A Preliminary Study of A New Sampling Approach

by

Agus Budiyono

S.B., Bandung Institute of Technology (1992)

S.M. Aero-Astro, Massachusetts Institute of Technology (1998)

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Author . . . .
Department of Aeronautics and Astronautics
January 14, 2000

Certified by . . . .
Rudrapatna V. Ramnath
Senior Lecturer, Department of Aeronautics and Astronautics
Thesis Supervisor

Certified by . . . .
Manuel Martinez-Sanchez
Professor of Aeronautics and Astronautics
Chairman. Engineer in Aeronautics and Astronautics Program

Accepted by . . . .
Nesbitt Hagood
Chairman, Departmental Committee on Graduate Students

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Abstract

This work addresses the problem of tracking a nonstationary slowly varying signal. Reconstruction of the signal by judicious but nonuniform sampling using Ramnath's approach [1] is studied by means of illustrative examples. The top-level feasibility of efficient tracking of several signals of widely separate frequencies by a multiplexing approach is also studied. The benefits of the scheme including the multipexing technique for the onboard avionics system are evaluated and presented through simulation.

Thesis Supervisor: Rudrapatna V. Ramnath
Title: Senior Lecturer, Department of Aeronautics and Astronautics

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Chapter 1

Introduction

1.1 Background

Most if not all computational engines today are digital. The incorporation of a digital computer in a system design has made advances in many sophisticated engineering applications ranging from fiber-optics based telecommunication to modern avionics systems. At the core of this technology are sampling and interpolation theory. The theory addresses the fundamental questions of regaining the original signal from samples of a continuous signal or assesing the information lost in the sampling process.

It has been more than fifty years since Shannon introduced the sampling theorem to communication theory. Historically, the interest of communications engineers in sampling theorem may be traced back to Nyquist [11]. Some people credit Cauchy for recognition of the mechanics of band-limited signal sampling in 1841. A summary of key events in the development of the sampling theorem is listed in Table 1.1 [12].

The main contribution of Shannon Sampling theorem to information theory is that it allows the replacement of a continuous band-limited signal by a discrete sequence of its samples without loss of any information. Nowadays, the theorem has been extended and generalized to many forms suitable for more general applications. The extensions include sampling of functions of more than one variable, random processes, nonuniform sampling, nonband- limited functions, implicit sampling, generalized functions (distributions), sampling with the function and its derivatives (as suggested by Shannon in his original paper), and sampling of functions represented as general integral transforms [11]. The notion of nonuniform sampling of a nonstationary signal at judiciously chosen instants was developed by Ramnath [1]. The objective of the research presented here is to demonstrate the applicability of Ramnath's approach through illustrative examples. When there is a need for tracking many signals of widely separated frequencies, Ramnath [1] developed the idea of multiplexing an on-board computer appropriately. By this means the limited capacity of an on-board computer can be substantially enhanced so that many signals can be tracked by the same computer thereby resulting in grest efficiency. This approach is

Table 1.1: History of sampling theorem development[12]

Year	Progress
1841	Cauchy's recognition of the Nyquist rate
1897	Borel's recognition of the feasibility of regaining a band-limited signal from its samples
1915	E.T. Whittaker publishes his highly cited paper on the sampling theorem
1928	H. Nyquist establishes the time-bandwidth product of a signal
1929	J.M. Whittaker coins the term cardinal series
1933	A. Kotel'nikov publishes the samping theorem in the Soviet literature
1948	C.E. Shannon publishes a paper which establishes the field of information theory. The sampling theorem is included
1959	H.P. Kramer generalizes the sampling theorem to functions that are bandlimited in other than Fourier sense
1962	D.P. Peterson and D. Middleton extend the sampling theorem to higher dimensions
1968	A. Papoulis first publishes his generalization of the sampling theorem. A number of previously published extensions are shown to be special cases.

rendered possible by means of Ramnath's Generalized Multiple Scales theory. [2].

1.2 Problem Definition

1.2.1 Motivations

The original Shannon sampling theorem was based on the assumption of a time-invariant system. In general, however, we are dealt with time-varying systems. For a more general system, we can categorize the sampling scheme as follows:

- **Constant sampling**. The signal is sampled into discrete uniformly-spaced sequences using a uniform sampling rate.

- **Periodic constant sampling**. The scheme is intended for signals with a periodically-changing frequency. The signal is sampled into discrete uniformly-spaced sequences using a single sampling rate within a certain time interval and the pattern is repeated in accordance with the signal's frequency periodicity.

