₀(1 + .2 ln h/40)
ALTITUDE (IN FT.)
WIND VELOCITY (IN FT./SEC.)
0
10
20
30
40


Curves Showing
Wind Velocity Gradient against Altitude
For V = 10 m/hr. at 40'
Using
dV/dh = 1.4 1/∜h
dV/dh = .2 V40/h
Altitude (in ft.)
Wind Velocity Gradient (in ft./sec./ft.)
40
30
20
10
0
1
2
3

III.

THE EFFECT OF THE WIND VELOCITY GRADIENT ON LIFT.

The object of this paper is to determine the effect of the velocity gradient on the flying characteristics of airplanes, in level flight, in a climb and when turning.

Applying first the principles of applied mechanics:

It is obvious that, regardless of the altitude or wind velocity, if an airplane is flying level, the velocity gradient will have no effect and it should make absolutely no difference whether the airplane wise flying with the wind, against it, or across it.

If the airplane is climbing into the wind at a constant air speed the increasing velocity of the wind acts as a force assisting in the climb. Conversely, if the airplane is climbing with the wind this force tends to decrease the rate of climb.

The time effect of this force in the direction of motion is

$$\int_0^t \frac{F}{\cos\theta}\, dt = m(V_1 - V_0) \qquad (6)$$

and the space effect is

$$\int_{s_0}^{s_1} \frac{F}{\cos\,\theta}\, ds = \frac{1}{2}\, m(V_1^2 - V_0^2) \qquad (7)$$

where F is the component of force in the direction of motion, θ is the angle between the flight path and the wind direction, which we assume horizontal, and V_0 and V_1 the wind velocities at two altitudes.

Formula (6) gives the change in momentum resulting from the force and formula (7) the change in kinetic energy resulting from the work done by the force.

A more useful form of (6) may be obtained as follows:

$$F = m\ a = m\,\frac{dV}{dt} = m\,\frac{dV}{dh} \cdot \frac{dh}{dt} \qquad (8)$$

That is, the force which acts on an airplane, as a result of the wind velocity gradient, is the combined product of the mass of the airplane, the wind velocity gradient and the rate of climb. The component of this force in the direction of motion would be $F \cos\theta$ where θ is the angle between the flight path and the horizontal.

As an example of the effect of the wind velocity gradient on a given plane let us assume an airplane having the following characteristics:

Weight	=	4000 lbs.
Rate of climb	=	1000'/' = 16.67'/"
Speed of climb	=	110'/"
HP	=	400
Propeller efficiency	=	65%

The thrust is then

$$\frac{400 \cdot 550 \cdot .65}{110} = \underline{1300 \text{ lbs}}.$$

The accelerating force at 10' in a 50'/" wind is

$$\frac{124.2 \times (.2 \cdot \frac{50}{10}) \times 16.67}{110} \; .988 = \underline{18.6 \text{ lbs.}}$$

This is 1.43% of the total thrust; and as the rate of climb is a function of the excess HP, which in this case would be about 70% of the total available HP, the increase in rate of climb would be about 2%.

The force at 200' under similar conditions is

$$\frac{124.2 \times (.2 - \frac{69}{200}) \times 16.67}{110} .988 = 1.28 \text{ lbs.}$$

a negligible amount.

As the velocity gradient and the rate of climb decrease much more rapidly than the ratio of available HP to excess HP increases, the effect of this force will continue to decrease as the altitude increases.

In a turn, during which the altitude of the center of gravity of the airplane is held constant, the main effect of the wind velocity gradient is to cause over banking when the plane is turning with the wind and under banking when the plane is turning into the wind. This is due to the increased wind velocity acting on the upper wing and would be measurable only when very near the ground.

When turning away from a head wind, so close to the ground that the velocity gradient can no longer be considered a straight line over a vertical distance equal to the projected height of the banked airplane there will be a net loss in lift for the first 90° of turn, resulting from the increase in the lift of the upper wing, due to the gradient, having an absolute value less than that of the decrease in lift of the lower wing; and there will be a net gain in lift for the next 90°. If the turn is continued there will be a net gain in lift for the third 90° and another loss for the fourth 90°.

In a normal turn near the ground there is a tendency to climb during the turn. If a turn is started immediately after taking off into a strong wind, the velocity gradient will aid the climb for the first 90° and oppose it for the next 90°. If the turn is continued, the velocity gradient will oppose the climb for the third 90° and aid it for the fourth. This effect is exactly the opposite of that cited in the paragraph immediately preceding, and would overrule it if the climb was steep or the curvature of the gradient slight. This is the general case, and accounts, in part, for the feeling of loss of speed, or sinking, after the first 90°. The pilot endeavors to keep his angle of climb constant, and this can be accomplished only by increasing the angle of attack, which in turn decreases the speed and, if the plane is already near its best angle of climb, causes stalling.

IV.

CIRCULATION.

Air is a fluid of slight viscosity which, under the conditions ordinarily met with in aerodynamics, may be considered perfect; that is, devoid of viscosity and incompressable. This is not strictly true, but the compression in front of a body moving at 250'/" is only about 3%, and the consideration of viscosity renders the mathematical treatment extremely difficult and would have very little effect on the final results. Ordinarily we consider the air viscosity only long enough to set up a desired form of flow, and then neglect the viscosity in the study of the flow. This, at first, appears to be a very approximate manner of approach, but the check between theory and experiment justifies the means.

The simplest types of flow are:

(1) That in which the fluid particles move along parallel lines at constant velocity. The angular velocity is zero.

(2) That in which the fluid particles move in concentric circles so that the velocity is always proportional to the distance from the center. The angular velocity is constant.

(3) That in which the fluid particles move along straight parallel lines but with velocities proportional

to their distances from a fixed line where the velocity is zero. The angular velocity parallel to a streamline is zero, and a maximum across it.

(4) That in which the fluid particles move in concentric circles so that the velocity is always inversely proportional to the distance from the center. The angular velocity is in one direction along the streamlines and in the opposite direction across them.

If we call the rotation the sum of the angular velocities of any two directions perpendicular to each other we see that (1) and (4) are irrotational and (2) and (3) rotational.

Flows (1) and (4) can be produced by a difference in pressure alone and (2) and (3) only by friction or viscosity. This explains why we must consider viscosity in the setting up of certain types of flow even tho we are forced to neglect it thereafter.

The fourth type of flow is extremely important in that it is the simplest representation of a vortex. We stated, in this type of flow, that the velocity was inversely proportional to the distance from the center, or

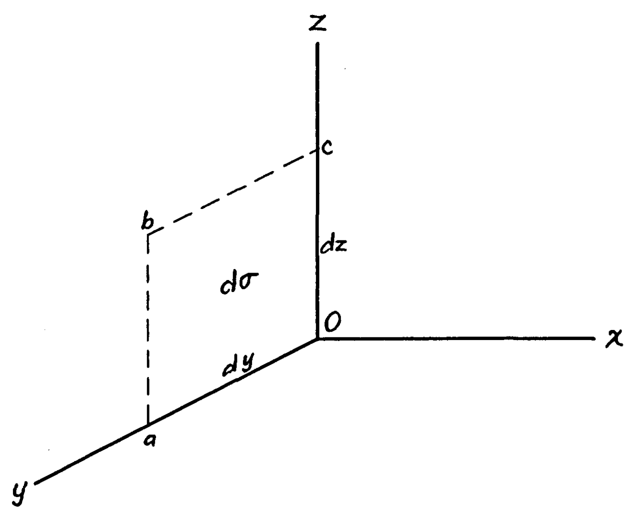
$$V \propto \frac{1}{r}$$

At the center, then, where $r = 0$, the velocity would be infinite. As we cannot deal with infinite velocities and, in fact, do not admit any such thing, we must exclude from our calculation a small region near the center, which we will consider rotating as a solid and neglect. In the irrotational part of this flow (all of it except the negligibly small center) it is only necessary to know the tangential velocity at any point and the distance of this point from the center in order to completely determine the flow. More generally we speak of the circulation, Γ, of a vortex which is 2π times the velocity at a unit distance from the center. The circulation of a vortex is constant around every streamline, as the velocity decreases in the same ratio that the radius increases. This enables us to determine the strength of any vortex, knowing the velocity and the location of the nucleus, as

$$\Gamma = 2\pi r V \tag{9}$$

If, at any point in a fluid, we determine the instantaneous axis of rotation and then move along this axis an infinitesimal length, we shall find a new direction of the instantaneous axis of rotation. The curve traced out by successive movements in this manner is called a vortex line. The surface generated by a vortex line which is moved along a small closed contour is called a vortex tube, and the contents of a vortex tube is termed a vortex filament.

There is a geometric analogy between vortex lines and streamlines in that the former correspond everywhere with the axes of rotation and the latter everywhere with the direction of the velocity.



In order to determine the circulation around the elementary rectangle dy dz, let the components of velocity at O be u,v,w. Then at a the velocity along z will be

$$w + \frac{\partial w}{\partial y}dy$$

The circulation along a b is

$$(w + \frac{\partial w}{\partial y}dy)dz$$

and along c O

$$w(-dz)$$

The velocity along y at b is

$$v + \frac{\partial v}{\partial z} dz$$

The circulation along $b\,c$ is then

$$(v + \frac{\partial v}{\partial z} dz)(-dy)$$

and along $O\,a$

$$v\ dy$$

The sum of these circulations gives

$$\gamma = dy\ dz(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}) = \xi$$

where ξ is the rotational component about the x axis so

$$\gamma = 2\ \xi\ d\sigma$$

That is, the circulation along the contour $d\sigma$, normal to the x axis, is equal to the surface of the contour by twice the intensity of rotation about the x axis. If ω is the angular velocity of rotation about any axis and α the angle between this axis and the x axis, the circulation around the x axis is then twice the product of the projection of this velocity on the x axis and the cross sectional area of the vortex tube, or

$$\gamma = 2\omega \cos \alpha \, d\sigma \tag{10}$$

A simply connected body is one in which any closed curve in the body can be reduced to a point without going outside the body. The volume of a fluid is simply connected, as any contour can be reduced to a point without going outside the fluid. If we take a closed contour and break up the surface into infinitesimal rectangles, there will be two equal and opposite circulations, which will cancel each other, along every line except the bounding contour. We may then say that the circulation around the contour is equal to the sum of the circulations of the elementary rectangles, or

$$\Gamma = \Sigma\gamma$$

From (10)

$$\Gamma = 2\Sigma \, \omega \cos \alpha \, d\sigma$$

$$\text{or } \Gamma = 2\iint \omega \cos \alpha \, d\sigma \tag{11}$$

This is Stokes Theorem and states that:

The circulation around a closed contour is equal to twice the sum of the intensities of all the vortex tubes passing thru the contour.

Helmholtz's Theorems will be stated without proof and are as follows:

I. *The movement of a fluid particle is composed of translation, deformation, and rotation.*

II. *The strength of the vortex, along a vortex tube, is constant.*

III. *A vortex tube is always composed of the same fluid particles.*

IV. *The intensity of a vortex tube is constant throughout all of its motion.*

Thompson's Theorem states, in substance, that:

A closed contour, made up of the same fluid particles, can never cross a vortex tube.

(9) may be derived by means of Stokes's theorem, and from Helmholtz's second theorem

$$\Gamma = 2\pi\, r_o V_o \qquad (12)$$

where V_o is the velocity at the surface of the vortex tube and r_o its radius.

The velocity at any point is then

$$V = \frac{r_0}{r} V_0 \qquad (13)$$

and the pressure, from Bernoulli's equation, is

$$p = \text{const.} - \frac{V^2}{r} = \text{const.} - \frac{r_0^2 V_0^2}{r^2} \qquad (14)$$

It is seen from this that the pressure decreases as r decreases.

We may also consider circulation as the line integral of a velocity, in the same way that work is the line integral of a force. The circulation along a curve S from A to B is then

$$\Gamma = \int_A^B V \cos\theta \, ds \qquad (15)$$

where V is the instantaneous velocity, θ the angle between the direction of the velocity and the curve, and ds an infinitesimal distance along the curve.

If there is a velocity potential φ, then

$$V \cos\theta = \frac{\partial\varphi}{\partial s} \qquad (16)$$

and

$$\Gamma = \int_A^B \frac{\partial\varphi}{\partial s} ds = \varphi_B - \varphi_A \qquad (17)$$

where φ_A is the velocity potential at the point A
and φ_B the velocity potential at the point B.

In a plane parallel flow in which the fluid is moving along a smooth level surface, the particles in immediate contact with the surface are at rest. If there were no viscosity, the succeeding particles would slide over them and there would be no rotation. What actually happens is that, due to viscosity, the next layer of particles is retarded and each succeeding layer retarded less until at some distance the rotation becomes negligible. In the immediate neighborhood of the surface it has a considerable value. The thinner the boundary layer the more rapid the transformation and the greater the rotation.

The case with which we are to deal is of this general nature and the velocity has previously been shown to follow the formula

$$V = 7\ h^{\frac{1}{5}} \qquad (3)$$

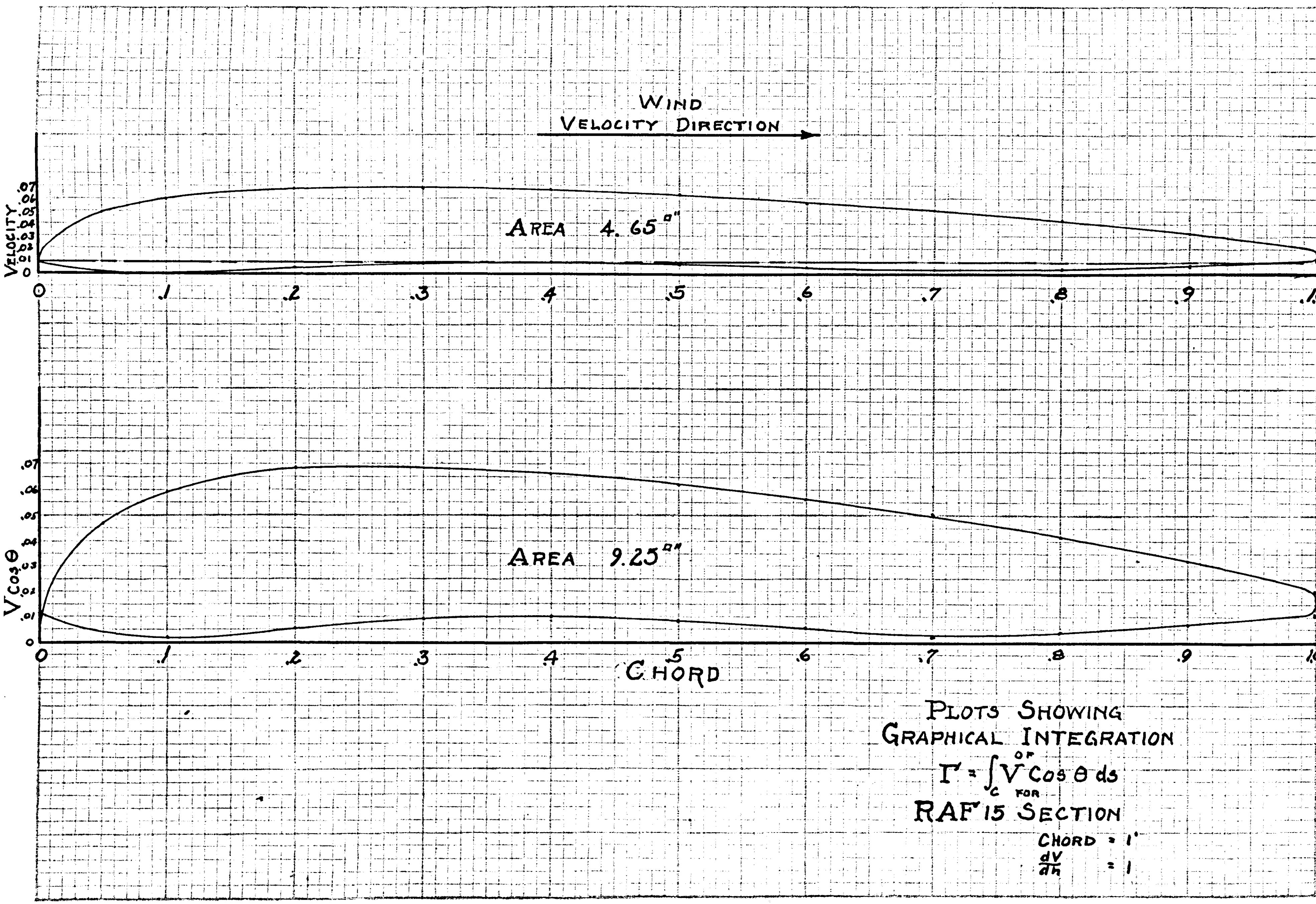
or

$$V = V_{40'}(1 + .2 \ln \frac{h}{40}) \qquad (5)$$

We may then determine the circulation around any wing section, due to the wind velocity gradient, by using (15) and integrating around the outside boundary of the section. As this boundary curve is not readily expressed as an analytic function the easiest method is to plot the wing section, determine the values of the function, V cos θ, at various points, plot these to some convenient scale, and obtain the area under the curve with a planimeter.

This was done for an R.A.F. 15 section, similar to that used in the DH4M1 airplane. The plot is shown on page 28 and the tabulation is given below.

Chord	Ordinate	Velocity	θ	Cos θ	V Cos θ
0 (top)	.0033	.013	−57°	.545	.0071
.1 "	.0500	.060	−10°	.985	.059
.2 "	.0578	.068	− 4°	.997	.068
.3 "	.0581	.068	+ 1°	1.000	.068
.4 "	.0560	.066	+ 2°	1.000	.066
.5 "	.0517	.062	+ 3°	.999	.062
.6 "	.0463	.056	+ 4°	.998	.056
.7 "	.0399	.050	+ 5°	.996	.050
.8 "	.0323	.042	+ 6°	.994	.042
.9 "	.0225	.0325	+ 7°	.992	.032
1.0 "	.0100	.020	+ 9°	.988	.020
1.0 (Bottom)	.0005	.0105	+174°	− .994	−.0105
.9 "	.0035	.0065	+177°	− .999	−.0065
.8 "	.0065	.0035	+179°	−1.000	−.0035
.7 "	.0069	.003	+181°	−1.000	−.002
.6 "	.0052	.005	+185°	− .996	−.005
.5 "	.0023	.008	+182°	−1.000	−.008
.4 "	.0002	.010	+180°	−1.000	−.010
.3 "	.0009	.009	+177°	− .999	−.009
.2 "	.0050	.005	+174°	− .994	−.005
.1 "	.0102	.002	+179°	−1.000	−.002
.0 "	.0033	.013	+205°	− .906	−.012



Wind
Velocity Direction
Area 4.65 □"
Velocity
0 .01 .02 .03 .04 .05 .06 .07
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
Area 9.25 □"
V cos θ
0 .01 .02 .03 .04 .05 .06 .07
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
Chord
Plots Showing
Graphical Integration
of
$\Gamma = \int_c V \cos\theta \, ds$
for
RAF 15 Section
Chord = 1"
$\frac{dV}{dh} = 1$

The area under the curve is 9.25"².

The scale is 1"² = .1 × .05 = .005

As we considered a unit chord and unit gradient, the effect on any R.A.F. 15 wing would be expressed by

$$\Gamma_a = .0465 \times \frac{dV}{dh} \times (\text{chord in feet})^2 \quad (18)$$

From the foregoing discussion and the similarity of the diagrams shown on page 28 we are led to suspect that the circulation, due to any given wind velocity gradient is independent of the shape of the wing section and depends only upon the area included within its bounding contour. The proof that this is actually the case, provided the wind velocity varies as a linear function of the altitude, follows. We know that the wind velocity does not vary as a straight line, but, over the distance affected by the wing,(its projected height), the approximation is very close. If we consider the wind directed along the positive direction of the x axis, and zero at the origin, then

$$V = k\,y$$

where k is a function of $\frac{dV}{dh}$ and is constant when the wind velocity varies as a straight line. Consider two points A and B in the x,y plane joined by any curve whatever. The area under this curve is

$$A_1 = \int_a^b y_1 dx_1$$

The area under any other curve joining these points is

$$A_2 = \int_a^b y_2 dx_2$$

If we consider the first curve to be the upper boundary of a wing section and the second curve the lower boundary, then the area of the wing section is

$$A = A_1 - A_2 = \int_a^b y_1 dx_1 - \int_a^b y_2\, dx_2$$

but

$$y = \frac{V}{k} \qquad \text{and} \qquad dx = \cos\,\theta\; ds$$

so

$$A = \frac{1}{k}\left[\int_a^b V_1 \cos\,\theta_1 ds_1 - \int_a^b V_2 \cos\,\theta_2 ds_2\right]$$

but this integral is exactly the circulation around the wing due to the wind velocity gradient.

The circulation around any wing due to the gradient is then

$$\Gamma_G = k\ A$$

From (18) we have then, for any wing:

$$\Gamma_G = k_1 \frac{dV}{dh}\, c^2 \qquad (19)$$

where k_1 is a constant depending only on the area of a wing section of unit chord and c is the chord in feet.

The values of the constant k_1, calculated for some of the more common wing sections, are tabulated below

Section	k_1
R A F 15 (mod.)	.0461
R A F 15	.0465
Eiffel 36	.0465
Albatross	.0467
Curtiss C62	.0541
U S A 27	.0796
Clark Y	.0811
Gott 387	.1035

The preceding reasoning also shows that the circulation due to gradient is unaffected by changing the angle of attack, as long as we consider a straight line variation.

VI.
THE KUTTA-JOUKOWSKI THEORY

In the work which follows, the notation adopted by the National Advisory Committee for Aeronautics will be employed. In this system the origin is taken at the center of gravity of the airplane, the x axis corresponds to the longitudinal axis and is positive toward the tail, the y axis is the lateral axis and is positive to the left, and the z axis is mutually perpendicular to these and positive upwards. Furthermore

X is the component of force along the x axis,
Y is the component of force along the y axis,
Z is the component of force along the z axis,

u is the component of velocity along the x axis,
v is the component of velocity along the v axis,
w is the component of velocity along the z axis,

A flow, such that all the particles of a fluid flow parallel to a fixed plane is called "plane parallel flow". As the fluid flows parallel to the plane, the component of velocity normal to the plane is zero. If there is no rotation, there exists a velocity potential. If we use the fixed plane for the xy plane and φ for the velocity potential, the components of velocity along the axes are

$$u = \frac{\partial \varphi}{\partial x} \quad \text{and} \quad v = \frac{\partial \varphi}{\partial y} \qquad (20)$$

differentiating

$$\frac{du}{dx} = \frac{\partial^2 \varphi}{\partial x^2} \quad \text{and} \quad \frac{dv}{dx} = \frac{\partial^2 \varphi}{\partial y^2} \qquad (21)$$

The equation of continuity in plane parallel flow is

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

But if the fluid is considered incompressible the density is constant, and this becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (22)$$

and from (21)

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad (23)$$

This is the equation of Laplace for a function of two variables.

As the velocity, at any point in a fluid, is directed along the tangent to the streamline at that point, the projections of the component velocities, u and v, on the coordinate axes, are proportional to the projections of dx and dy, which are the components of ds, an element of the curve representing the streamline. That is

$$\frac{dx}{u} = \frac{dy}{v}$$

$$u \; dx - v \; dy = 0 \qquad (24)$$

This is the differential equation of the streamline. If we let

$$u \; dx - u \; dy = d \; \varphi \qquad (25)$$

then

$$d\varphi = 0 \quad \text{or} \quad \psi = \text{const.}$$

This function ψ is called the stream function. from (25)

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = - \frac{\partial \psi}{\partial x} \qquad (26)$$

and from (20)

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y} \quad \text{and} \quad \frac{\partial \varphi}{\partial y} = - \frac{\partial \psi}{\partial x} \qquad (27)$$

from this

$$\frac{\partial \varphi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \varphi}{\partial y} \frac{\partial \psi}{\partial y} = 0$$

or

$$\frac{\frac{\partial\varphi}{\partial y}}{\frac{\partial\varphi}{\partial x}} = -\frac{\frac{\partial\psi}{\partial x}}{\frac{\partial\psi}{\partial y}}$$

This is the condition for orthogonality, and if we consider the two families of curves, φ = const. and ψ = const., they form an orthogonal net work.

If we consider any closed contour, exposed to a non-rotational current having a velocity V_0 at infinity and directed along $-x$, and take a velocity potential

$$\varphi = -V_0 x + f \qquad (28)$$

where f satisfies Laplace's equation (23) then

$$\left.\begin{aligned} u &= \frac{\partial\varphi}{\partial x} = -V_0 + \frac{\partial f}{\partial x} \\ v &= \frac{\partial\varphi}{\partial y} = \frac{\partial f}{\partial y} \end{aligned}\right) \qquad (29)$$

As we go farther and farther away from the center of the contour, the circulation approaches more and more nearly that around a rectilinear vortex filament, in which the individual particles describe concentric circles. If then, we consider a circle of large radius R, concentric with the contour, and remembering that at infinity

$$u = -V_o, \quad \text{and} \quad v = 0$$

we see that for any point on this circle of large radius

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$$

The principle of the conservation of momentum states that no momentum can be lost and therefore any momentum which the body, represented by the original contour, has must be accounted for by the momentum imparted to the airstream. But momentum is mass times velocity, or force divided by time. Force is then the rate of change of momentum or the change in momentum per unit of time. The reason for making a point of this simple relation is that in the work which follows we use the principle of the conservation of momentum to prove a point, in the equations which are set up we are dealing with forces and not momentums.

If we compute the rate of change of momentum of the fluid bounded by the circle of radius R and by the original contour, the forces which act on the fluid must be equilibrated by the forces due to the rate of change of momentum of the fluid.

Let X and Y be the components of pressure which the fluid exerts on the body, p the hydrodynamic pressure acting on an element, ds, of the large circle and α and β the angles made, by an interior drawn normal from this element, with the x and y axes. The velocity at ds is V, which may be broken up into its components u and v where

$$V = \sqrt{u^2 + v^2}$$

The component of u perpendicular to ds is u cos α, and the component of v perpendicular to ds is v cos β. The mass of the fluid which enters the circle thru ds, in unit time, or the rate at which the fluid enters, is

$$\rho(u \cos \alpha + v \cos \beta)ds$$

but at infinity

$$u = -V_0 + \frac{\partial f}{\partial x} \quad \text{and} \quad v = \frac{\partial f}{\partial y}$$

so on the circle of large radius

$$\rho(u \cos \alpha + v \cos \beta)ds = \rho[(-V_0+\frac{\partial f}{\partial x})\cos \alpha+\frac{\partial f}{\partial y}\cos \beta]ds$$

If we multiply this latter quantity by the components of velocity

$$V_\sigma + \frac{\partial f}{\partial x} \qquad \text{and} \qquad \frac{\partial f}{\partial y}$$

we obtain the projections of the rate of change of momentum along the x and y axes. Considering all the forces acting, we have

$$-X+\int p \cos \alpha \, ds+\rho\int(-V_\sigma+\frac{\partial f}{\partial x})[(-V_0+\frac{\partial f}{\partial x})\cos \alpha+\frac{\partial f}{\partial y}\cos \beta]ds = 0 \qquad (30)$$

and

$$-Y+\int p \cos \beta \, ds+\rho\int(\frac{\partial f}{\partial y})[(-V_\sigma+\frac{\partial f}{\partial x})\cos \alpha+\frac{\partial f}{\partial y}\cos \beta]ds = 0 \qquad (31)$$

In (30) $-X$ is the reaction of the body against the fluid, a force. $\int p \cos \alpha \, ds$ is the x component of the hydrodynamic pressure, a force. The last integral is the product of the x component of velocity and the rate of change of momentum, another force.

The hydrodynamic pressure is expressed by the formula

$$p = \text{const.} - \frac{1}{2}\rho V^2$$

$$= \text{const.} - \frac{1}{2}\rho[u^2 + v^2]$$

$$= \text{const} - \frac{1}{2}\rho[V_o^2 - 2V_o\frac{\partial f}{\partial x} + (\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2]$$

Substituting this value for p in the force equations we obtain

$$X = -\frac{\rho}{2}V_o^2\int\cos\alpha\, ds + \rho V_o\int\frac{\partial f}{\partial x}\cos\alpha\, ds - \frac{\rho}{2}\int(\frac{\partial f}{\partial x})^2\cos\alpha\, ds - \frac{\rho}{2}$$

$$-\frac{\rho}{2}\int(\frac{\partial f}{\partial y})^2\cos\alpha\, ds - \rho V_o\int\frac{\partial f}{\partial x}\cos\alpha\, ds + \rho\int(\frac{\partial t}{\partial x})^2\cos\alpha\, ds$$

$$+\rho\int\frac{\partial f^2}{\partial x\partial y}\cos\beta\, ds - \rho V_o\int[(-V_o + \frac{\partial f}{\partial x})\cos\alpha + \frac{\partial f}{\partial y}\cos\beta]ds$$

$$Y = -\frac{\rho}{2}V_o^2\int\cos\beta\, ds + \rho V_o\int\frac{\partial f}{\partial x}\cos\beta\, ds - \frac{\rho}{2}\int(\frac{\partial f}{\partial x})^2\cos\beta\, ds$$

$$-\frac{\rho}{2}\int(\frac{\partial f}{\partial y})^2\cos\beta\, ds - \rho V_o\int\frac{\partial f}{\partial y}\cos\alpha\, ds + \rho\int\frac{\partial f^2}{\partial x\partial y}\cos\alpha\, ds$$

$$+ \rho\int(\frac{\partial f}{\partial y})^2\cos\beta\, ds$$

Neglecting second order differentials and terms of the form $\int \cos\alpha\, ds$, as the former are <u>small</u> and the latter zero, from

$$\int_0^{2\pi} \cos\alpha\, ds = R\int_0^{2\pi} \cos\alpha\, d\alpha = 0 ,$$

we obtain

$$X = -\rho V_0 \int [(-V_0 + \frac{\partial f}{\partial x})\cos\alpha + \frac{\partial f}{\partial y} \cos \beta] ds$$

But this is the total mass of fluid entering and leaving the circle of radius R and, as the fluid is considered incompressible, the total amount cannot change; so the same amount leaves as enters, and therefore

$$X = 0$$

similarly

$$Y = -\rho\, V_0 \int \frac{\partial f}{\partial y} \cos\alpha - \frac{\partial f}{\partial x} \cos \beta) ds$$

but $\frac{\partial f}{\partial y}\cos\alpha + \frac{\partial f}{\partial x}\cos\beta$ is the projection on a tangent to the circle of that part of the velocity which depends upon the function f. Therefore, the integral represents the circulation around the circle of radius R. As we started with the assumption that there were no vortices between the contour and the large circle, this is also the circulation around the contour; so

$$Y = -\rho\, V_0 \Gamma$$

This is the Kutta-Joukowski theorem and states that:

When a current, whose velocity is V_0 at infinity, flows along any closed contour, and the circulation around the contour is Γ, the resultant of the pressures of the fluid is equal to the product of the vector, representing the velocity at infinity, by the circulation and by the density of the fluid. The direction of the force is obtained by turning the vector, V_0, 90^0 in a direction opposite to the circulation.

Putting this in other words, we have, for any portion of a wing of infinite span, in a perfect fluid

$$\text{Drag} = 0 \qquad (32)$$

$$\text{Lift} = \rho\, \Gamma V_0 l \qquad (33)$$

where l is the length of the position considered.

Another method of arriving at the same result is by means of the complex variable. Suppose we take

$$z = x - i\, y$$

and consider

$$F(z) = \varphi + i\psi$$

where φ and ψ are real functions of x and y.

Taking partials we have

$$\frac{\partial\varphi}{\partial x} + i\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial y} - i\,\frac{\partial\varphi}{\partial y}$$

equating reals and imaginaries

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y} \quad \text{and} \quad \frac{\partial\varphi}{\partial y} = -\frac{\partial\psi}{\partial x}$$

From this we see that there exists, between the functions of φ and ψ, when considered as functions of the complex variable, z, the same relation that existed when they represented the velocity potential and the stream function. This property of the complex variable furnishes a means of obtaining any desired type of flow. In order to do this, we take different functions of z and group the real part φ and the imaginary part $i\psi$ where, as before, φ is the velocity potential and ψ is the stream function.

Joukowski shows that we may take any simple flow or combination of simple flows and by a conformal transformation develop any complicated flow we may desire. That is, if we take

$$z = x + i\,y$$

and let

$$F(z) = \varphi + i\,\psi$$

represent the original flow then the transformed flow may be expressed by

$$\bar{\zeta} = \xi + i\,\eta$$

where ζ is some function of z such that for each point, (x,y) in the original flow there corresponds a point (ξ,η) in the transformed flow. Then if the equipotential lines φ = const. and the streamlines ψ = const. formed an orthogonal network in the original current, the transformed lines will also form an orthogonal net work. At infinity the two currents have the same direction and the velocity of the transformed current is some constant times the velocity of the original current. Critical points in the transformed correspond to critical points in the original current and the circulation is unaltered by the transformation.

For an example we could take the flow around a circular cylinder and transform the cylinder into a wing. The flow is also transformed, and a study of the simple flow around the cylinder gives us all the information we desire regarding the complicated flow around the wing. The method employed in the selection of the proper function is to try a great many and choose those that appear to fit the case in hand. The really useful ones are not so numerous but that a mathematician can at once select the one he desires, unless the flow is unusual.

The function used by Joukowski, in the development of his series of airfoils, was

$$f(z) = z + \frac{b^2}{z}$$

He developed his wings, calculated their lift, and then measured the lift and drag in a wind tunnel, simulating conditions for infinite span by using a wing which went entirely across the tunnel and came as close to the sides as was possible without touching them. Joukowski's experiments checked his theory very closely except that there was a slight drag, due to the viscosity of the air, which was practically constant for all angles of attack as long as the flow remained smooth, and varied with the exposed wing surface and the shape of the wing section.

The production of circulation around a wing and the lift incident thereto may be explained as follows: Suppose the wing to be set, in an air flow, at the angle of attack for zero lift. The circulation is zero, and there is no lift. Now let the angle of attack be increased. The rest point, which was at the trailing edge, moves around to a point, slightly forward of the trailing edge, on top of the wing. The fluid on the lower side of the wing must flow around the trailing edge, and leave the wing at this point. At the trailing edge the velocity is very high, and a vortex is immediately formed. This vortex, according to Helmholtz's third theorem, goes off with the fluid, leaving a circulation around the wing which exactly balances it. If the angle of attack, and consequently the circulation and the lift, is not changed, no more vortéces are given off; but if the angle of attack is changed, positive or negative vortáces (according to the sense of the change) are given off until the algebraic sum of the strengths of all the vortáces given off just equals the required circulation, after which conditions become steady again and we have a smooth flow off the trailing edge.

VI.

PRANDTL'S INDUCED DRAG THEORY.

In the preceeding chapter we considered a wing of infinite span in which the circulation, and consequently the lift distribution, is constant along the span.

In a wing of finite span the lift usually falls off to zero at the wing tips, and the distribution along the span depends upon the shape of the wing, the wing section used, and the angle of attack. The distribution of circulation must be the same as the distribution of lift; so the circulation will be a maximum at the center and fall off to zero at the wing tips. The circulation, however, cannot change without cutting vortices; consequently there will be vortices, whose axes are roughly parallel to the direction of flight, coming off the trailing edge. Ordinarily these vortices will be weak near the center of the wing, where the change in circulation is slight, and very strong at the tips, where the change in circulation is a maximum. The sum of all the vortices coming off from either half of the wing is equal to the circulation around the center of the wing. We may consider the vortices as coming off the trailing edge of the wing in the form of a sheet or ribbon. This ribbon will be constant in width, normally the width of the wing, and thickest where the rate of change of circulation is greatest. At some distance behind the wing this ribbon starts to curl up on itself from both sides and finally exists only as two rather weak cores.

If the axes are taken as in the N.A.C.A. notation, and we consider a wing of span, s, for any lift distribution, the

$$\text{Total Circulation} = \Gamma_{tot} = \int_{-\frac{s}{2}}^{+\frac{s}{2}} \Gamma \, dy \qquad (34)$$

where Γ is the circulation at any point along the wing and dy is an element of length. From the Kutta-Joukowski Theorem, (32)

$$dR = \rho \, V_o \, \Gamma \, dy$$

The total reaction against the wing is then

$$R_{tot} = \rho \, V_o \, \Gamma_{tot} = \rho \, V_o \int_{-\frac{s}{2}}^{+\frac{s}{2}} \Gamma \, dy \qquad (35)$$

The strength of the vortices flowing off from any element dy, along the trailing edge is

$$\frac{d\Gamma}{dy} \, dy \qquad (36)$$

In the x, y plane there is no component of velocity parallel to the plane; so, if we consider u, v, and w the components of velocity due to the circulation ,

$$u = 0 \quad \text{and} \quad v = 0$$

The component of the disturbance velocity normal to the x,y plane may be calculated in the same way that Boit-Savart computed the magnetic field around a conductor. We may show that the velocity at any point P, due to a vortex element ds is

$$dv = \frac{\Gamma\ ds\ \sin\ \alpha}{4\ \pi\ r^2} \qquad (37)$$

where α is the angle between the vortex element and a line joining it with P, and r is the distance from P to ds. A proof of this is given in Ramsey's "Hydrodynamics".

If we let h be the perpendicular distance from the vortex to the point,

$$s = h \cot \alpha$$

$$ds = -\frac{h}{\sin^2\alpha}\, d\, \alpha$$

and

$$r = \frac{h}{\sin\ \alpha}$$

The velocity at any point, P, due to an infinitely long rectilinear vortex is then

$$v = \frac{1}{4\pi} \int_{-\infty}^{+\infty} \frac{ds \sin \alpha}{r^2} = \frac{1}{2\pi h} \qquad (38)$$

If the vortex goes to infinity in one direction only we have

$$v = \frac{1}{4\pi} \int_{0}^{\infty} \frac{ds \sin \alpha}{r^2} = \frac{1}{4\pi h} \qquad (39)$$

and for a finite length of a vortex

$$v = \frac{1}{4\pi h} \int_{\alpha_1}^{\alpha_2} \sin \alpha \, d\alpha = \frac{1}{4\pi h}[\cos \alpha_1 - \cos \alpha_2] \qquad (40)$$

where α_1 and α_2 are the angles made by the vortex with the lines joining its ends to P. From this

$$w = \frac{\Gamma}{4\pi h} (\cos \alpha_1 - \cos \alpha_2)$$

Along the lifting line representing the wing itself, the component due to the transverse circulation is zero. The vertical velocity due to the element dy of the longitudinal vortex, is, from (39),

$$w = \frac{\Gamma}{4\pi h}$$

$$dw = \frac{1}{4\pi} \frac{d\Gamma}{dy} \frac{dy}{(y'-y)} \qquad (41)$$

where y' is the point on the lifting line at which we are measuring the velocity and y is the point at which the elementary vortex dy is considered to come off. The total vertical velocity at y' is then

$$w = \frac{1}{4\pi} \int_{-\frac{s}{2}}^{+\frac{s}{2}} \frac{d\Gamma}{dy} \frac{dy}{(y'-y)} \qquad (42)$$

For $y = y'$ the function passes thru infinity; so we must consider what Prandtl calls the "chief value" of the integral, or

$$\lim_{\varepsilon \to 0} \left(\int_{-\frac{s}{2}}^{y'+\varepsilon} \frac{d\Gamma}{dy} \frac{dy}{(y'-y)} + \int_{y'-\varepsilon}^{+\frac{s}{2}} \frac{d\Gamma}{dy} \frac{dy}{(y'-y)} \right) \qquad (43)$$

We can show, in a similar manner, that the vertical velocity at any point $x'y'$, in the x,y plane but not on the lifting line is

$$w = \frac{1}{4\pi} \int_{-\frac{s}{2}}^{+\frac{s}{2}} \int_{x'=0}^{x'=\varepsilon} \frac{d\Gamma}{dy} \frac{(y'-\delta)\, dy\, dx'}{[(y'-y)^2 + x'^2]^{\frac{3}{2}}} \qquad (44)$$

As a result of the downward velocity, or downwash, the flow past the wing is not in the direction of the velocity V_0 but is inclined downward, to the rear, by an amount φ where

$$\varphi = \tan^{-1} \frac{w}{V_0} \tag{45}$$

The resultant pressure on the wing, R, is normal to the air stream; so it is inclined backward an amount φ. The lift and drag, however, are to be taken, respectively, perpendicular to and along V_0; so

$$\text{Lift} = L = R \cos \varphi$$

$$\text{Induced Drag} = D_{ind} = R \sin \varphi = L \tan \varphi$$

As long as the angle φ is not large, we may consider $\tan \varphi = \sin \varphi$ and $\cos \varphi = 1$, whence

$$\text{Lift} = R$$

$$\text{Induced Drag} = R \tan \varphi$$

Every element of lift

$$d\Gamma' = \rho V_0 \Gamma' dy'$$

then contributes to the drag an amount

$$dD_{ind} = \tan \varphi \, dL' = \rho \Gamma' w \, dy'$$

The total drag is then

$$D_{ind} = \rho \int_{-\frac{s}{2}}^{+\frac{s}{2}} \Gamma' \, w \, dy' = \frac{\rho}{4\pi} \int_{-\frac{s}{2}}^{+\frac{s}{2}} \int_{-\frac{s}{2}}^{\frac{s}{2}} \frac{\Gamma' \frac{d\Gamma}{dy} \, dy \, dy'}{y' - y} \qquad (48)$$

where the primes refer to a functional relation with y'.

These formulas represent a first approximation, and are valid only as long as the disturbance velocity, due to the vortices, is small compared to V_o, and as long as the circulation at the tips is zero. This is almost always true with the wings ordinarily employed. It is difficult to conceive a case where there could be a positive circulation out past the wing tip set up by the wing, but we could easily have the circulation fall off to zero any desired distance inside the tips by a suitable variation of the angle of attack along the span.

For rectangular wings, of the aspect ratios commonly used, and at normal angles of attack, the lift distribution is approximately a semi-ellipse. The formulas given above may be greatly simplified by this approximation, as for the ordinates of the ellipse we have the various values of Γ, and the principal semi-axes are $\frac{b}{2}$ and Γ_{max}. The equation for this ellipse, which has its center at the origin, is

$$\frac{y^2}{(\frac{s}{2})^2} + \frac{\Gamma^2}{(\Gamma_{max})^2} = 1$$

$$\therefore \quad \Gamma = \Gamma_{max}\sqrt{1 - (\frac{y}{s})^2} = \Gamma_{max}\frac{2}{s}\sqrt{(\frac{s}{2})^2 - y^2} \qquad (47)$$

differentiating

$$\frac{d\Gamma}{dy} = \frac{-\Gamma_{max}\ y}{\frac{s}{2}\sqrt{(\frac{s}{2})^2 - y^2}} \qquad (48)$$

Substituting the values of (58) in equations (42) we obtain

$$w = \frac{\Gamma_{max}}{2\pi s}\int_{-\frac{s}{2}}^{+\frac{s}{2}} \frac{y\ dy}{\sqrt{(\frac{s}{2})^2 - y^2}\ (y'-y)} = \frac{\Gamma_{max}}{2\ s}$$

That is, the downwash is constant over the entire span.

Substituting (47) in (35)

$$L = \rho\ V\ \Gamma_{max} \frac{2}{s}\int_{-\frac{s}{2}}^{+\frac{s}{2}} \sqrt{\left(\frac{s}{2}\right)^2 - y^2}\ dy = \rho\ V_0 \Gamma_{max} \frac{\pi}{4} s \qquad (50)$$

$\therefore$ substituting the value of Γ_{max} from (50) in (49) we obtain

$$w = \frac{2L}{\pi\ \rho\ s^2 V_0} \qquad (52)$$

$$D_{ind} = \frac{2L^2}{\pi\ \rho\ s^2 V_0^2} \qquad (53)$$

(It can be proved that the elliptic lift distribution, treated above, gives the least induced drag, for any given lift, that it is possible to obtain.)