- **Periodic variable sampling**. The sampling rate is continuously changing with certain periodicity.
- **Non-periodic variable sampling**. The sampling rate is non-uniformly changing according to the frequency variation of the continuous signal.
- **Multirate-sampling**. The method handles the case where different signals with different frequencies need to be sampled using a single processing unit. This gives rise to a signal multiplexing technique.

The Shannon sampling theorem can be applied to the first two categories. In order to have an efficient sampling algorithm for a more general form of signals a new formulation is required. It is in response to this requirement that a new sampling scheme using asymptotic theory is proposed. Asymptotic theory allows systematic separation between high and low frequency signals and, thus, can lead naturally to an optimal non-uniform sampling scheme. The research was focused on the last two categories of sampling problems.

1.2.2 Literature Survey

Techniques for non-uniform sampling and its properties have not yet been extensively explored. In his celebrated paper [13], Shannon pointed out that in order to specify the function, samples of the original signal need not be equally spaced. However, for unequally spaced samples, their location must be known accurately in order to adequately reconstruct the function. The reconstruction process is also more involved with unequal spacing. It is also known that sampling rates below Nyquist [15] maybe used to perfectly recontruct a band-limited signal. It is possible to sample at even lower average rate using non-uniform sampling if some assumptions about the signal are made. The work by Nohrden [14] explored the various techniques for obtaining such a sample train. The filter bank method and interpolation methods implemented for frequency estimation were proposed and evaluated. However, both of these methods require assumptions to be made about the spectrum. If these assumptions are not met, the results become totally unreliable. It was shown in this work that taking extra samples is necessary to alleviate the problem. A different approach using the notion of system function defined in [17] was proposed by Jerri [16] in addressing non-uniform sampling for time-varying systems. This paper, however, focuses on theoretical and mathematical issues and, thus, is of a limited practical importance.

1.2.3 Applications

The principles of non-uniform sampling and signal multiplexing have been implemented in diverse areas of engineering. The applications include multirate control system design [27][28][32][35][36], automotive technology, signal processing [26][29][33], image processing [23], pattern recognition [20], optics [21], telecommunications [24][10], information theory [22], queueing systems [31], and storage device technology [30].

Many new developments in engineering are driven by current needs in the industry. As an illustration, the idea of multiplexing has been implemented to accomodate an increasingly higher demand for bandwidth in the area of telecommunications. Data, video, voice signals and bandwidth- hogging megabytes of animated graphics have crowded transmission systems that had ample of space a few years ago and now demand much higher bandwidth. This demand triggers the development of a new technology called wavelength division multiplexing, or WDM, which represents the second major fiber-optic revolution in telecommunications. This technology makes use of fiber optics cables that existed since the 1980's. It multiplies the potential capacity of each fiber by filling it with not just one but many wavelengths of light, each capable of carrying a separate signal.

In this work, an application is made of Ramnath's approach of nonuniform sampling and multiplexing through illustrative examples. These include analytical examples as well as nonstationary signals representing the responses of time varying flight vehicle systems such as the generic hypersonic vehicle (GHAME) [5] and the XC-142 VTOL vehicle [4].

Chapter 2

Sampling of Time-Varying Signals

2.1 Introduction to Sampling Theorem

Shannon's original statement of the sampling theorem is as follows:

Theorem 1 *If a function $f(t)$ contains no frequencies higher than W cps, it is completely determined by giving its ordinates at a series of points spaced $1/2W$ seconds apart*

Intuitively, we can justify the theorem by observation that, if $f(t)$ contains no frequencies higher than W, it cannot change to a substantially different value in a time less than one-half of a cycle of the highest frequency, that is, $1/2W$. Shannon's mathematical proof starts by writing

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega = \frac{1}{2\pi}\int_{-2\pi W}^{2\pi W} F(\omega)e^{i\omega t}d\omega \tag{2.1}$$

The spectrum of $f(t)$, $F(\omega)$, is assumed to be zero outside the band $(-2\pi W, 2\pi W)$. The Fourier series expansion of $F(\omega)$ on the fundamental period $-2\pi W < \omega < 2\pi W$ is

$$F(\omega) = \sum_{n=-\infty}^{\infty} c_n e^{-i\omega n/2W} \tag{2.2}$$

$$c_n = \frac{1}{4\pi W}\int_{-2\pi W}^{2\pi W} F(\omega)e^{i\omega n/2W}d\omega = \frac{1}{2W}f(\frac{n}{2W}) \tag{2.3}$$