From (50) we see that the lift of a wing varies directly as the density, the velocity, the circulation, and the span. From (52) the drag varies as the square of the lift and inversely as the density, the aspect ratio and the square of the velocity. These formulas were developed for a monoplane, but Prandtl has shown that for every multiplane there is a monoplane having the same characteristics, so they are applicable to biplanes as well. Also, we have been dealing throughout with the induced drag only, but inasmuch as the total drag is the sum of the induced drag and the profile drag, and the profile drag is practically constant for ordinary angles of attack this is satisfactory.

In any given airplane, the span is constant; and in level flight the lift must equal the weight. At any given altitude, neglecting atmospheric variations, ρ is constant. Then for any given V_0, Γ is determined. But the drag, for any given V_0, is determined by the same constants as the lift, so the drag is the same regardless of how the circulation is produced. Therefore, in level flight, we would expect that tho the wind velocity gradient produces circulation around the wing, this circulation cannot be utilized either in the production of lift or the diminution of drag. We should expect, however, since the difference between the circulation required and the circulation due to the wind velocity gradient must be generated by the wing itself, and since this circulation is a function of the angle of attack, that the angle of attack would be slightly less when flying into the wind than when flying with the wind.

Prandtl shows that, due to the downward velocity imparted to the air by the vortices around a wing and flowing off the wing, there is an increased pressure under the wing. In order to prove this it is convenient and sufficiently accurate, except in the immediate vicinity of the wing, to consider the lift distribution ~~constant~~ across the wing constant and a somewhat decreased span. We then have a simplified representation called a horseshoe vortex. The amount of this pressure increase is

$$p' = \frac{L}{2\pi}\frac{h}{R^3} \qquad (54)$$

where L is the total lift, h the vertical height and R the distance from the plane to the point on the ground at which the pressure is being measured. The maximum value of this pressure increase, for any given h, is

$$p'_{max} = \frac{L}{2\pi h^2} \qquad (55)$$

and is directly under the plane.

The effect of increasing the density is to increase the lift and reduce the drag. Both of these changes are favorable; so we should expect an increase in efficiency at very low altitudes, especially when flying near minimum power.

VII.

FREE FLIGHT TESTS.

In order to determine whether the foregoing principles of Applied Mechanics and Hydrodynamics suffice to explain the effect of wind velocity on airplane performance, free flight tests were made with both land and sea planes. The important characteristics of the planes used are given below.

(1) The JN6HS is a remodeled JN4H, (an obsolete Army Training Plane,) incorporating certain structural changes but no important aerodynamic alterations. Its characteristics are:

Type		Biplane	- Training
Motor		Wright I	150 HP
Weight		2017#	
Wing area		352'²	
Span	Upper	43'7 3/8"	
	Lower	33' 11 1/4"	
Length		27' 0 1/2"	
Height		9' 10 5/8"	
Chord		4' 11 1/2"	
Gap		5' 1 1/2"	
Stagger		16°	
Dihedral		1°	

Incidence	2°
Airfoil used	Eiffel 36
High Speed	93 m/hr
Landing speed	44 m/hr

(2) The DH4M1 is the steel fuselage Corps Observation Plane built by the Boeing Airplane Company and designed to take the ordinary DH wings. Its characteristics are:

Type	Biplane - Observation
Motor	Liberty 12 400 HP
Weight	3876#
Wing Area	440'²
Span	42' 5 15/32"
Length	30' 1 13/14"
Height	10' 6"
Chord	5' 6"
Gap	5' 10"
Stagger	12"
Dihedral	3°
Incidence	3°
Airfoil used	R A F 15 (mod)
High Speed	123 m/hr
Landing Speed	55 m/hr.

(3) The TG4 is a single pontoon seaplane designed and built by the Naval Aircraft Factory for training. It has the following characteristics:

Type	Biplane - Training Seaplane
Motor	Aeromarine T6 200 HP
Weight	2985#
Wing Area	370'²
Span	36'
Length	30' 1 "7/32"
Height	11' 9 11/32"
Chord	5' 6"
Gap	5' 6"
Stagger	12°
Dihedral	1°
Incidence	2°
Airfoil used	Albatros
High speed	98.5 m/hr
Landing speed	47 m/hr.

(4) The TS is a single seater fighting plane developed by the Naval Aircraft Factory and the Curtiss Airplane and Motor Company, in 1921 - 1922, as a shipboard fighter for Navy use. Its characteristics are:

Type		Biplane - Fighter
Motor		Lawrence J1 200 HP
Weight		1920#
Wing Area		227.8'²
Span		25'
Length		21'6"
Height		9'
Chord		4'9"
Gap		5'6"
Stagger		0°
Dihedral	Upper	0
	Lower	3°
Incidence		0°
Airfoil used		U S A 27
High speed		124 m/hr
Landing speed		48 m/hr

In order to eliminate, as far as possible, any turbulence or vertical currents in the air, an effort was made to run the tests when the sky was overcast and when the wind was from the east or north east, as in the latter case it had a long unobstructed approach over the ocean, and so had the tendency of minimizing fluctuations in the flow. This was not always possible, as it was very difficult to get an ideal set of conditions at a time when advantage could be taken of them. For example, a strong east wind almost always preceded a storm; so ideal conditions were maintained, at best, only for a very short period of time. Furthermore, if one went out over the ocean a sufficient distance to get away from the turbulence due to the land, the ground swell, in a wind of sufficient strength to give conclusive results, was so large as to seriously disturb the air flow. This inequality in flow was indicated by the smaller waves superimposed on the ground swells. These areas of increased velocity were small in extent and recurrent in nature. They usually moved along with the wind for a short distance, disappeared, and then reappeared at approximately the same place. Occasionally a small patch of disturbed water would remain in the same place and retain practically the same shape thru a considerable period of time.

All of the seaplane tests and the majority of the land plane tests were made over the water at a considerable distance from land. This gave the most even flow that it was possible to obtain on any given day, in velocity, gradient, and direction. There was an obvious psychological effect, especially with the land planes, when going with the wind, which made it very difficult to get minimum readings at low altitudes.

This disadvantage over flying over land was more than overcome by the greater smoothness of air flow.

The altitudes given refer to the height of the "mean lifting surface" above the ground (or water). The "mean lifting surface" is that single surface, which, at the altitude considered ~~would~~ represents exactly all the conditions of lift, drag, moment etc., resulting from the two wings of the biplane. Thus, when we say the plane was held at an altitude of ten feet, the wheels or pontoons were from three to five feet above the earth's surface.

Measurements of the wind velocity were made on a Robinson cup animometer and checked, in certain cases, by means of light hydrogen balloons which were photographed with a constant speed moving-picture camera. Their displacement over any given time interval gives an extremely accurate measure of the wind velocity. This, taken together with the vertical velocity of the balloon, assuming the absence of vertical currents, gives a means of checking over previously cited formulas for velocity gradient.

Measurements were taken at various altitudes while the plane was flying against the wind and with the wind:

(a) In level flight.

(b) During climb.

(c) While turning.

a. The plane was flown at various constant air speeds and the motor revolutions per minute required to maintain level flight at a given altitude noted. It was very difficult to obtain accurate readings at points near minimum power, as a considerable difference in air speed has only a slight effect on the power required. Futhermore, the rate of climb is so slow at this point that any slight air disturbance completely upsets equilibrium conditions and allows the plane to settle. Any vertical velocity downward, however small, causes a momentum not readily overcome by the lift, and since increasing the angle of attack increases the drag more than the lift, the forward velocity of the plane decreases, and the plane drops with increasing rapidity until checked with the motor.

The method used in obtaining the air speed for level flight at 200' was to select an approximate speed and hold it constant over a considerable time. If the plane climbed the speed was too low and a slightly increased speed was tried. If one speed was too high and another, two miles less, too low the mean speed was accepted without further check.

The following tables and graphs show the motor revolutions per minute and corresponding air speeds for the various conditions cited.

Run 1.

Date	May 20
Time	3:30 P.M.
Place	North of Boston Light Ship.
Wind	30 m/hr
Altitude	10'
Plane	JN6HS (AS 24-244)

Up Wind		Down Wind	
RPM	Air Speed	RPM	Air Speed
1530	84	1530	83
1430	76	1440	77
1350	71	1350	71
1265	65	1260	64
1180	57	1180	57
1150	53	1150	54
1110	48	1110	48

At 1110 RPM the plane would settle if a turn was attempted, or if it was disturbed by a bump, and full motor was required to keep the plane from dropping into the water.

Run 2.

Date	May 20
Time	4:00
Place	North of Boston Light Ship
Wind	30 m/hr
Altitude	200'
Plane	JN6HS (AS 24-244)

Up Wind		Down Wind	
RPM	Air Speed	RPM	Air Speed
1540	84	1540	84
1450	77.5	1450	77.5
1300	66.5	1300	67
1200	57	1200	57
1180	55	1180	54
1150	52	1150	52
1170	42	1170	43
1250	41	1250	41

An attempt was made to take readings at 40 mi-hr but it was impossible to obtain steady conditions as, at the very large angle of attack required, the controls were not effective.

Run 3.

Date	May 2.
Time	2:30
Place	The Graves Lighthouse.
Wind	40 m/hr.
Altitude	10'
Plane	DH4M1 (AS 31432)

Up Wind		Down Wind	
RPM	Air Speed	RPM	Air Speed
1650	117	1650	116.5
1500	105	1500	105
1400	97	1400	97
1260	85	1265	86
1130	70	1130	70
1120	60	1110	60

This appeared to be very close to minimum power, and it was not considered advisable to attempt a lower speed at this altitude as the plane was beginning to answer the controls very sluggishly.

Run 4.

Date	May 2
Time	3:00 P.M.
Place	The Graves Light House.
Wind	35 m/hr
Altitude	200'
Plane	DH4M1 (AS 31432)

Up Wind		Down Wind	
RPM	Air Speed	RPM	Air Speed
1650	116	1650	116
1490	103	1480	102
1350	91	1350	91
1220	80	1220	80
1170	73	1160	72
1150	65	1150	65
1140	60	1150	60

Settled slowly at all speeds from 60 to 70 at 1130 RPM.

Run 5.

Date	May 22
Time	10 A.M.
Place	Boston Harbor
Wind	20 m/hr
Altitude	10'
Plane	TG4 (A 6348)

Up Wind		Down Wind	
RPM	Air Speed	RPM	Air Speed
1780	90	1780	90
1690	83.5	1695	84
1600	76	1590	75.5
1510	69	1500	68
1460	65.5	1470	65.5
1400	56.5	1400	59
1360	53	1360	53
1340	48	1340	48
1400	43	1410	43.5

The seaplane tests were made at an average altitude of about two feet less than the land plane tests, as no harm greater than spoiling a run came from occasionally touching the water, while this had to be assiduously guarded against in the land planes.

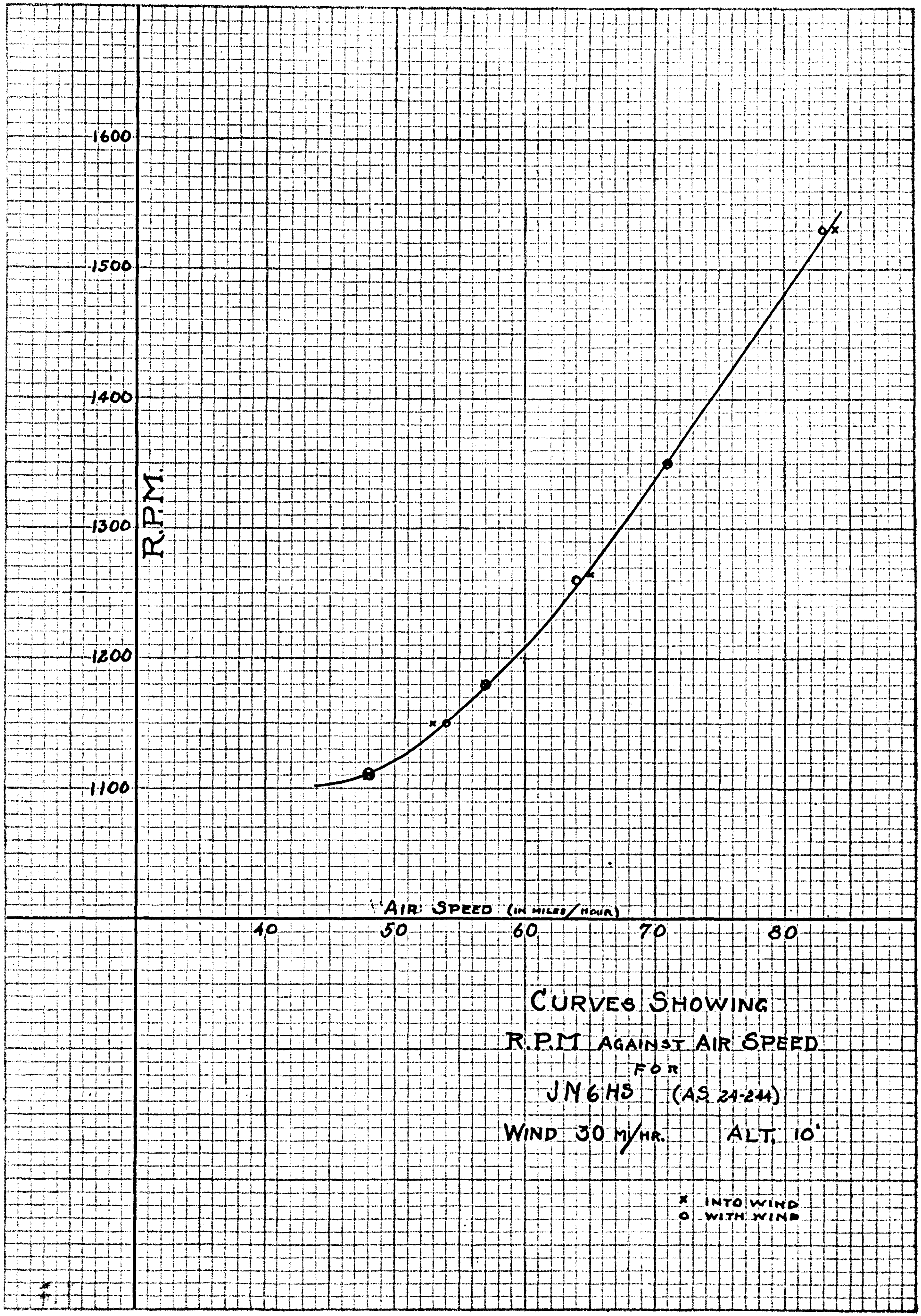
Run 6.

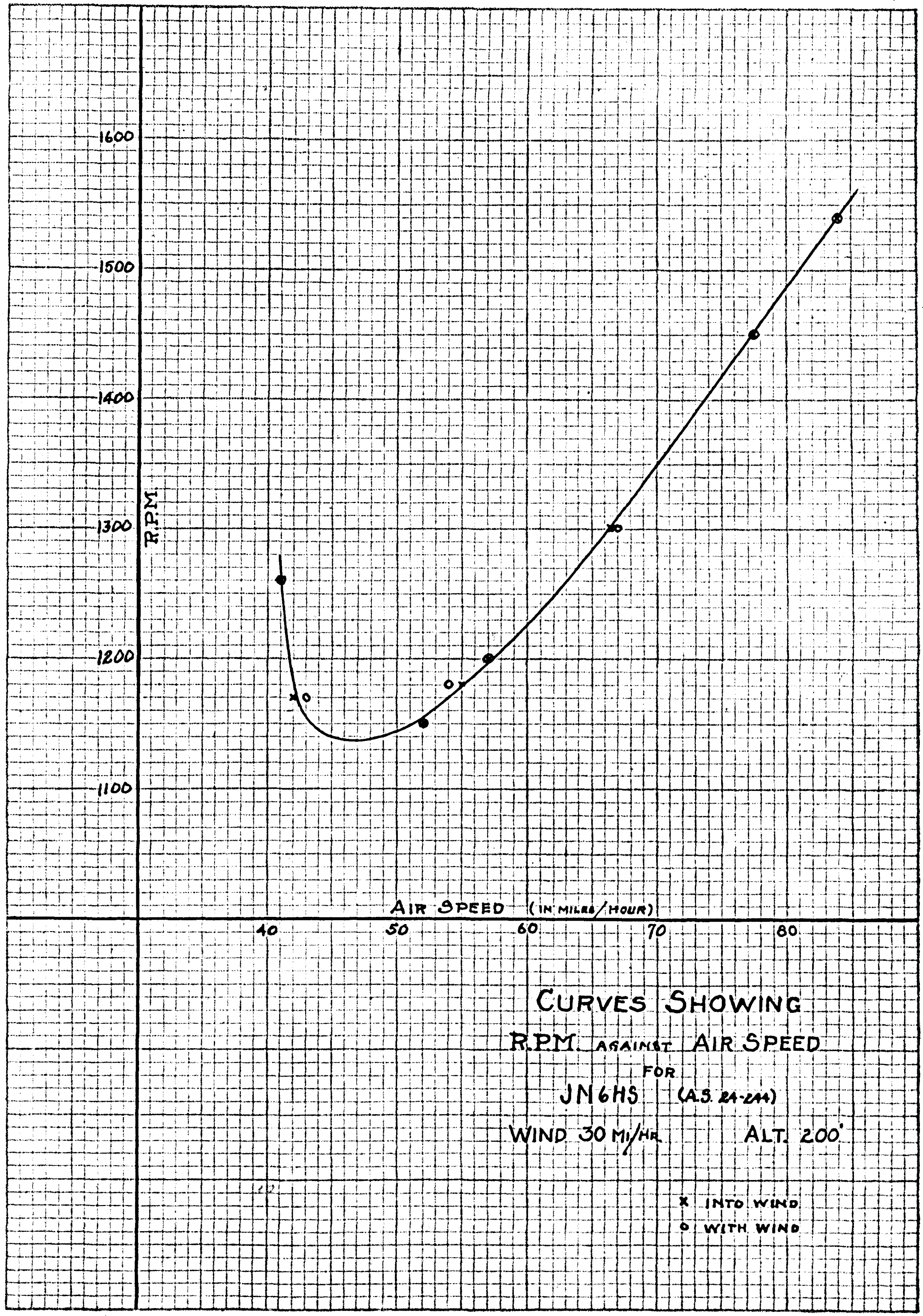
Date	May 22
Time	10:30 A.M.
Place	Boston Harbor
Wind	20 m/hr
Altitude	200'
Plane	TG4 (A 6348)

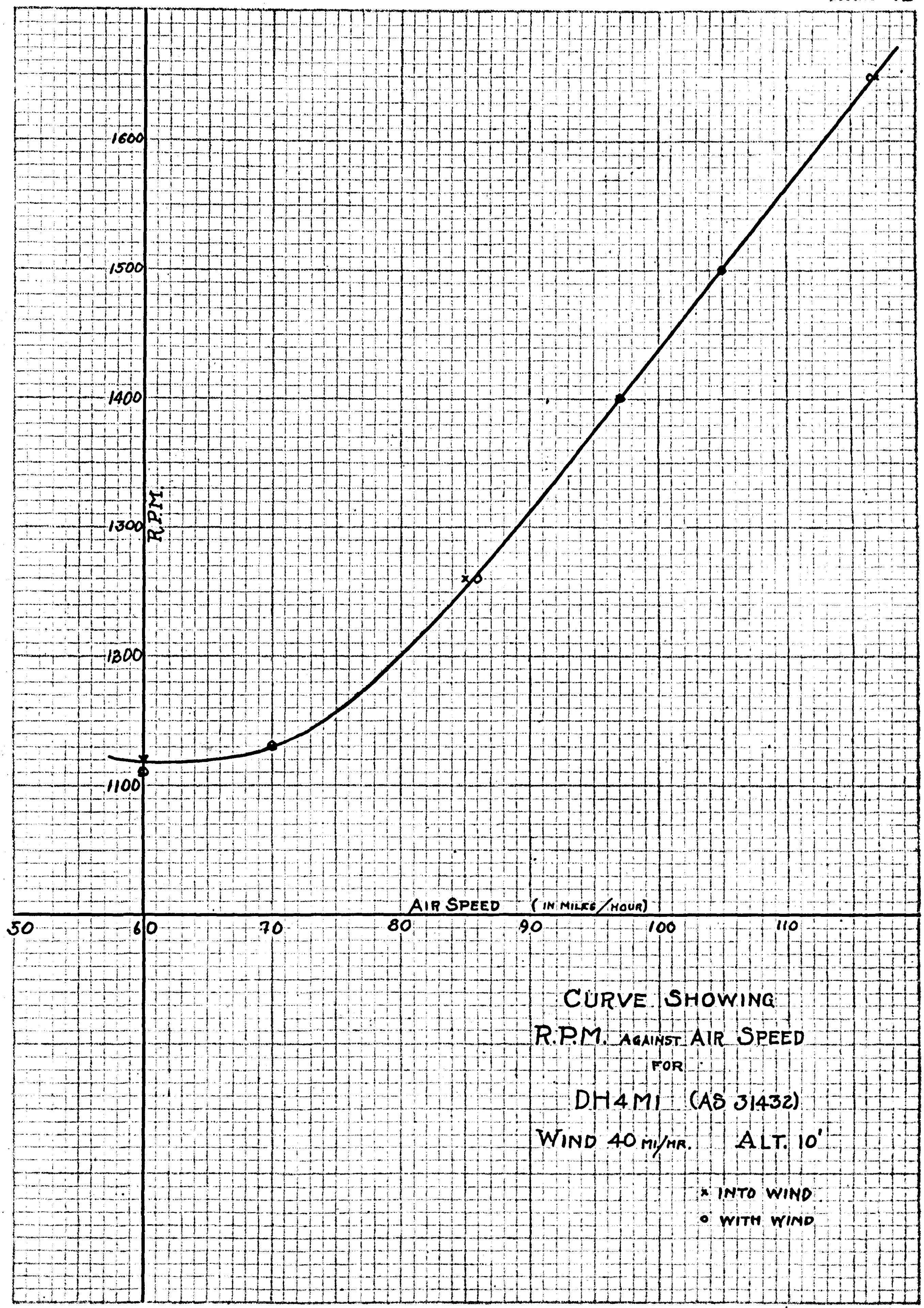
Up Wind		Down Wind	
RPM	Air Speed	RPM	Air Speed
1780	89	1780	89
1700	83	1700	83.5
1600	75	1600	75
1500	65.5	1510	66
1405	53	1410	55
1500	43	1450	43.5

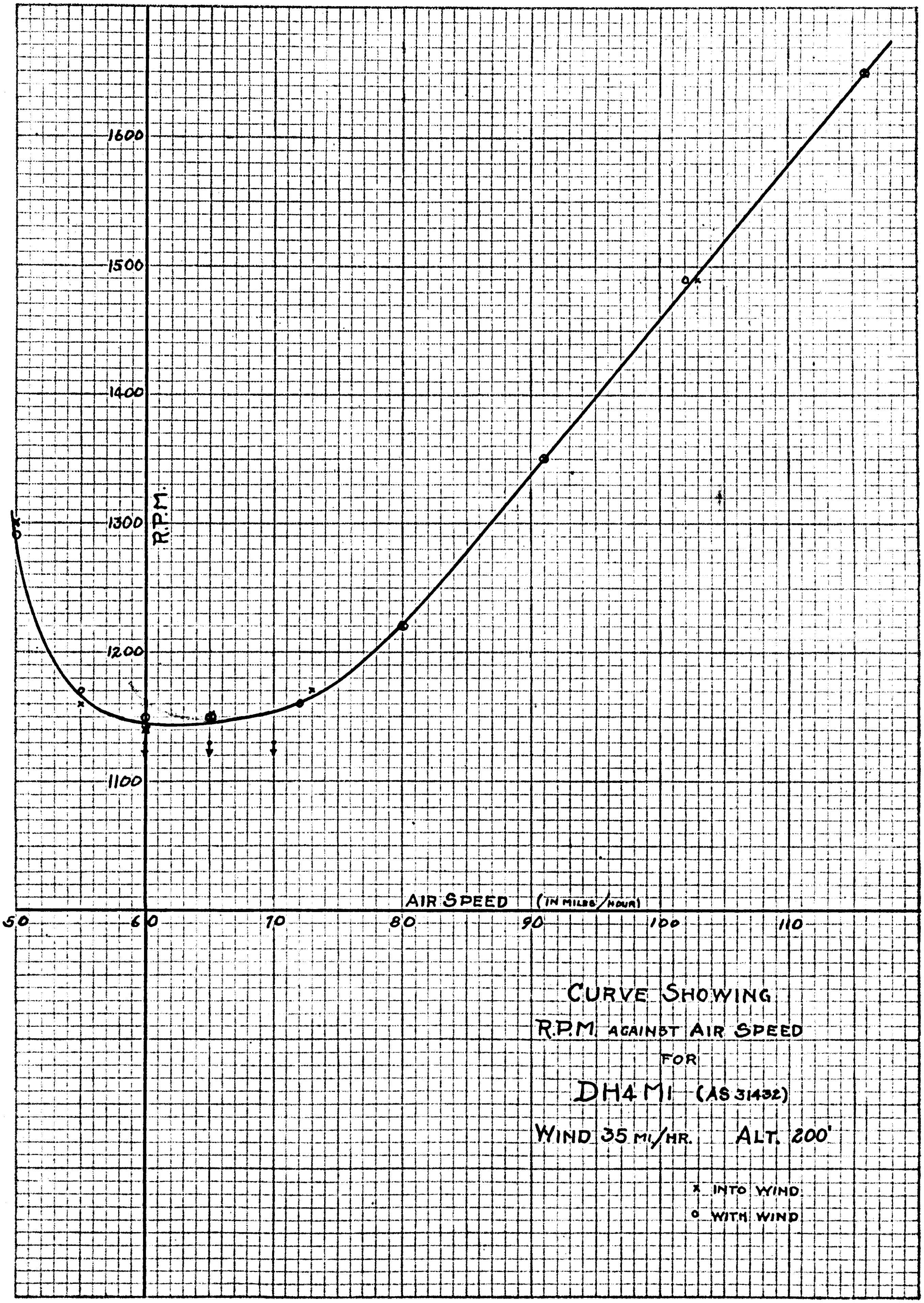
Plane would not fly level at any RPM under 1400.

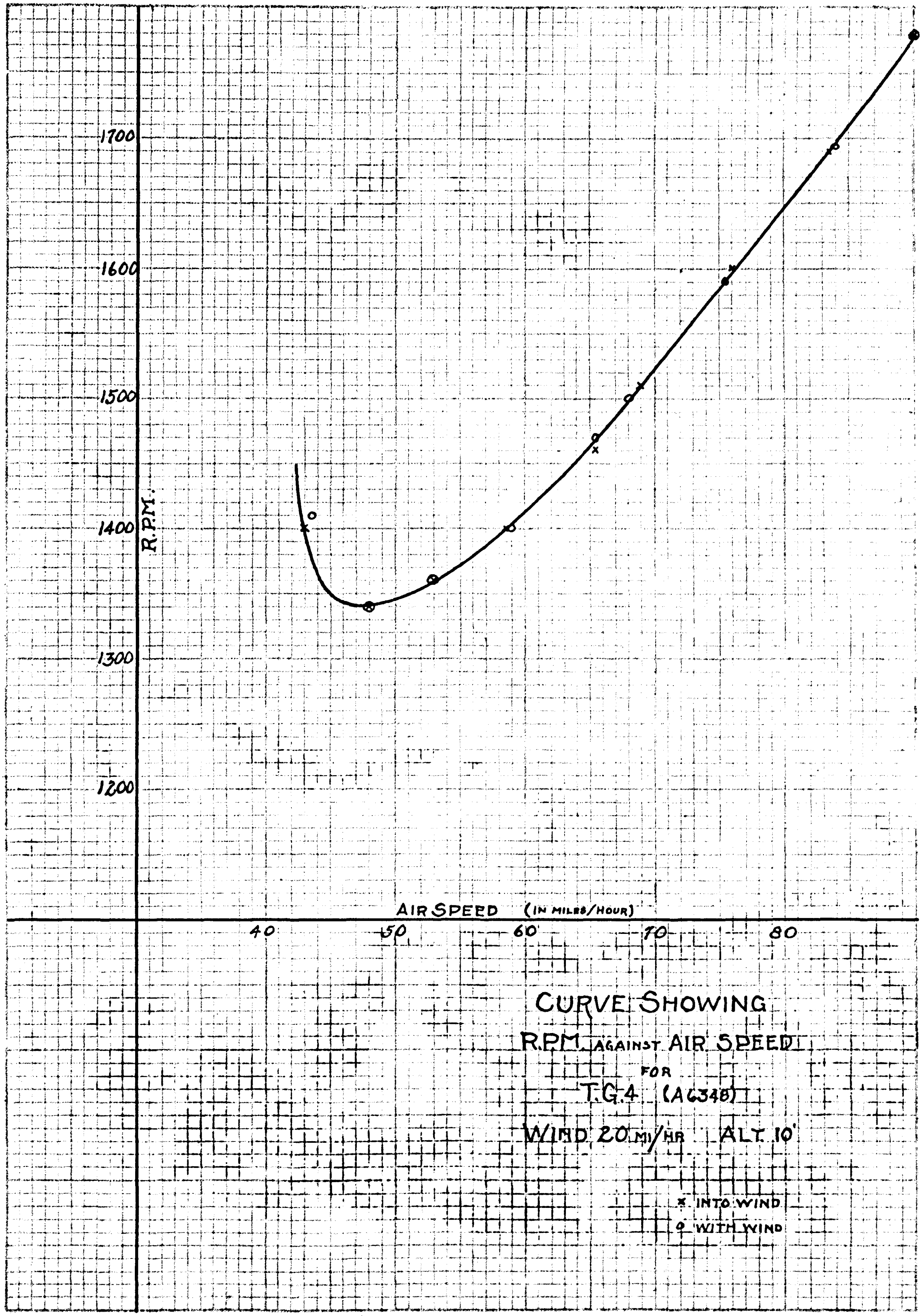
Runs 5 and 6 were made by Lt. Reginald Thomas of the U.S. Naval Air Station at Squantum, Mass. The care and skill with which he executed these two tests are indicated by the smoothness of the curves obtained.

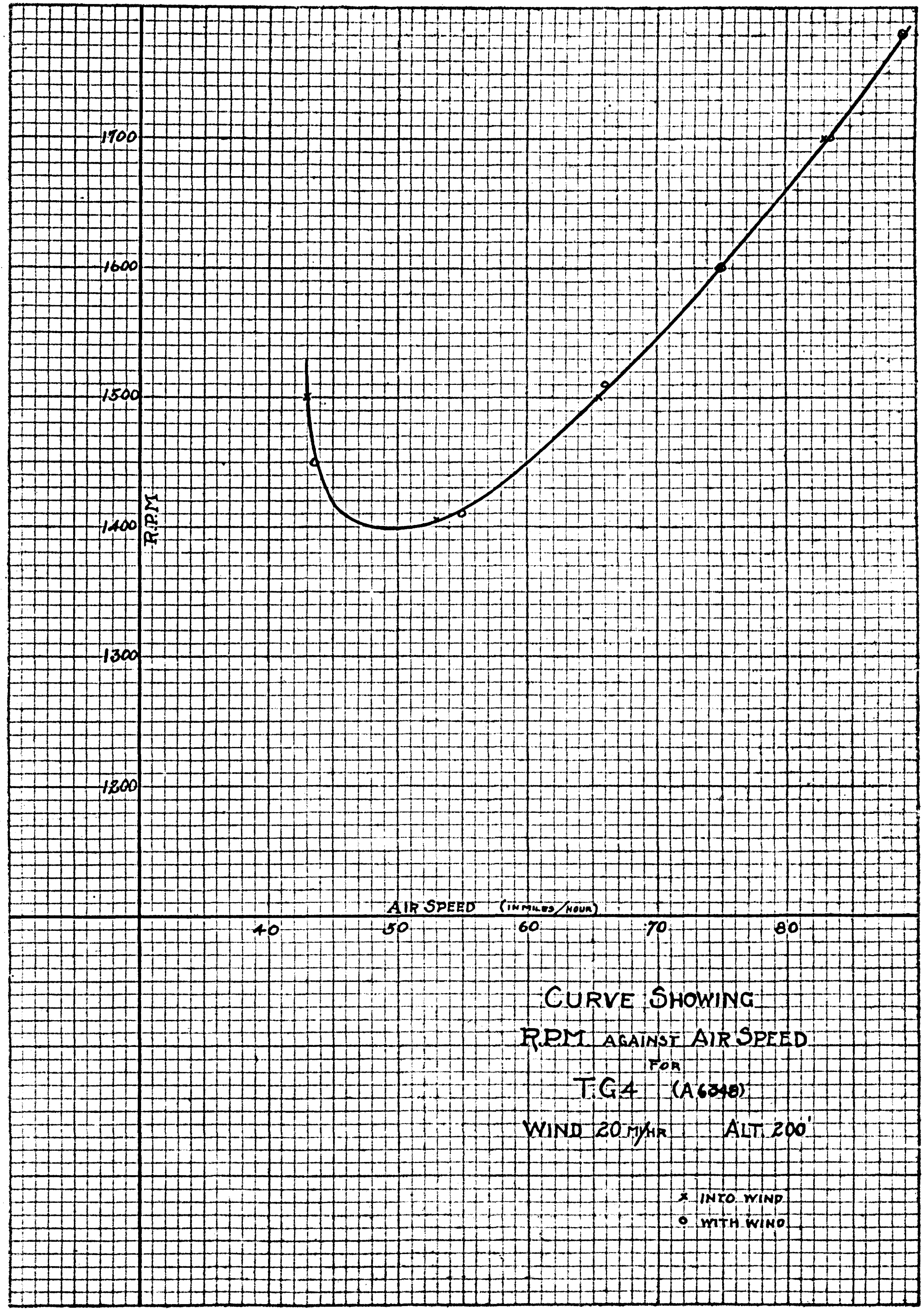

1600
1500
1400
1300
1200
1100
R.P.M.
AIR SPEED (IN MILES/HOUR)
40
50
60
70
80
CURVES SHOWING
R.P.M AGAINST AIR SPEED
FOR
JN 6 HS (AS 24-244)
WIND 30 M/HR. ALT. 10'
x INTO WIND
o WITH WIND


1600
1500
1400
1300
1200
1100
R.P.M.
AIR SPEED (IN MILES/HOUR)
40
50
60
70
80
CURVES SHOWING
R.P.M. AGAINST AIR SPEED
FOR
JN6HS (A.S. 24-244)
WIND 30 MI/HR. ALT. 200'
x INTO WIND
o WITH WIND


1600
1500
1400
1300
R.P.M.
1200
1100
AIR SPEED (IN MILES/HOUR)
50
60
70
80
90
100
110
CURVE SHOWING
R.P.M. AGAINST AIR SPEED
FOR
DH4M1 (AS 31432)
WIND 40 MI/HR. ALT. 10'
× INTO WIND
○ WITH WIND


1600
1500
1400
1300
1200
1100
R.P.M.
AIR SPEED (IN MILES/HOUR)
50
60
70
80
90
100
110
CURVE SHOWING
R.P.M. AGAINST AIR SPEED
FOR
DH4 M1 (AS 31432)
WIND 35 MI/HR. ALT. 200'
x INTO WIND
o WITH WIND


1700
1600
1500
1400
1300
1200
R.P.M.
AIR SPEED (IN MILES/HOUR)
40
50
60
70
80
CURVE SHOWING
R.P.M. AGAINST AIR SPEED
FOR
T.G.4 (A6348)
WIND 20 MI/HR ALT. 10'
× INTO WIND
o WITH WIND


1700
1600
1500
1400
1300
1200
R.P.M.
AIR SPEED (IN MILES/HOUR)
40
50
60
70
80
CURVE SHOWING
R.P.M. AGAINST AIR SPEED
FOR
T.G.4 (A6348)
WIND 20 M/HR ALT. 200'
x INTO WIND
o WITH WIND

A sensitive bubble inclinometer was constructed, and placed so that the zero reading coincided with the longitudinal axis of the ship. It had a range of over five degrees and an adjustment which allowed the setting to be changed thru five degree increments thus making the total range as great as was desired. Three sensitivities were tried. In the first a change of angle of .5° could be detected. In the second a change of angle of .2° could be readily detected and the nearest .1° approximated. In the third angles could be read directly to the nearest .1°. The second setting was chosen for the final tests as the first was not sufficiently precise and the third was effected too much by slight variations in wind and piloting, which produced instantaneous accelerations and caused bubble displacements. It was found that, even with the most sensitive bubble, engine vibration had no effect except at one certain critical speed of the ship and motor.

The object of this was to determine the variation of angle of attack with air speed. Runs were made in winds of different velocities; but as irregularities of flow increased rapidily with increased velocity, and the bubble recorded the smallest accelerations even though they could not be felt and did not show on the air speed indicator, it was found that any speed above 15 m/hr could not be used.

The tables and graphs follow.

Run 7

Date	May 26
Time	1:30 P.M.
Place	Boston Harbor
Wind	15 m/hr
Altitude	10'
Plane	JN6HS (AS 24-244)

Up Wind		Down Wind	
Air Speed	α	Air Speed	α
85	-1.9	84	-1.7
75	- .5	74	- .5
67	+ .8	67	+ .8
59	+2.1	60	+2.1
53	+4.5	53	+4.6
45	+8.4	45	+8.4

Run 8.

Date	May 26
Time	2:00 P.M.
Place	Boston Harbor
Wind	15 m/hr
Altitude	200'
Plane	JN6HS (AS 24-244)

Up Wind		Down Wind	
Air Speed	α	Air Speed	α
84	-1.2	84	-1.4
75	- .5	75	- .4
67	+1.0	67	+1.0
60	+2.5	60	+2.5
53	+4.5	53	+4.6
50	+6.7	50	+0.5
45	+9.3	45	+9.3
42	+13.4	42	+13.3

The motor revolutions per minute corresponding to 42 mi/hr in this run were the same as those corresponding to 41 mi/hr in Run 2.

Run 9.

Date	May 26
Time	3:30 P.M.
Place	Boston Harbor
Wind	15 m/hr
Altitude	10'
Plane	DH4M1 (AS 31432)

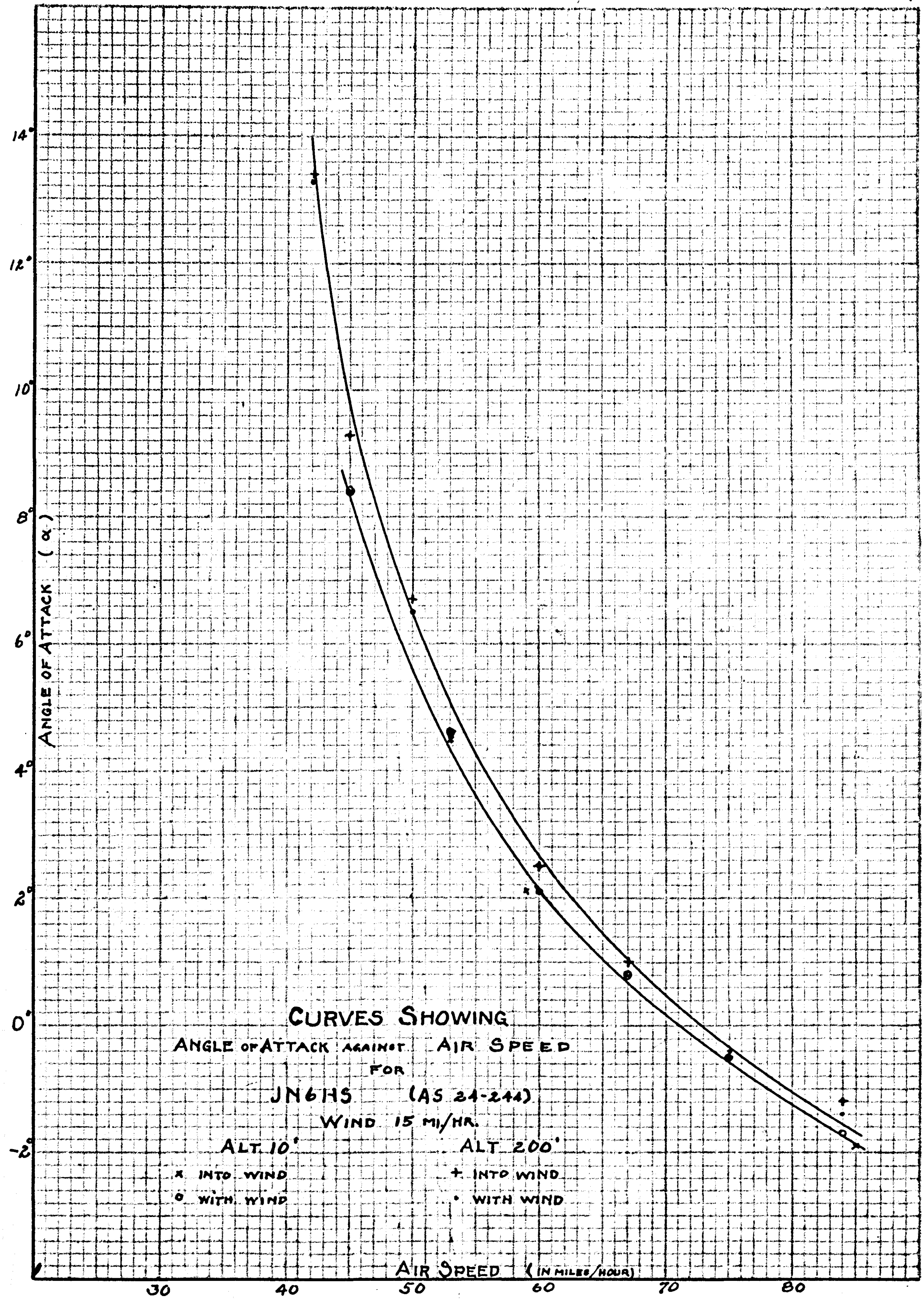
Up Wind		Down Wind	
Air Speed	α	Air Speed	α
118	-1.2	116	-.9
95	+ .5	95	+.5
85	+1.9	85	+2.1
70	+5.0	70	+5.0
60	+8.9	60	+8.7

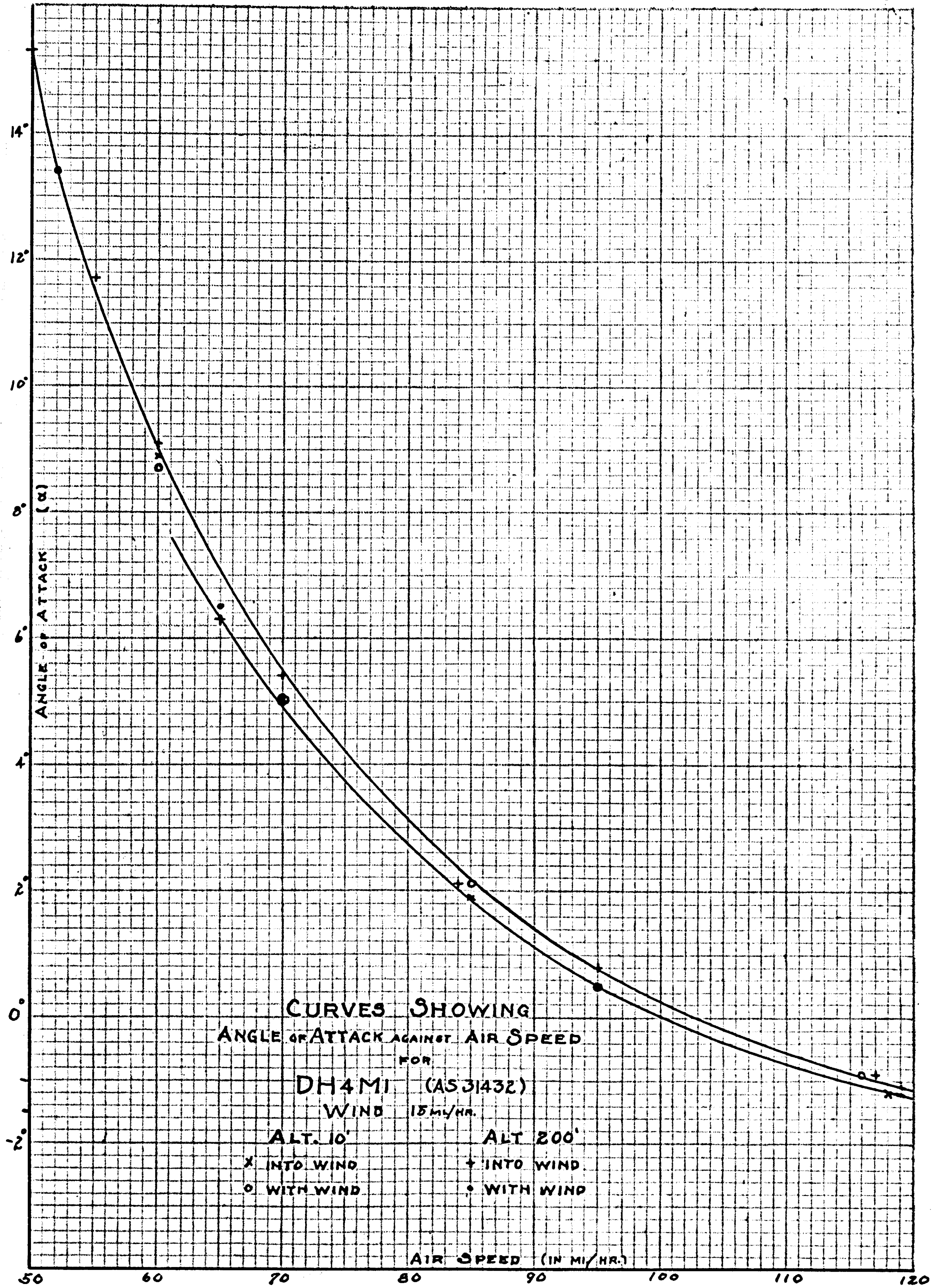
It is very difficult to obtain accurate bubble readings near minimum power as at this point, a considerable angular change in the ship's attitude causes no immediate change in its lift. When the plane does start to settle power must be applied to correct the settling, and the ensuing longitudinal acceleration disturbs the bubble.