We observe that the Fourier coefficient c_n is proportional to $f(\frac{n}{2W})$, the values of the signal $f(t)$ at the sampling points. Also, $\{c_n\}$ determines $F(\omega)$, hence, by uniqueness property of the Fourier transform, $f(t)$ is determined. The signal reconstruction is given by:

$$f(t) = \sum_{n=-\infty}^{\infty} f(\frac{n}{2W}) Sinc(\pi(2Wt - n)) \tag{2.4}$$

where $Sinc$ function is defined as:

$$Sinc(\pi(2Wt - n)) = \frac{sin\pi(2Wt - n)}{\pi(2Wt - n)} \tag{2.5}$$

2.2 An Asymptotic Approach to Non-uniform Sampling

As noted earlier, the original Shannon sampling theorem is valid only for time-invariant systems. In this work, a new approach to sampling of time- varying systems is considered, based on the asymptotic method of generalized multiple scales developed by Ramnath [1]-[5]. A comprehensive description of the technique can be found in references [1][2]. The method enables us to develop asymptotic solutions to dynamic problems by systematically separating the fast and the slow parts of the dynamics. The fast scale solutions in this case describe variations in frequency and phase of the solutions. This gives rise to the continuously-changing instants at which the corresponding time- varying signal can be sampled. In the following sections, we illustrate the benefit of a continuously changing sampling rate over the constant sampling scheme. We describe the technique by first considering a simple sinusoidal signal with linearly changing frequency. Later, we will implement the same technique for the angle of attack dynamics of GHAME vehicle during reentry and VTOL aircraft during the transition phase.

2.2.1 The Case of Prescribed Signals

As a preliminary description, suppose that we know the signal a priori. The analog signal used is $sin(t^2)$. Efficiencies of the three possible methods are compared.

- Constant rate sampling: In this technique, the analog signal is sampled using the Nyquist sampling frequency i.e. $2 \times f_h$, where f_h is the highest frequency present in the signal.

- Discretely-changing rate sampling: The analog signal is sampled within three different regions of frequency (low, medium and high). In each interval, the

Nyquist sampling rate is used.

- Continuously-changing rate sampling: In this method, the sampling rate is changing continuously in accordance with the changes in the analog signal frequency, $f(t) = t/\pi$. In our simulation, we have a time varying signal from $t = 0$ to $t = 20s$, stored in 2000 data points. We divide the interval into non-uniform subintervals. Each subinterval represents one cycle and, thus, each has a unique duration if the signal frequency changes in time. We assume that within each subinterval the frequency increment is small and so we can use the Nyquist rate $2 \times f(t)$ where $f(t)$ is known at all times at each instance we carry out the sampling. As a result, we have a continuously changing sampling rate over the entire time interval. The algorithm for this sampling procedure can be summarized as follows:

  1. Choose a time interval T of the signal to be sampled.
  2. Divide the interval into n subintervals (Δt_i) each of them representing one cycle.
  3. For each subinterval Δt_i, perform sampling by taking two samples. This corresponds to the Nyquist sampling rate. To minimize the error it is suggested that the maxima and minima points be chosen as samples. For practical sampling, intermediate points can be added as samples.
  4. Repeat the procedure for all subintervals, $\Delta t_0 ... \Delta t_n$.

  The result of a continuously-changing rate sampling procedure is presented in Figure 2-4.

The last two methods lead to significant savings in both memory and CPU time since less data is to be processed. The memory usage is measured by the size of the vectors that represent the signals. The CPU time is measured by processing the sampled signal using some signal processing techniques and comparing the required time. The processing can be done by doing FFT, IFFT, interpolation (over- sampling), decimation (under-sampling) or any other common signal processing techniques. The comparison is made between the three approaches and the results are presented indicating the potential savings. It is important to note that the comparison shown in the table indicates the least potential saving. In general we might not need to use the entire signal for this purpose (e.g. display, time-history, or control). Thus, the amount of savings depends on the fraction of signal which is actually used or processed. The actual benefit of the technique is shown in Figure 2-5. This figure indicates that we gain most savings in the early time intervals. For these intervals, if a constant sampling rate is used we would have had to sample the lowest frequency portion of the signal at a much higher sampling rate than is required. We can also observe that the larger is the separation between the lowest and the highest frequencies, the more savings are gained.

Methods of sampling	Avg. CPU time (seconds)	Memory (number of samples)
Constant rate	0.548	134
Discretely-changing rate	0.283	90
Continuously-changing rate	0.258	76

Table 2.1: Comparison of CPU time and Memory Allocation between Different Sampling Schemes

2.2.2 The sampling technique for more general signals: frequency estimation using the discrete Fourier transform

In general, we do not know the analytical expression of the signals that we want to sample. All we have, for instance, is a sequence obtained from measurement (e.g. Doppler measurement). In a prescribed signal we know the frequency at any point in time, so the Nyquist rate at every instances of sampling can be easily determined. In a more general case, we need to know how the frequency behaves over the required time interval.