Run 10

Date	May 26
Time	4:00 P.M.
Place	Boston Harbor
Wind	15 m/hr
Altitude	200'
Plane	DH4M1 (AS 31432)

Up Wind		Down Wind	
Air Speed	α	Air Speed	α
117	- .9	117	- .9
95	+ .8	95	+ .8
84	+2.1	85	+1.9
70	+5.4	70	+5.4
65	+6.3	65	+6.5
60	+9.1	60	+9.1
55	+11.7	55	+11.7
50	+15.5	52	+13.4


CURVES SHOWING
ANGLE OF ATTACK AGAINST AIR SPEED
FOR
JN6HS (AS 24-244)
WIND 15 MI/HR.
ALT 10'
x INTO WIND
o WITH WIND
ALT 200'
+ INTO WIND
• WITH WIND
ANGLE OF ATTACK (α)
14°
12°
10°
8°
6°
4°
2°
0°
-2°
AIR SPEED (IN MILES/HOUR)
30
40
50
60
70
80


CURVES SHOWING
ANGLE OF ATTACK AGAINST AIR SPEED
FOR
DH4M1 (AS 31432)
WIND 15 MI/HR.
ALT. 10'
X INTO WIND
O WITH WIND
ALT 200'
+ INTO WIND
• WITH WIND
ANGLE OF ATTACK (α)
AIR SPEED (IN MI/HR.)
14°
12°
10°
8°
6°
4°
2°
0°
-2°
50
60
70
80
90
100
110
120

b. In order to determine the effect of the wind velocity gradient on rate of climb a motor driven motion picture camera was set up, on a riged base, so the axis was normal to the wind direction. The plane was flown into the field of view at constant speed and at a constant altitude. The throttle was advanced to full on and the air speed kept constant by increasing the angle of attack. The plane started to climb, rapidly at first, and soon settled down to a steady climb. The air speed was kept constant at all times. Considerable difficulty was encountered in maintaining the air speed constant while the angle of attack was changing and the plane accelerating, (vertically), but after a few hours practice the pilot was able to overcome the tendency to kill too much speed at first and then let it pick up too rapidly.

A certain time interval was required for the motor to accelerate but, as the throttle was pushed forward rapidly, and the carburetors set so that the mixture was very rich, this was short and practically constant, for any given air speed, regardless of the wind direction. Some time was required to accelerate the plane vertically but, as we only want comparative results, this may also be neglected.

The plane was flown in front of the camera with and against the wind, at various speeds, and the climbs started at two different altitudes. The tests were made over Dorchester Bay on the afternoon of May 26, 1925.

The pictures were projected, as stills, on a sheet of tracing paper. The center of gravity and length of the plane were marked on this paper for each second of travel. The horizontal displacement of the C.G. from its position at zero second and its distance above the horizon were measured for each second. The ratio of the length of the plane to the measured length of its image gives a conversion factor by means of which we can convert displacements on the pictures to distances and elevations in relation to the earth. If the horizontal distances are then corrected for the wind velocity and the vertical distance from the horizon to a point on the earth's surface, directly under the plane, added to the elevation we have the distance above the earth's surface and the horizontal displacements in relation to the medium in which we are working. The next step was to take a common zero from which to measure horizontal displacements in all runs.

The point at which the plane first began its climb was chosen. There was a slight discrepancy in the altitudes at which different 10' and 60' runs were started. As this difference was always small the curves were elevated or lowered slightly in order to make comparison easier. The corrected displacements are tabulated below:

Run 11. (Up Wind)

Plane	JN6HS	(AS 24-244)
Altitude (at start)	60'	
Wind	10 m/hr	
Air Speed	60 m/hr	

Time	Distance	Altitude
0	0	60.0
1	100	64.2
2	187	72.3
3	279	80.2
4	372	89.8
5	463	96.7
6	549	104.0
7.	642	112.3
8	730	119.8

	Run 12.	(down wind)
Plane	JN6HS	(AS 24-244)
Altitude (at start)	10	
Wind	15 m/hr	
Air Speed	60 m/hr	

Time	Distance	Altitude
0	0	10.0
1	97	12.4
2	185	21.1
3	263	34.9
4	354	52.6
5	455	64.8
6	547	80.2
6.5	593	84.3

This run was actually started at an altitude of exactly 10' so there is no vertical displacement of the curve in the tabulation above or in the plot on pages 102 and 103.

Run 13 (Up Wind)

Plane	JN6HS	(As 24-244)
Altitude (at Start)	60'	
Wind	8 m/hr	
Air Speed	55 m/hr	

Time	Distance	Altitude
0	0	60.0
.5	45	61.7
1.5	130	67.9
2.5	213	76.2
3.5	295	84.6
4.5	380	93.8
5.5	469	100.8
6.5	543	105.5
7.5	623	113.6
8.5	700	121.6

This run was started at 70' so it was necessary to lower the curve ten feet.

Run 14. (Down Wind)

Plane	JN6HS	(AS 24-244)
Altitude	10'	
Wind	5 m/hr	
Air Speed	55 m/hr	

Time	Distance	Altitude
0	0	10.0
1	90	15.2
2	190	28.0
3	252	46.3
4	332	62.0
5	413	75.2
6	499	84.6
7	584	94.4
8	670	102.

This run was started at 12' and the smoothness, at the start, interrupted by a short rapid air current.

Run 15 (Up Wind)

Plane	JN6HS	(AS 24-244)
Altitude (at start)	60'	
Wind	5 m/hr	
Air Speed	50 m/hr	

Time	Distance	Altitude
0	0	60.0
1	85	65.9
2	160	75.2
3	235	84.1
4	308	94.0
5	381	101.0
6	453	109.0
7	528	117.0
8	602	123.6
9	680	131.8
10	754	140.2

The curve was lowered 15' but as the gradient and density change is very small, at this altitude, this should not effect the shape of the curve.

Run 16. (Down Wind)

Plane	JN6HS	(AS 24-244)
Altitude (at start)	10'	
Wind	5 m/hr	
Air Speed	50 m/hr	

Time	Distance	Altitude
0	0	10.0
1	74	15.5
2	147	29.6
3	219	44.4
4	298	61.0
5	367	72.8
6	443	84.0
7	521	92.0

The curve was moved down three feet, which is the maximum permissable change at this low altitude.

Run 17. (Up Wind)

Plane	JN6HS	(AS 24-244)
Altitude	10'	
Wind	15 m/hr	
Air Speed	60 m/hr	

Time	Distance	Altitude
0	0	10.0
1	95	14.2
2	181	24.3
3	270	35.3
4	357	52.3
5	460	70.7
6	548	83.2
7	640	90.1

The reason for waiting for this run was to allow the wind to pick up. The wind speed was roughly checked by flying over a Robinson Cup animometer and observing its speed of rotation.

Run 18. (Down Wind)

Plane	DH4M1	(AS 31432)
Altitude (at start)	10'	
Wind	3 m/hr	
Air Speed	70 m/hr	

Time	Distance	Altitude
0	0	10.0
.75	91	13.6
1.75	208	36.6
2.75	293	67.5
3.75	400	102.5
4.75	491	132.2
5.75	592	158.2
6.75	688	181.3
7.75	788	203.5

It was found to be much more difficult to hold a constant air speed in the DH.

pgs. 93 - 97 do not exist
Mis-numbering error by the author

Run 19. (Up Wind)

Plane	DH4 M1	(AS 31432)
Altitude	60'	
Wind	3 m/hr	
Air Speed	70 m/hr	

Time	Distance	Altitude
0	0	60.0
1.25	140	77.0
2.25	255	102.2
3.25	362	130.0
4.25	463	160.2
5.25	558	189.0
6.25	646	212.0
7.25	733	234.0

This felt, to the pilot, to be the smoothest run and so should be one of the most accurate.

Run 20 (Down Wind)

Plane	DH4 M1	(AS 31432)
Altitude	10'	
Wind	3 m/hr	
Air Speed	65 m/hr	

Time	Distance	Altitude
0	0	10.0
1.25	120	26.5
2.25	215	59.5
3.25	307	96.0
4.25	397	148.0
5.25	489	190.0
6.25	578	223.0
7.25	668	250.0

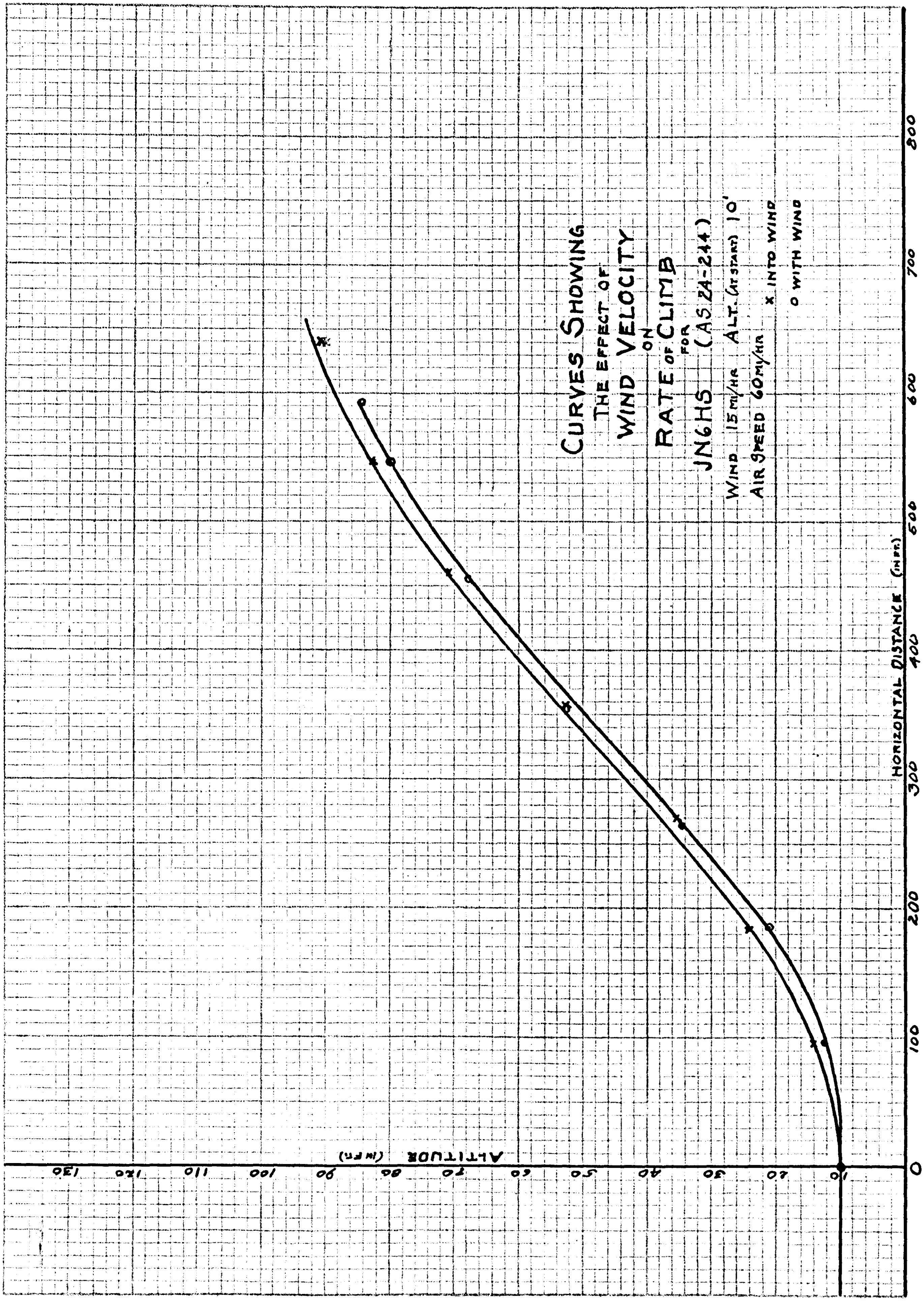
This run was started at 15' so when lowered bodily to 10' is only roughly comparative.

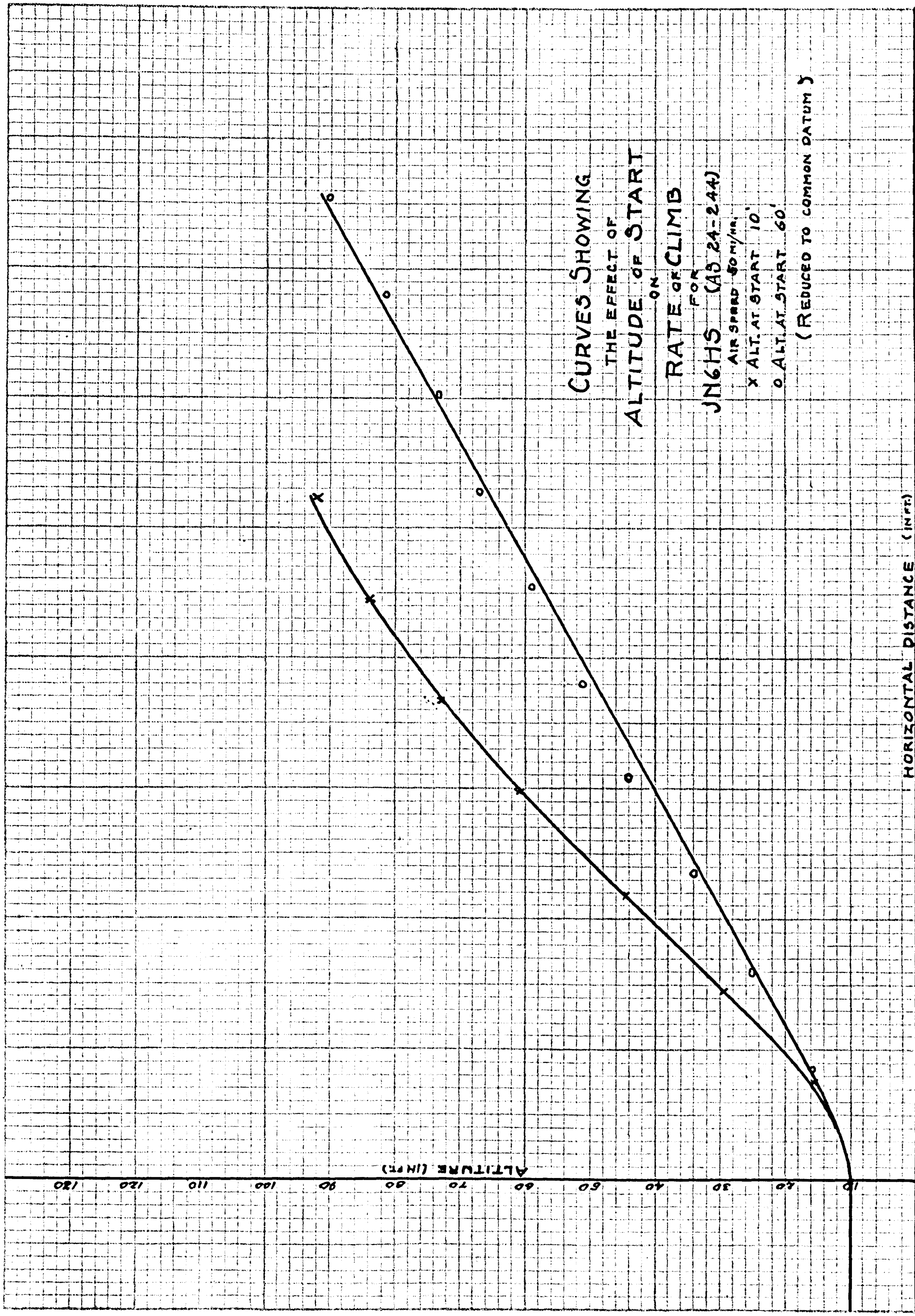
Run 21. (Up Wind)

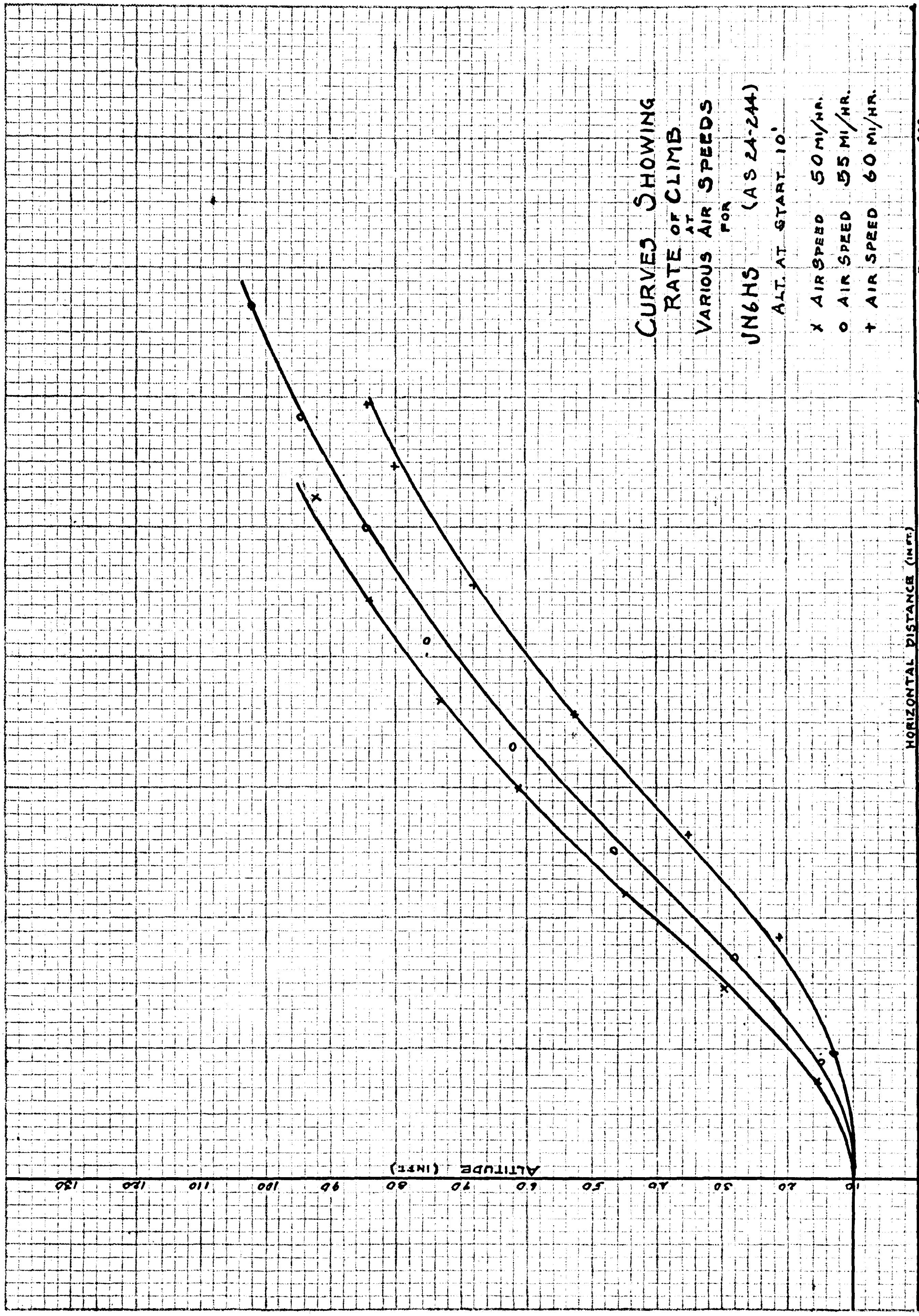
Plane	DH4 M1	(AS 31432)
Altitude	60'	
Wind	3m/hr	
Air Speed	80 m/hr	

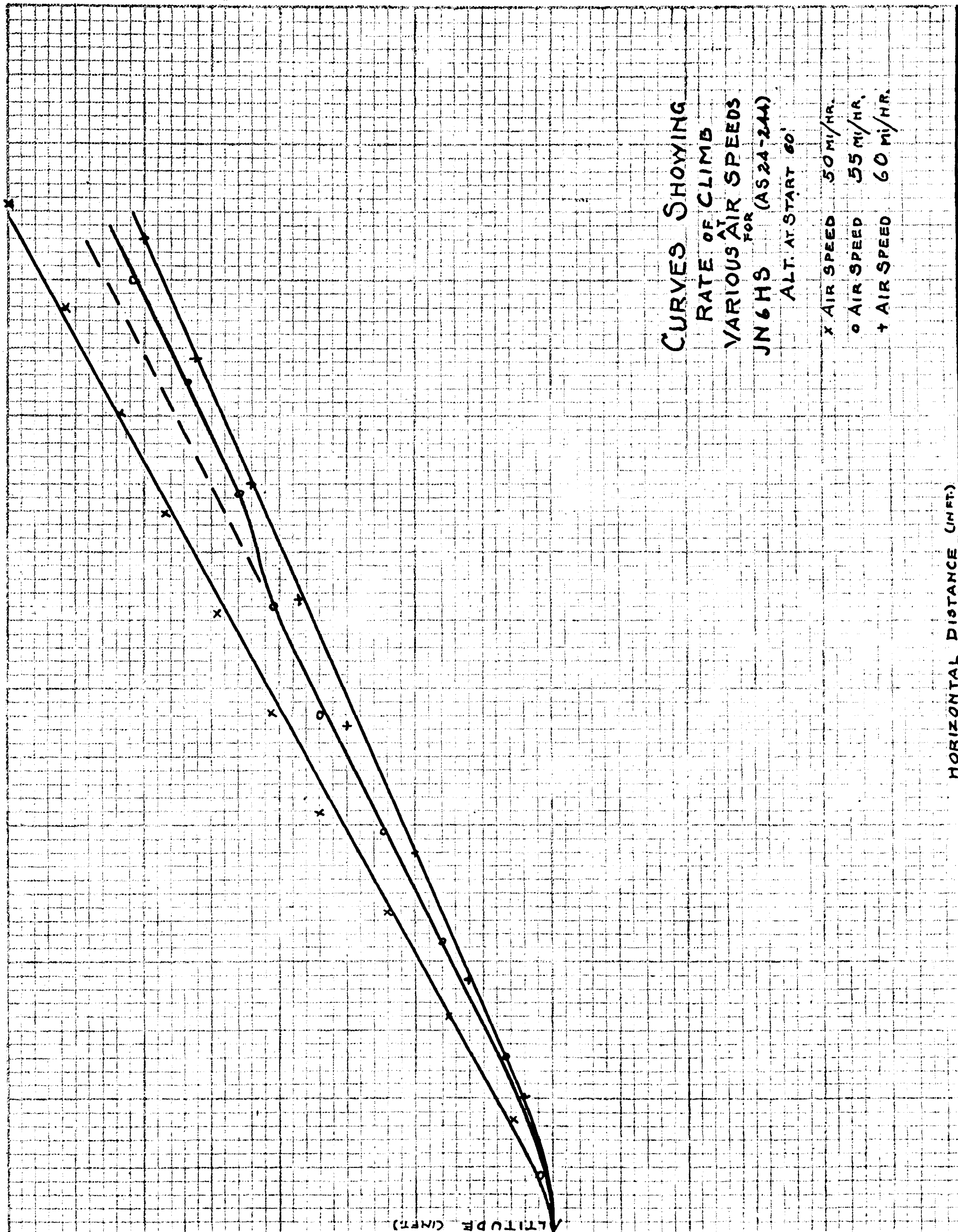
Time	Distance	Altitude
0	0	60.0
1	125	65.7
2	249	79.5
3	370	95.5
4	491	111.6
5	614	122.0

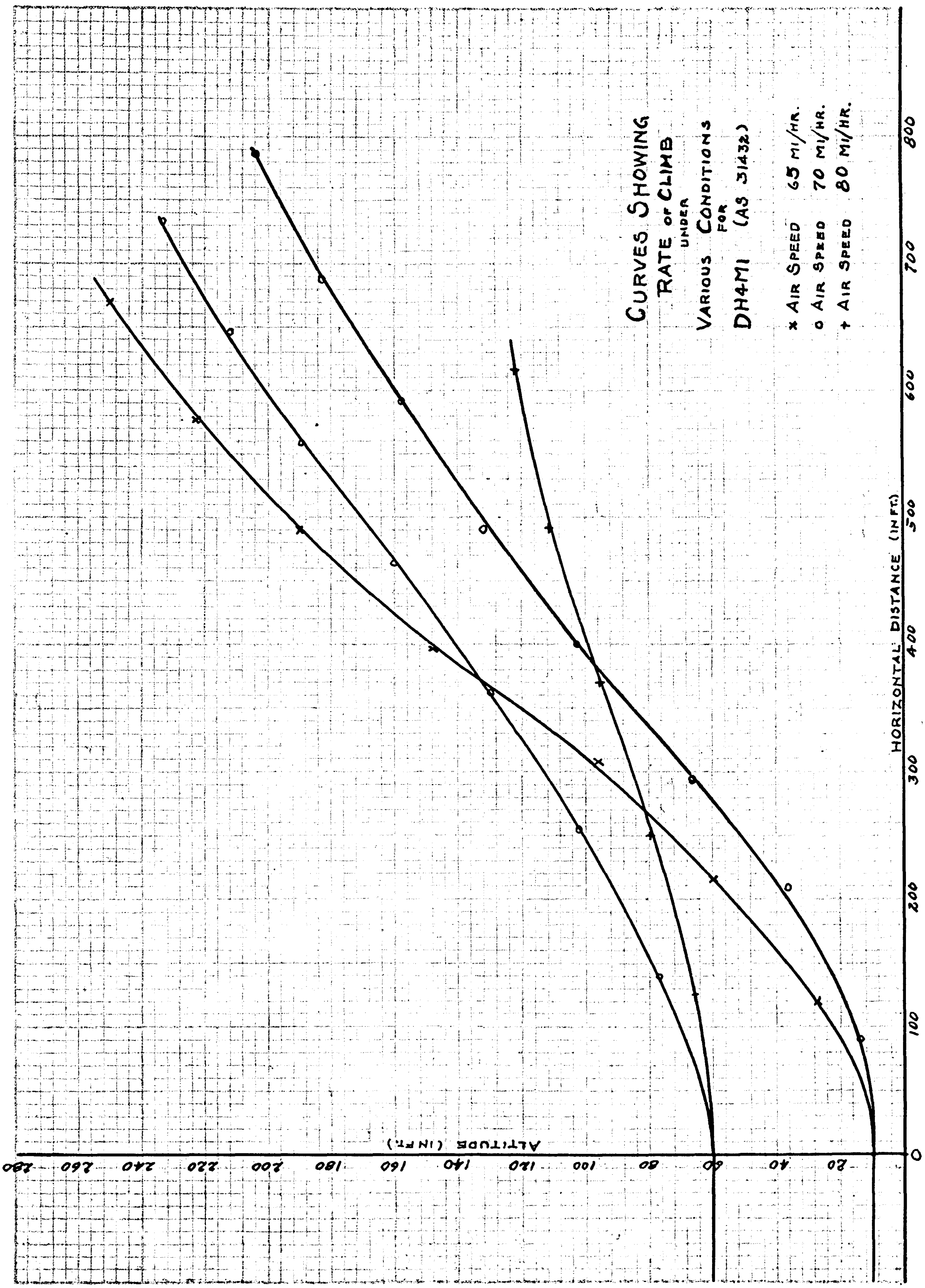
This run was made much closer to the camera and, while the image was clearer and easier to work with, the field was so small that the plane passed out of view before the climb became steady.


CURVES SHOWING
THE EFFECT OF
WIND VELOCITY
ON
RATE OF CLIMB
FOR
JN6H5 (AS 24-244)
WIND 15 MI/HR ALT. (AT START) 10'
AIR SPEED 60 MI/HR
X INTO WIND
O WITH WIND
HORIZONTAL DISTANCE (IN FT.)
ALTITUDE (IN FT.)
0
100
200
300
400
500
600
700
800
10
20
30
40
50
60
70
80
90
100
110
120
130


CURVES SHOWING
THE EFFECT OF
ALTITUDE OF START
ON
RATE OF CLIMB
FOR
JN6HS (AB 24-24A)
AIR SPEED 50 MI/HR.
x ALT. AT START 10'
o ALT. AT START 60'
(REDUCED TO COMMON DATUM)
HORIZONTAL DISTANCE (IN FT.)
0
100
200
300
400
500
600
700
800
ALTITURE (IN FT.)
10
20
30
40
50
60
70
80
90
100
110
120
120


CURVES SHOWING
RATE OF CLIMB
AT
VARIOUS AIR SPEEDS
FOR
JN6HS (AS 24-244)
ALT. AT START 10'
x AIR SPEED 50 MI/HR.
o AIR SPEED 55 MI/HR.
+ AIR SPEED 60 MI/HR.
HORIZONTAL DISTANCE (IN FT.)
ALTITUDE (IN FT.)
0
100
200
300
400
500
600
700
800
10
20
30
40
50
60
70
80
90
100
110
120
130


CURVES SHOWING
RATE OF CLIMB
AT VARIOUS AIR SPEEDS
FOR JN6HS (AS 24-244)
ALT. AT START 60'
x AIR SPEED 50 MI/HR.
o AIR SPEED 55 MI/HR.
+ AIR SPEED 60 MI/HR.
HORIZONTAL DISTANCE (IN FT.)
ALTITUDE (IN FT.)
0
100
200
300
400
500
600
700
800
10
20
30
40
50
60
70
80
90
100
110
120
130


CURVES SHOWING
RATE OF CLIMB
UNDER
VARIOUS CONDITIONS
FOR
DH4MI (AS 31432)
x AIR SPEED 65 MI/HR.
o AIR SPEED 70 MI/HR.
+ AIR SPEED 80 MI/HR.
HORIZONTAL DISTANCE (IN FT.)
ALTITUDE (IN FT.)

c. In order to determine the effect of the wind velocity grsdient on angular velocity, in a normal turn, a plane was equipped with an air speed recorder, an angular velocity recorder, and a sighting wire set parallel to the YZ plane and 15° from the Y axis. The air speed recorder was of the standard NACA optical recording type. The Angular Velocity Recorder consisted of a motor dirven gyroscope mounted between two pivots and working against two coiled springs. The displacement of the gyroscope actuated a small mirror and these displacements were recorded optically. The sensitivity could be adjusted by the spring tension and by changing the lever arm on the mirror. The range of sensitivity was from .04 to 1.5 radians per second per inch of displacement on the recording film. In order to obtain this range three sets of springs, of different strengths, were required. In the following tests the greatest possible sensitivity was used. The instrument was so sensitive that in spite of the damping it was affected by vibration and the lines on the records were quite wide. This made it more difficult to measure displacements but, as a mean reading was taken, had very little effect on the final results. Synchronized timing lines at one second intervals facilitated the study of the records. In plotting up the results of the angular velocity measurements highly approximate averages had to be taken as, ordinarily, the displacements due to unsteady flying and air conditions were much greater than those due to gradient.

In Run 1 the altitude, throttle setting, and angle of bank were held constant and the turn made as smoothly as possible. The plane was allowed to turn until conditions became steady and as soon, after this, as it again headed directly into the wind the instruments were started and one 360° turn executed. This turn was also timed with a stop watch. The method used in determining when exactly 360° had been turned thru was to set to sight on a distant object, (the balloon hangar at Langley Field was used.) when starting the turn and then turn until the object again appeared in the same relative position. The air speed and angular velocity were allowed to change at will and these changes were recorded by the instruments. The altitude was held constant by flying as close to the ground as possible. This method can be used only on very low runs.

In Runs 2 and 3 the altitude, angle of bank and air speed were held constant and the angular velocity recorded. In run 2, by a sufficient stretch of the imagination, it was possible to detect a slight tendency in the airplane to change altitude during the turn. This tendency exhibited itself as a slight settling for the first 90° after coming into the wind and a gradual rise there after. Absolutely no effect of the wind was observed, by the pilot, at an altitude of 1000'.

All runs were made over Chesapeake Bay, at a point ten miles east of Langley Field, on the afternoon of May 23, 1925. A TS land plane (A6249), borrowed from the N.A.C.A., was used.

Run 1.

Altitude 10'

Wind 20 m/hr

RPM	Air Speed (Av)	Time	ω
(α) 1670	103	99.5	.0631
(β) 1450	85	85.0	.0739
(γ) 1300	70	74.0	.0849

Measurements, taken from the records, for each 45° of turn are tabulated below.

(α)

Angle	Displacement	(Air Speed)	Displacement	(ω)
0	1.28	105	.51	.065
45°	1.22	102	.51	.065
90°	1.20	101	.49	.064
135°	1.20	101	.49	.064
180°	1.20	101	.47	.063
225°	1.22	102	.47	.063
270°	1.22	102	.45	.062
315°	1.22	102	.43	.067
360°	1.225	103	.41	.060

(β)

Angle	Displacement	(Air Speed)	Displacement	(ω)
0	.94	88	.61	.070
45°	.90	86	.65	.072
90°	.87	84	.66	.0725
135°	.85	83	.66	.0725
180°	.87	84	.61	.070
225°	.90	86	.61	.070
270°	.94	88	.61	.070
315°	.92	87	.61	.070
360°	.94	88	.63	.071

Angle	Displacement	(Air Speed)	Displacement	(ω)
0	.68	72	.82	.082
45°	.655	70	.88	.085
90°	.655	70	.90	.086
135°	.63	68	.90	.086
180°	.64	69	.82	.082
225°	.65	70	.82	.082
270°	.70	73	.96	.090
315	.685	72	.80	.080
360°	.67	71	.80	.080

Run 2.

Altitude 200'

Wind 20 m/hr

RPM	Air Speed	Time	ω
1370	102	100	.0628
1460	85	87	.0721
1320	70	78	.0806

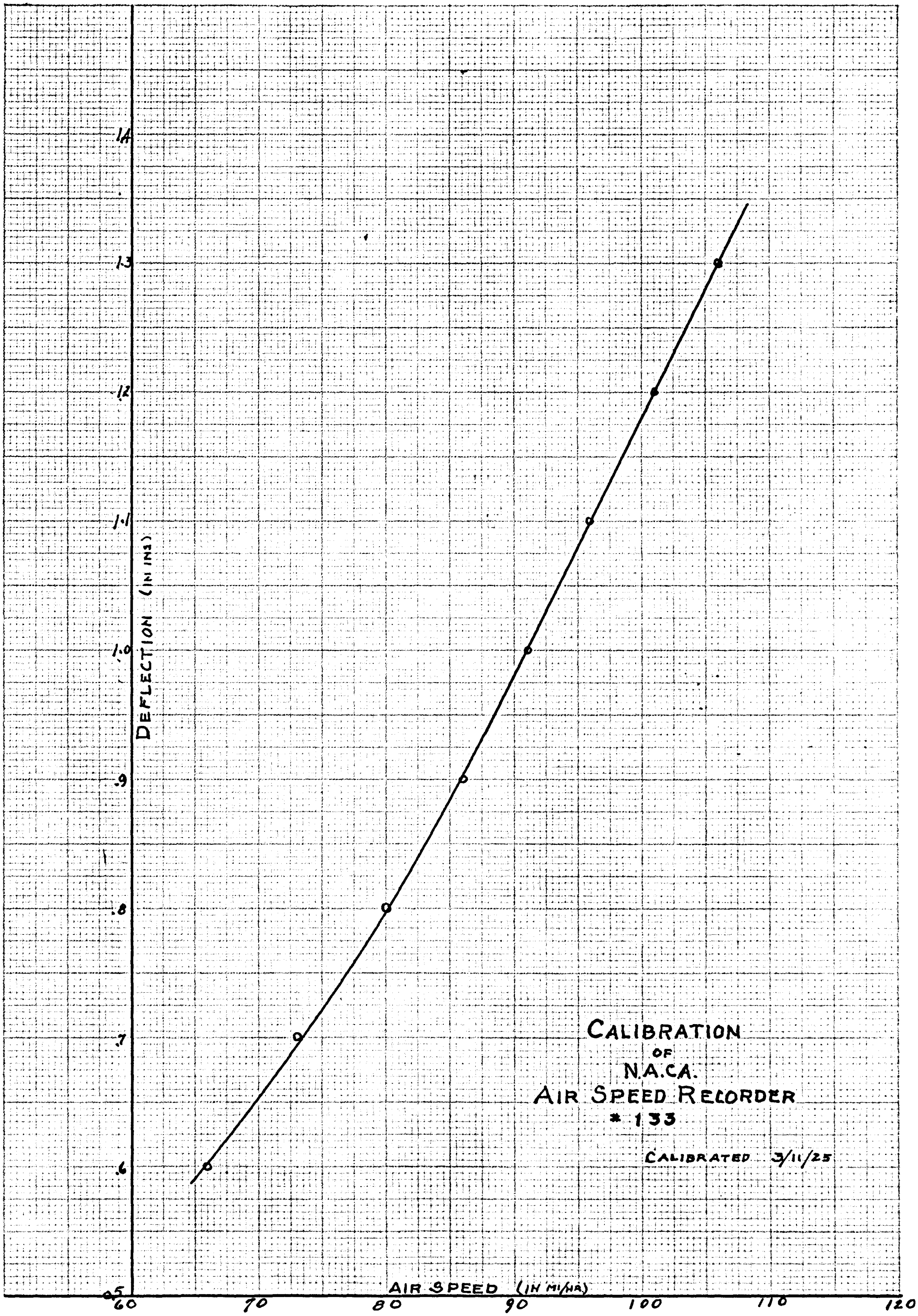
Run 3

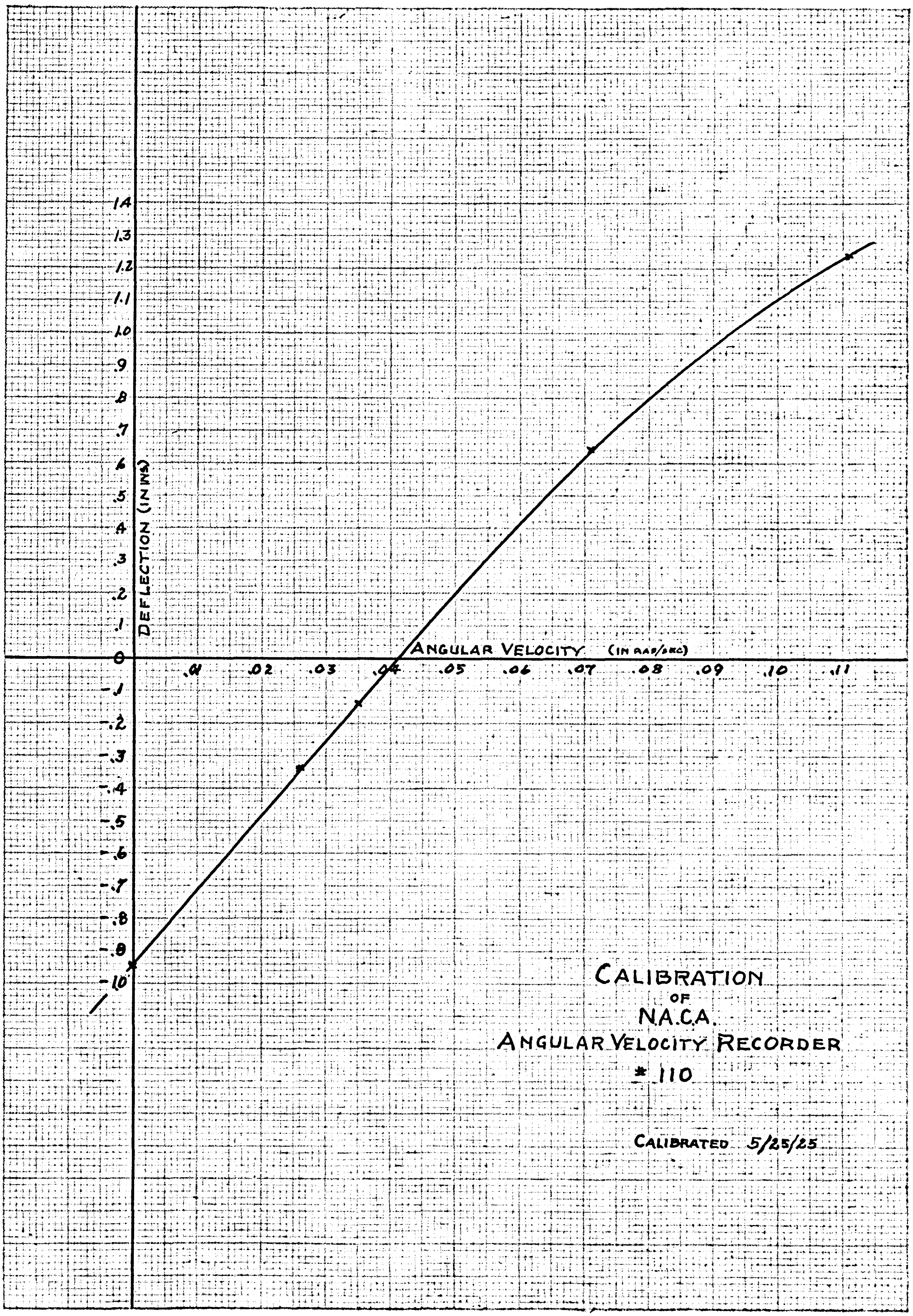
Altitude	200'
Wind	20 m/hr

RPM	Air Speed	Time	ω
1550	90	90	.0698
1470	80	80.5	.078

In α, of Run 1, there appeared to be a periodic change in air speed but no definite effect could be detected on the angular velocity record. In β, there is a suggestion of periodicity and in γ it becomes more pronounced.

In Runs 2 and 3 no change in angular velocity is indicated by the records.


1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
DEFLECTION (IN INS)
60
70
80
90
100
110
120
AIR SPEED (IN MI/HR)
CALIBRATION
OF
N.A.C.A.
AIR SPEED RECORDER
133
CALIBRATED 3/11/25


DEFLECTION (IN INS)
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
-.5
-.6
-.7
-.8
-.9
-1.0
ANGULAR VELOCITY (IN RAD/SEC)
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
CALIBRATION
OF
N.A.C.A.
ANGULAR VELOCITY RECORDER
* 110
CALIBRATED 5/25/25

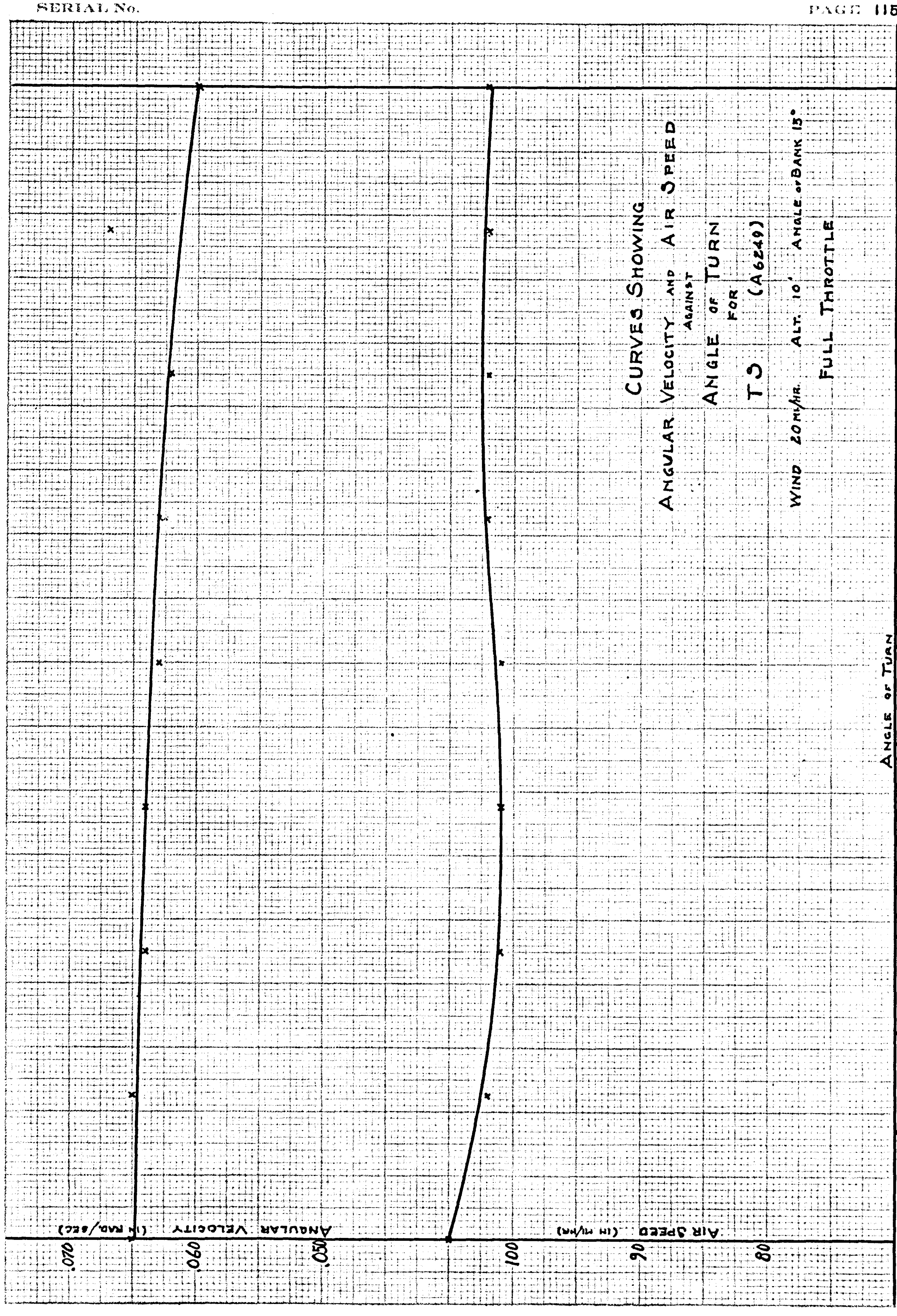
d. The immediate conclusions to be drawn from the flight tests are:

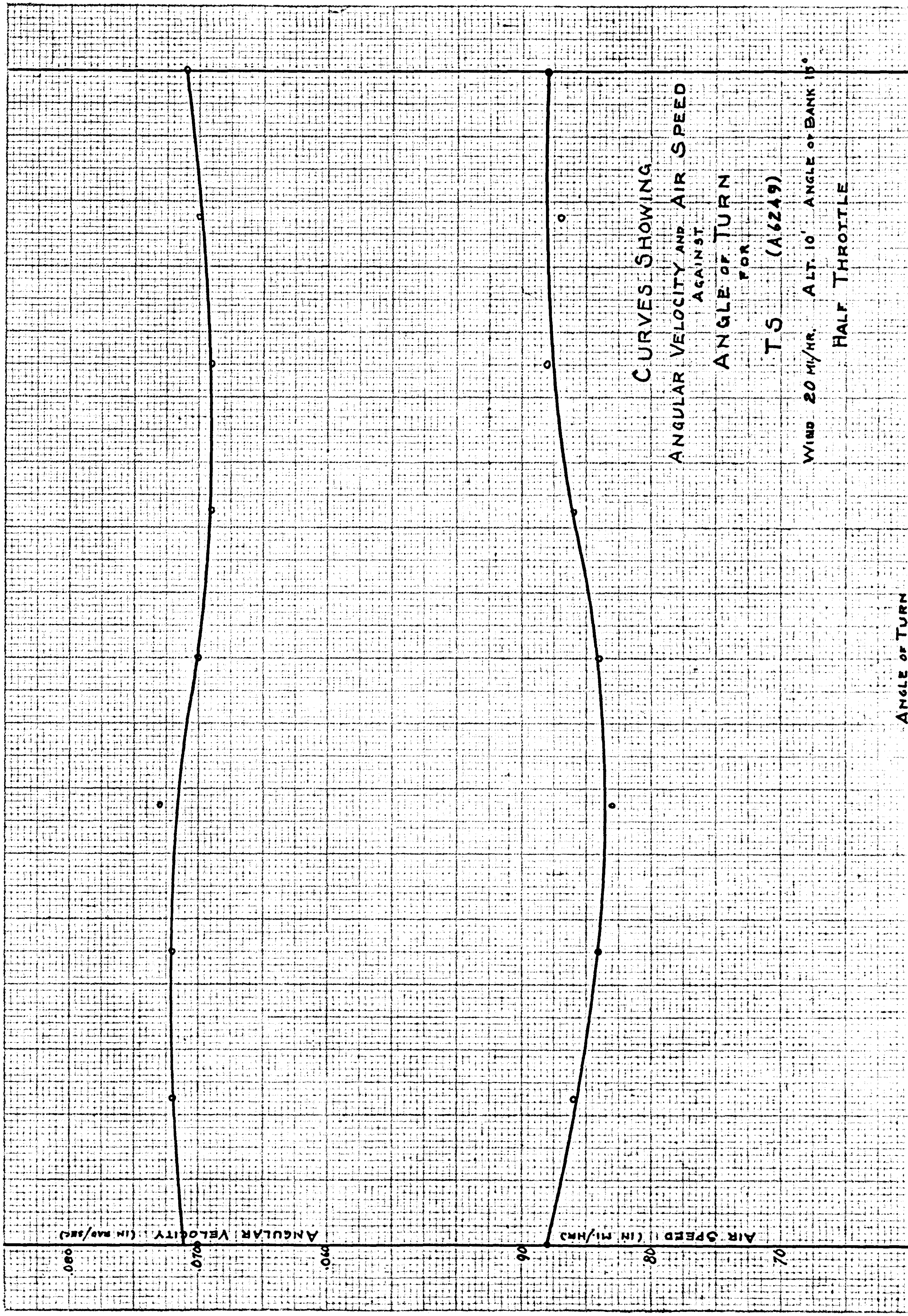
First. That there is no measurable effect, in level flight, due to wind.

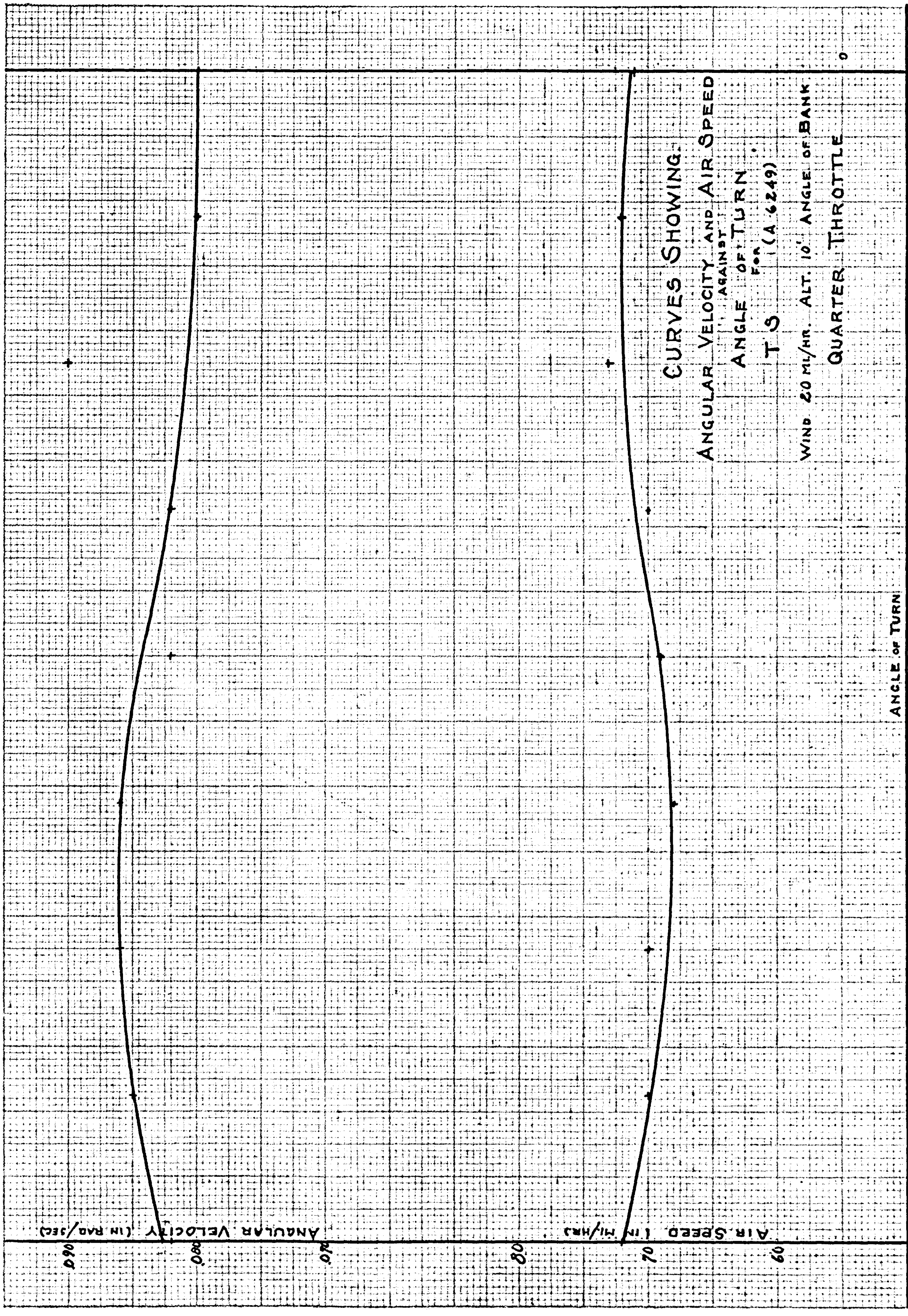
Second. That the effect of wind velocity on climb is to increase the rate of climb slightly, near the ground, when flying into the wind, and decrease the rate of climb, an equal amount when flying with the wind.

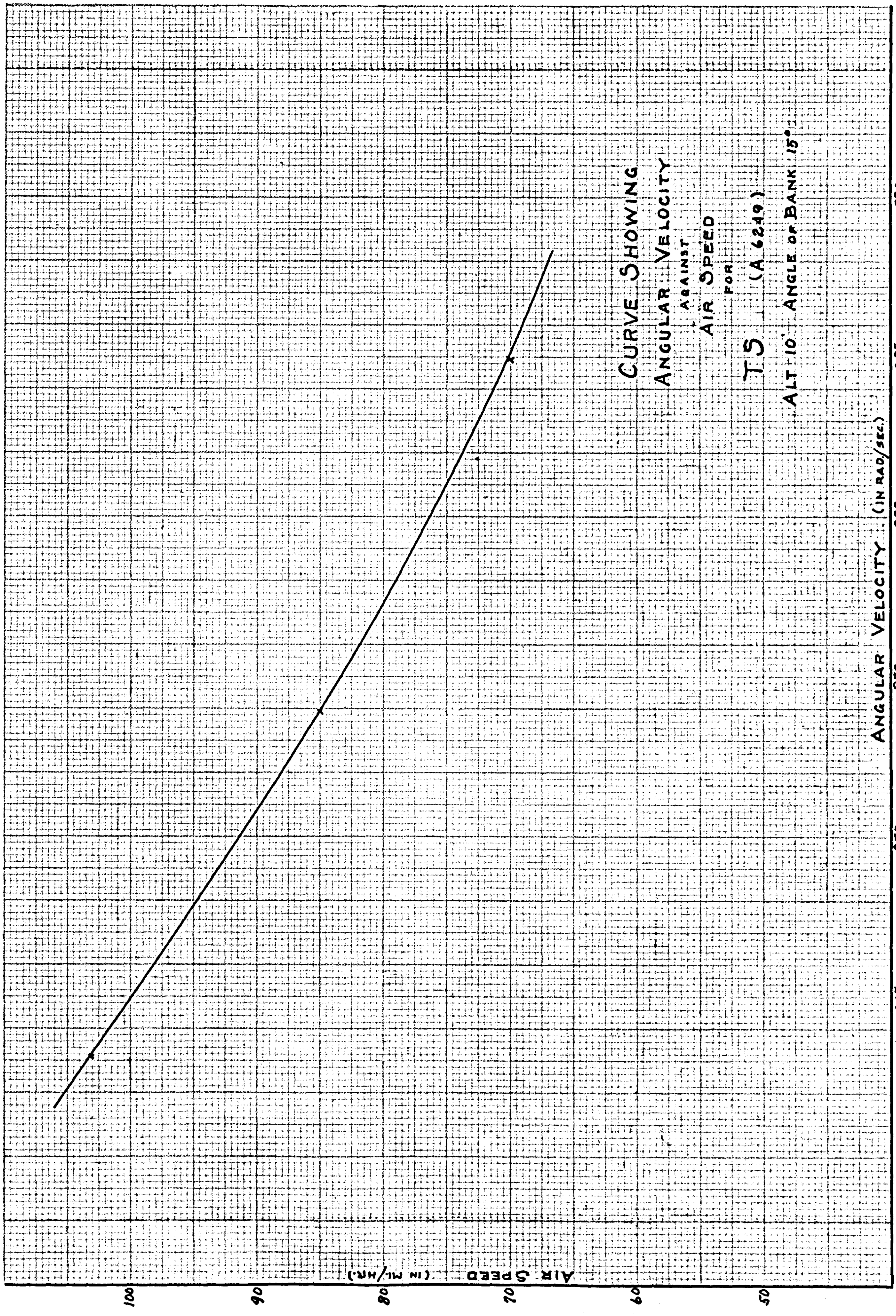
Third. That there is a tendency to settle when turning away from a head wind and a tendency to climb when turning into it, but this effect is so slight as not to be noticeable except in extremely strong winds, at very low altitudes, or when flying at a large angle of attack or near minimum power.

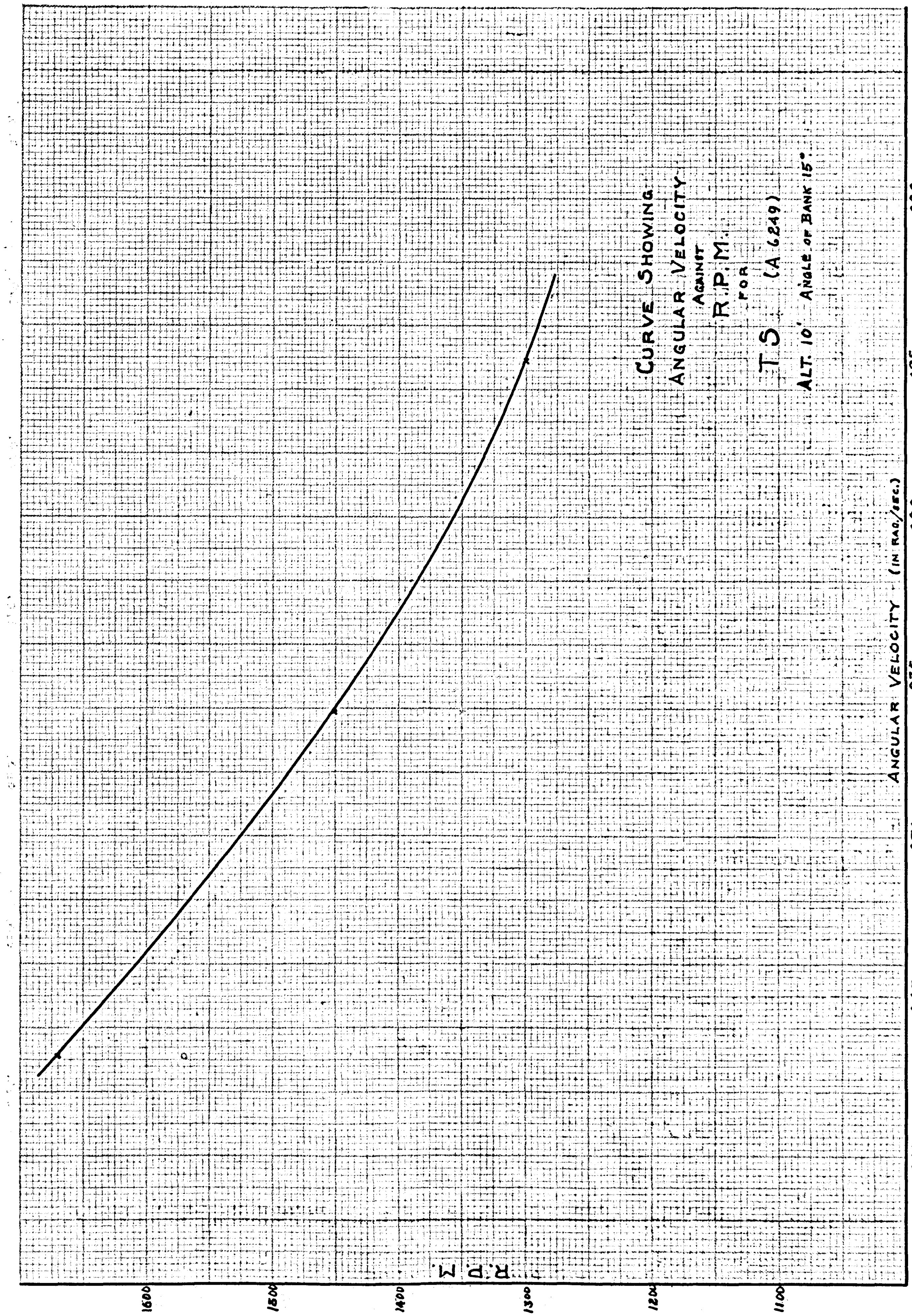
Fourth. That under normal wind conditions the effect of gradient is secondary to the effect of unsteady wind velocity, and at very low altitudes the air density under an airplane is increased, and this increase in density exerts a predominating influence on its lift and drag characteristics.


CURVES SHOWING
ANGULAR VELOCITY AND AIR SPEED
AGAINST
ANGLE OF TURN
FOR
T.S (A6249)
WIND 20 MI/HR. ALT. 10' ANGLE OF BANK 15°
FULL THROTTLE
ANGULAR VELOCITY (IN RAD./SEC.)
.070
.060
.050
AIR SPEED (IN MI/HR)
100
90
80
ANGLE OF TURN
0
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°


CURVES SHOWING
ANGULAR VELOCITY AND AIR SPEED
AGAINST
ANGLE OF TURN
FOR
T.S. (A6249)
WIND 20 MI/HR. ALT. 10' ANGLE OF BANK 15°
HALF THROTTLE
ANGLE OF TURN
ANGULAR VELOCITY (IN RAD/SEC)
AIR SPEED (IN MI/HR)
.080
.070
.060
90
80
70
0
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°


CURVES SHOWING
ANGULAR VELOCITY AND AIR SPEED
AGAINST
ANGLE OF TURN
FOR
T S (A 6249)
WIND 20 MI/HR. ALT. 10' ANGLE OF BANK
QUARTER THROTTLE
ANGLE OF TURN
0 30° 60° 90° 120° 150° 180° 210° 240° 270° 300° 330° 360°
ANGULAR VELOCITY (IN RAD/SEC)
.090 .080 .070
AIR SPEED (IN MI/HR.)
80 70 60


Curve Showing
Angular Velocity
against
Air Speed
for
T5 (A 6249)
Alt 10' Angle of Bank 15°
Angular Velocity (in rad/sec.)
.060
.065
.070
.075
.080
.085
.090
Air Speed (in mi/hr.)
100
90
80
70
60
50


CURVE SHOWING
ANGULAR VELOCITY
AGAINST
R.P.M.
FOR
TS (A 6249)
ALT. 10' ANGLE OF BANK 15°
ANGULAR VELOCITY (IN RAD./SEC.)
R.P.M.
.060
.065
.070
.075
.080
.085
.090
1600
1500
1400
1300
1200
1100

VIII.

CONCLUSIONS.

Both theory and experiment indicate that neither wind velocity nor wind velocity gradient exert an influence on airplane performance in straight level flight. In spite of this, numerous cases may be cited where planes have been known to settle when flying with a strong wind and to climb when flying into it. A possible explanation is that when a pilot is flying with a strong tail wind his ground speed is increased by the amount of the wind velocity and his angle of climb, with respect to the ground, decreased. The natural tendency is to pull the nose up to increase the climb. This tendency is even greater if there is an obstacle to clear, and when altitude must be obtained in as short a time as possible. If the plane is lightly loaded no trouble is experienced, but if heavily loaded and already flying near its most efficient climbing angle, the increase in angle of attack causes a decreased rate of climb and, if the rate was already low, causes settling. Instead of nosing over and picking up some speed, the average pilot, experiencing a panicky feeling, continues to pull up and hope. An inexperienced pilot often spins into the ground at this point. If the plane can be brought around into the wind the apparent increase in the rate of climb, resulting from the lower ground speed, makes the plane appear to be climbing much better.

From the theory, we should expect a slight difference in the attitude of the plane: a decrease in the angle of attack when flying into the wind and an increase when flying with the wind. This is due to the effect of the gradient on circulation and may be calculated as follows: Consider the case of a DH 4 M1 plane flying at 100 m/hr in a 15 m/hr wind at an altitude of 10'. From (50)

$$\Gamma_{max} = 4.94$$

From page 82

$$\alpha = +.2^{\circ}$$

From (1) the gradient is

$$\frac{dV}{dh} = .44$$

From (19) the circulation due to the gradient is

$$\Gamma_G = .615$$

The difference in circulation, due to gradient, when flying against and with the wind is then twice this amount or 25% of the total circulation. As the lift, and therefore the circulation, is zero at -1,8°, the required circulation is induced by an angular change of 2°. We should expect, according to the theory, to find a difference in angle of attack of .5°, between flying with and into the wind. This difference was not observed in the free flight tests though the instrument used was sufficiently accurate to detect an angular change of half this amount. This discrepancy should not invalidate the theory, however, as that is checked in all other respects.

From the curves of RPM against Air Speed it appears that the break, at speeds below minimum power, is not as abrupt as is generally supposed but more gradual, especially in high powered planes where the maximum angle of attack is greater and the effect of the slip stream and of the vertical component of propeller thrust increased.

There does not appear to be any sharp break in the lift curve of a full size plane and the angle of attack for maximum lift is greater than in a geometrically similar model. In the DH 4 M1 used in the free flight tests the maximum lift had not been reached at 18.5°. Maximum lift occurs at 14° in a 36 × 6" model of the RAF 15 (mod) wing used on this plane.

A practical use for the curves of RPM against Air Speed, taken together with curves of RPM against fuel consumption, would be to determine the air speed to choose in order to cover a maximum distance in any given wind. The work would be complicated by the variation of speed with load and the variation of speed and fuel consumption, for any given RPM, with altitude, but the solution could be easily worked out by making runs with different loads and at various altitudes.

In flight tests, at high angles of attack, the lift would be steady for a time and then gradually fall off. The impression given was of a smooth, but unstable flow gradually becoming turbulent and destroying the lift. The probable cause for this apparent instability is that, while the plane was flying steadily, some variation in wind velocity or direction caused a slight settling of the plane which increased the angle of attack and if the plane was already at its angle of maximum lift, would cause a loss in lift as well as an increase in drag, and the plane would continue to settle and lose speed. In any case the effect of a downward velocity of the plane is unfavorable as the drag is increased more than the lift and it is very difficult to over_come any downward momentum when there is only a slight lift.

The effect of increased density completely overshadowed the effect of gradient when flying close to the ground. This circumstance, together with the fact that when the wind was strong enough to give an appreciable gradient effect the air was so bumpy that the bumps exerted a predominating influence, made the collection of experimental data very difficult.

The effect of the increased density, according to Prandtl, is to improve the L/D ratio. This is obvious if we admit that

$$\text{Lift} = \rho V_0 \Gamma_{max} \frac{\pi}{4} s$$

Then, holding V_0 constant, any increase in

ρ must cause a decrease in Γ_{max}, as the lift and span are constant. This decrease in Γ_{max} permits a decrease in angle of attack and a consequent decrease in drag. The net result is an increase in L/D or the ability to fly under reduced power. The effect is greater at high angles of attack as, in this case, the effect on drag, for a given change in angle of attack, is much greater. Near maximum lift the theory does not hold, as the profile drag becomes an important factor and we have been dealing only with induced drag, considering the profile drag constant.

The tests substantiated the theory expounded above and, further, showed that the ground effect was measurable only when the plane was very near the ground. This was to be expected; for in the formula for the increase in density the increase is inversely proportional to the square of the altitude.

In the work on the rate of climb and normal turns no mathematical check can be expected, as the effect of gradient is only a very small part of the total change from average conditions. The dominating effect is due to unsteady air flow and the task resolves itself, largely, into isolating the effect of gradient from the much larger effects of unsteady flow, piloting and, at very low altitudes, ground effect or density change.

Experiment shows that, near the ground, there is an increase in the rate of climb when flying into the wind and a decrease when flying with it, and the change is of the order/the theory would lead us to expect.

A point not mentioned on page 84 is that a seconds pendulum was allowed to swing in front of the camera. Its image at various points gave a means of checking the plane speed against the indicated air speed. In order to do this the wind velocity had to be considered. If we assume the wind velocity constant over the period of time required for two successive runs, half the difference in ground speed gives the wind velocity. Slight differences in spacing between various points on the same curve are due to the fact that exactly the same point in the pendulum swing was not always taken since exact timing was not required and some pictures were clearer than others and permitted more accurate measurements. It was therefore customary to take a picture one side or the other of the true point if the picture was better. The increase in the distance between points as the slope increase is due to the vertical scale being larger than the horizontal.

The effect of a sharp short down current is shown in the middle curve on page 104. The effect of a steadily increasing wind is shown in the 65 m/hr curve on page 105. This raises the apparent rate of climb, momentarily, to something over 2500 feet per minute; an impossible value for a DH. Part of this is due to the actual increase in rate of climb resulting from the wind, and part due to the slowing down of the ground speed increasing the apparent rate of climb, or rate in relation to the ground. This is the more unusual in that this run was made with a light wind; therefore there must have been an actual reversal of wind direction at this point.

The rate of climb was practically constant from 60' up, but in all cases was very steep when started from 10', and then gradually fell off until the normal rate was reached. This was due to the increased density improving the L/D near the ground; and the ensueing increased rate of climb fell off after the effect of the increased density could no longer be felt.

When using a moving picture camera in connection with airplane tests considerable time can be saved in working up the results, and the accuracy greatly increased if a few precautions are observed while taking the pictures.

The flight path should be in a plane which is normal to the axis of the camera. The image of the airplane is then constant in length, and all distances are immediately comparative. This is most readily brought about by flying between two points on the ground, or better still, where a great many runs are to be made, along a white line. This also enables the pilot to place the ship a given distance from the camera, and always to start his maneuvers at the same point. In flying over water, at least two buoys should be set out to mark the course, and the limits of the camera field.

In projecting the pictures, it was observed that the film did not fit snugly and so did not come to the same point each time. In centering the picture on the screen a displacement of .001" of the film caused a displacement of about .1" on the image. Such a displacement cannot be neglected when the plane is far away and the image small. It is therefore advisable, when taking the pictures, to have some stationary object, besides the horizon, upon which to center the picture. If there is no satisfactory object in the field of view it is advisable to place one there, say a thin post a short distance from the camera. In projecting stills, the films are not only blistered ~~by~~ but distorted, if left stationary in the projector over too long a period of time.

The simplest possible representation of the forces acting in a normal bank is the weight, W, acting down, a force, F, acting radially outward, of magnitude $\frac{WV^2}{gR}$ where R is the radius of turn, and a resultant force, M, opposing these two. In very flat banks, we are justified in saying that the wings take practically all the lift and that the resultant force, M, is normal to the wing and therefore of magnitude $\frac{W}{\cos\varphi}$ or $\frac{F}{\sin\varphi}$, where φ is the angle of bank. As the angle of bank becomes steeper, the angle of attack of the entire plane, with respect to the flight path, is increased in order to maintain level flight; and the fuselage, struts, wheels etc., begin to take a considerable part of the weight. This is especially true in very high speed ships where, in extreme cases, the plane can be flown for considerable distances in an absolutely vertical bank with the wings taking none of the load and $R = \infty$. In this case $F = 0$ and $M = -L$. At any point in between the first and second conditions, the resultant force lies somewhere between a normal to the wings and the backward extension of the line representing the weight. This vector is then the lift, not of the wings, but of the entire airplane. Ms changes in direction for each angle of bank and angle of attack, at large angles of bank, and greatly complicates the work. As we are primarily concerned with the effect on the wings a small angle of bank (15°) was taken. In this case, neglecting the small part played by the vertical component of the thrust, we have the following simple formulas

$$F = \frac{WV^2}{gR} = \frac{W\omega^2 R}{g} \qquad (56)$$

$$M = \sqrt{W^2 + F^2} = \frac{W}{\cos \varphi} = \frac{F}{\sin \varphi} = L_c A V^2 \qquad (57)$$

If we consider a plane turning near the ground, at constant altitude, angle of bank, and throttle setting, the lift falls off slightly as the plane starts to come into the wind. In order to maintain the lift constant, the angle of attack must be increased slightly. This causes a greater drag and the plane slows down, but as

$$V = \sqrt{g \tan \varphi R} \qquad (58)$$

and

$$V = \frac{g \tan \varphi}{\omega} \qquad (59)$$

we see that the radius of turn is decreased and the angular velocity is increased.

This effect was noted in the records and, as would be expected, was much smaller at high speeds than at low. The reason for this is that at the lower speeds the wing is working at a high angle of attack, and a given change in angle of attack at this point has a greater effect on the drag, thus tending to cause a still greater loss in speed and consequent increase in ω.

If, in a steady turn, the angle of bank, air speed, and throttle setting are held constant the loss in lift upon coming into the wind can exhibit itself only as a certain loss in altitude. This would ordinarily increase α and decrease V, but if we hold V constant there should be no change in R or ω and the extra power required would be obtained from the potential energy used in the slight drop.

It is obvious from the curves on pages 115-116-117 that, although V and ω are intimately related, the effect of any change in control position becomes almost immediately apparent in ω, but a certain time interval is required for the speed to change.

The periodic effect of gradient in a continued turn appeared to lag about 45° behind the theoretical position. This was to be expected, and was observed when making turns at low RPM while making the RPM against Air Speed tests. At very low altitudes and at minimum engine power it was not possible to turn away from a head wind. The plane settled thru the first 90° of turn even in a very flat bank. If the first 90° was accomplished no trouble was experienced in making the second 90°. In turning out of a tail wind, no trouble was experienced in the first 90°, but at about the 135° point the plane settled. If leveled up and a straight course flown until the speed picked up the turn could be completed. This leveling out process was usually accompanied with a slight loss of altitude allowing the plane to settle into the denser air.

The effect of settling, in turning out of a head wind, might persist even tho the turn was successfully executed, and thus add to the troubles ordinarily incedent to level flight in a tail wind.

If a plane is flying level at minimum power, it is usually possible to make a turn if the wind is not strong. Any decrease of altitude improves the L/D and this improvment makes up for the increased drag due to the turn. In case the plane is very low the lowered wing tip is working in a more favorable medium than the upper, and as the density varies inversely as the square of the height, there is a net gain. This again may overcome the loss due to increased α.

The curves of ω against Air Speed and RPM have the same general shape. This is to be expected, since we are working in the straight part of the lift curve which, from the curves on pages 70-75, gives a straight line variation of Air Speed against RPM.

The general conclusion to be drawn from this work is that the wind velocity gradient causes a measurable change in airplane performance but, as the effect is of an order of magnitude lower than the errors in ordinary experimental measurements, can be neglected in all routine performance tests.

IX.

WIND TUNNEL TESTS.

A study of the "ground effect" on the lift and drag characteristics of an airplane model was made by the British Advisory Committee and the results of their tests were published in July 1920 in Reports and Memoranda No. 754

Two methods of studying the effect of the ground were employed. In the first method a flat plate was placed in the tunnel and the model mounted directly above this plate. In the second, known as the reflection method, two identically similar models were mounted so that their under surfaces faced each other.

The models were biplanes, having no stagger, the gap equal to the chord, and 3"×18" R.A.F. 15 wings.

The results of the experiments showed a maximum L/D of 10.4 for the biplane model in free space, a maximum L/D of 15.1, in the same model when the flat plate was 1½" from the lower wing, 12.3 when the plate was 2¾" from the lower wing, 12.9 when two models, whose lower wings were separated by 3" wire used and 12.1 when the lower wings of the models were separated by 5½". This corresponds to an increase in efficiency of 45% in the first case, 18% in the second, 24% in the third, and 16% in the fourth. No satisfactory explanation was given for this increased efficiency and no good reason for the failure of the two methods to check when the model was 1 ½" from the plate and 3" between models and the close check when the model was 2 ¾" from the plate and 5 ½" between models.

If we admit that Prandtl's theoretical development represents the actual conditions met with in air flow the above conditions are roughly what we should expect to find.

The theoretical representation of the first method is similar to the second except that when we deal with the first we consider only the pressure increase, and consequently employ only the vorteces coming off the wing in the development of our formulas for vertical velocity, as directly under a wing, there is no vertical component of velocity due to the transverse circulation. Consequently we may consider the lift constant regardless of the altitude, and deal only with the effect of this circulation on drag. In this case we have merely <u>imagined</u> a mirrored image and have taken only the part of its effect which has a bearing on the case in hand.

The induced drag of a wing, if we assume an elliptical lift distribution, is represented by

$$D_{ind} = \frac{2 L^2}{\pi \rho V_0^2 s^2} \qquad (53)$$

The change in the induced drag of one wing due to the circulation around a second wing, providing the second wing lies directly above or below the first, is expressed by

$$D'_{ind} = \int_{-\frac{s}{2}}^{+\frac{s}{2}} \frac{w}{V_0} dL = \sigma \frac{2 L^2}{\pi \rho V_0^2 s^2} \qquad (60)$$

where w is the vertical component of velocity due to the first wing and σ is the influence coefficient which has been worked out for various values of the ratio of the span to the distance between the wings. The value of this integral is negative when the wings are lifting in opposite directions; so the effect is to reduce the drag.

If we consider the wings above, the total reduction in induced drag is the effect of the lower and upper wings of the imagined image on both the lower and upper wings of the model.

When the model is 1 ½" from the plate the lower wings are separated by 3", upper and lower by

6",and the upper wings by 9". If we call this distance h and the span s we may obtain from Prandtl's curve of h/s against σ the following values

h =	3"	5 ½"	6"	8 ½"	9"	11 ½"
$\frac{h}{s}$ =	.167	.306	.333	.477	.5	.639
σ =	.53	.37	.36	.24	.23	.18

In a biplane combination, having a gap chord ratio of 1:1, no stagger and working maximum L/D, the upper wing takes approximately the same proportion of the load as the lower; so the value of σ is

$$\frac{.53 + .36 + .23}{2} = .56$$

The drag coefficient at 6°, the point of maximum L/D, was given as .0249 and the lift coefficient as .260.

The lift is then

$$L = C_L \times \rho A V_0^2 = 12.48\#$$

and the drag

$$D = C_D \times \rho A V_0^2 = 1.195\#$$

The induced drag is, from (53)

$$D_{ind} = .354\#$$

The decrease in drag is then $.56 \times .354 = .198\#$.
The new drag is .997 and the new L/D 12.5, a considerably smaller value than the observed.

When the plane is $5\frac{3}{4}$" from the plate the L/D figures out to be 11.8, a closer check but still low.

In the case where we use another wing instead of the imaginary image we have the same effect on drag, due to the longitudinal vortices flowing off the wings, and also the effect of the transverse vortices. The circulation around each wing is in such a direction as to reduce the velocity and consequently the lift of the other wing.

The maximum circulation from (50) is

$$\Gamma_{max} = \frac{4\ L}{\pi\ \rho\ V\ b} \qquad (61)$$

The mean circulation is approximately $\frac{\pi}{4}$ times this, or

$$\Gamma_{mean} = \frac{L}{\rho\ V\ b} = 2.6$$

The velocity effect due to this circulation is

$$V = \frac{\Gamma}{4 \pi h} (\cos \alpha_1 - \cos \alpha_2)$$

When the wings are 3" apart the velocity at the tip of **one** wing, due to the circulation around the other wing, is .81 ft./sec. and at the center 1.56 ft/sec. The effective change in velocity is about 1.30 ft./sec. Taking the effect of both wings at the image on each wing of the model, we find that the total effective drop in velocity is 1.5 ft./sec. As V_o was 40 ft./sec., the loss in lift is 3.5%; so we should expect the same drop in L/D, giving a value of 12.1, which checks more closely with the experimental value of 12.8

When the wings are 5½" apart, the effective velocity drop is 1 ft./sec. and the L/D 12.0

The poor check between theory and experiment when the surfaces are close together is probably due to the theory being built up on the consideration of a lifting line which has a dimension of length only. When the distance between lifting surfaces is not greater

X.

REFERENCES.

The following references were used in the preparation of this Thesis.

Aerology — U.S. Naval Observatory, Pensacola, Fla.,

Traité de Mécanique — Appell.

Hydrodynamics — Lamb.

Hydrodynamics — Ramsey.

Flugtechnik — Proll.

Aérodynamique — Joukowski.

Sur la Théorie des Surfaces portantes — Maurice Roy.

Aerodynamik — Fuchs, Hopf.

Tragflugeltheorie — Prandtl.

Der induzierte Widerstand von Mehrdeckern — Prandtl.

Aeronautics — Wilson.

Wing Resistance near the Ground — Wieselsberger.

BRITISH ADVISORY COMMITTEE

Reports and Memoranda

No. 2[illegible] Variation of Wind Velocity close to the Ground. Taylor and Cave.

No. [illegible]31 Variation of Wind Speed near the Ground — Director of Naval Meteorological Service Hydrographic Dept. Admiralty.

BRITISH ADVISORY COMMITTEE

Reports and Memoranda

No. 754 Cushioning Effect on Aeroplanes close to the Ground. - W.L. Cowley, A.R.C.Sc. and C.N.H. Lock, B.A.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Technical Notes.

Reports

No. 3. Theory of an Airplane encountering gusts. - I Edwin Bidwell Wilson.

No. 13 Meteorology and Aeronautics - Wm R. Blair.

No. 21 Theory of an Airplane encountering gusts. - II Edwin Bidwell Wilson.

No. 27 Theory of an Airplane encountering gusts. - III Edwin Bidwell Wilson.

No. 97 General Theory of the steady motion of an Airplane. - George DeBothezat.

No. 112 Control in Circling Flight - F.H. Norton and E. T. Allen.

No. 116 Applications of Modern Hydrodynamics to Aeronautics. - L. Prandtl.

No. 121 The minimum Induced Drag Aerofoils Max M. Munk

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Techinal Notes.

No. 124 Aerodynamic Characteristics of Aerofoils II. By the National Advisory Committee for Aeronautics.

No. 142. General Theory of Thin Wing Sections. Max. M. Munk.

No. 151. General Biplane Theory - Max M. Munk.

No. 155. A study of Airplane Maneuvers with Special Reference to Angular Velocities. H.J.E. Reid.

No. 174. The Small Oscillations of Airplanes in Steady Flight. Max M. Munk

No 191 Elements of the Wing Section Theory and of the Wing Theory - Max M. Munk.

No. 213 A Résumé of the Advances in Theoretical Aeronautics made by Max M. Munk - Joseph S. Ames

XI.

BIOGRAPHICAL NOTE

The author was born December 14, 1896 in Alameda, California. Went thru the first six grades of grammar school in Nome, Alaska and the last two grades in Los Angeles, California, graduating in 1910. Graduated from the Manual Arts High School of Los Angeles in February 1914 and took a half year of post graduate work in English before entering college the following fall.

Enlisted in the U.S. Army Air Service, then the Aviation Section of the Signal Corps in 1917, instead of returning for his senior year at the University of California. It was at that time the policy of educational institutions, throughout the country to confer Bachelor degrees on seniors leaving school for the purpose of entering the military service of the United States, but the author's degree was not conferred until 1922.

Graduated from the Air Service Flying School in 1918, the Air Service Mechanics School in 1920 and the Air Service Engineering School in 1923.

Received the degree of Master of Science in Aeronautical Engineering in June 1924, at the Massachusetts Institute of Technology and was made a Fellow in Aeronautical Engineering at the Massachusetts Institute of Technology in November of the same year.

The author wishes to express his appreciation to Professor Warner of the Massachusetts Institute of Technology, Lieut. Guy Townsend, U.S.N., Lieut. G. F. Chapline, U.S.N., Lieut.Malcolm Schoeffel, U.S.N., Lieut.Reginald Thomas, U.S.N.R. and Mr. O. E. Kirchner, Mr. John Crowley, Mr. Henry Reid, Mr. George Bulisant and Mr. Hunsacker of the National Advisory Committee for Aeronautics for the assistance rendered in the collection of experimental data.