One way to calculate the frequency is to transform the signal into frequency domain using FFT. We know that FFT is a one-to-one mapping from the time domain to the frequency domain. Thus, it should be possible to obtain the information of the Nyquist frequency right away from one window of the FFT. In the frequency domain, we can determine the frequency by finding the location of the peak of the magnitude of the transformed signal. The peak indicates the frequency with the highest energy contribution. In conjunction to our previous simulation of continuously changing sampling rate, this problem can be recast into a problem of finding the frequency at each subinterval. We start by picking one of the subintervals and determining whether the frequency can be found. If that is the case, we can repeat the procedure over the entire interval. As a result, we determine how the frequency changes within the entire interval. The algorithm can be summarized as follows:

1. Choose a time interval of the signal to be sampled, T.
2. Divide the interval into n subintervals (Δt_i)
3. For each subinterval Δt_i, apply FFT to get the spectrum of the subinterval F_i.

4. Estimate the frequency of the signal in the subinterval Ω_i by identifying the location of the peak of the magnitude of the transformed signal. The continuous time frequency Ω_i (rad/s) is given by:

$$\Omega_i = \frac{2\pi j}{NT} \tag{2.6}$$

where	j	is j-th point of FFT where the peak occurs
	N	is the total length of FFT
	T	is the sampling period

5. Sample the subinterval using the sampling rate of $2 \times f_i$

6. Repeat the sampling for the all subintervals, $\Delta t_0 ... \Delta t_n$.

The algorithm is tested by implementing it on the previously known signal. The comparison between the exact frequency and the estimate is shown in Figure 2-1. The change in the frequency content of the signal over time is presented in the spectogram in Figure 2-2. The sampling results using this method are shown in Figure 2-3. The number of sampled data points using this scheme is 76. It is the same as the exact result (found in the previous case).

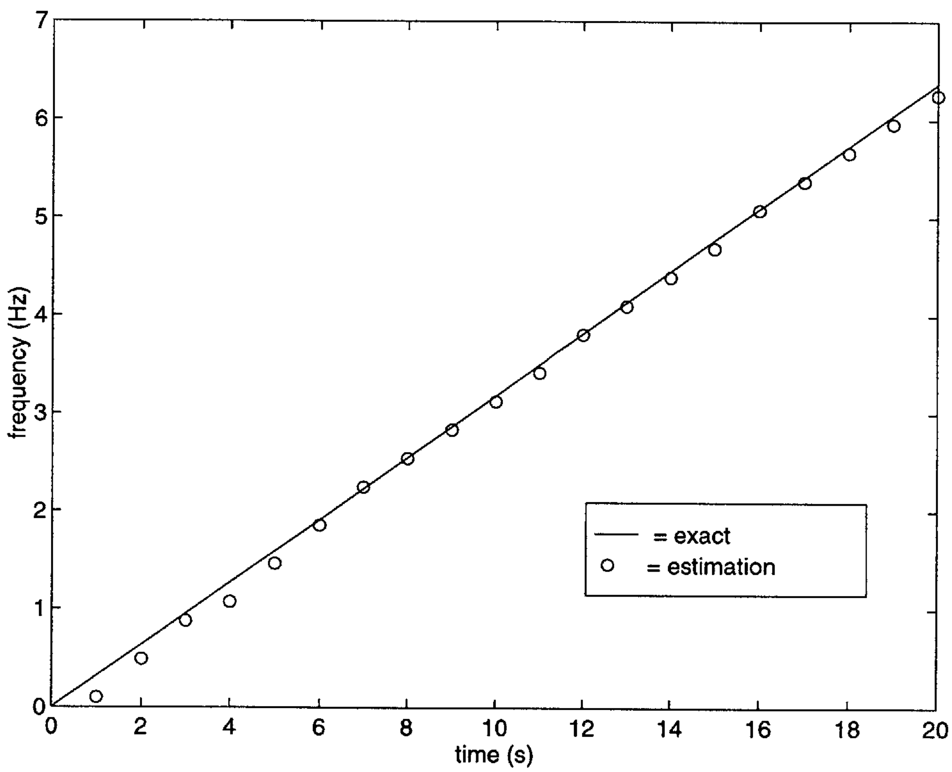

Figure 2-1: The comparison of the frequency: exact versus estimation

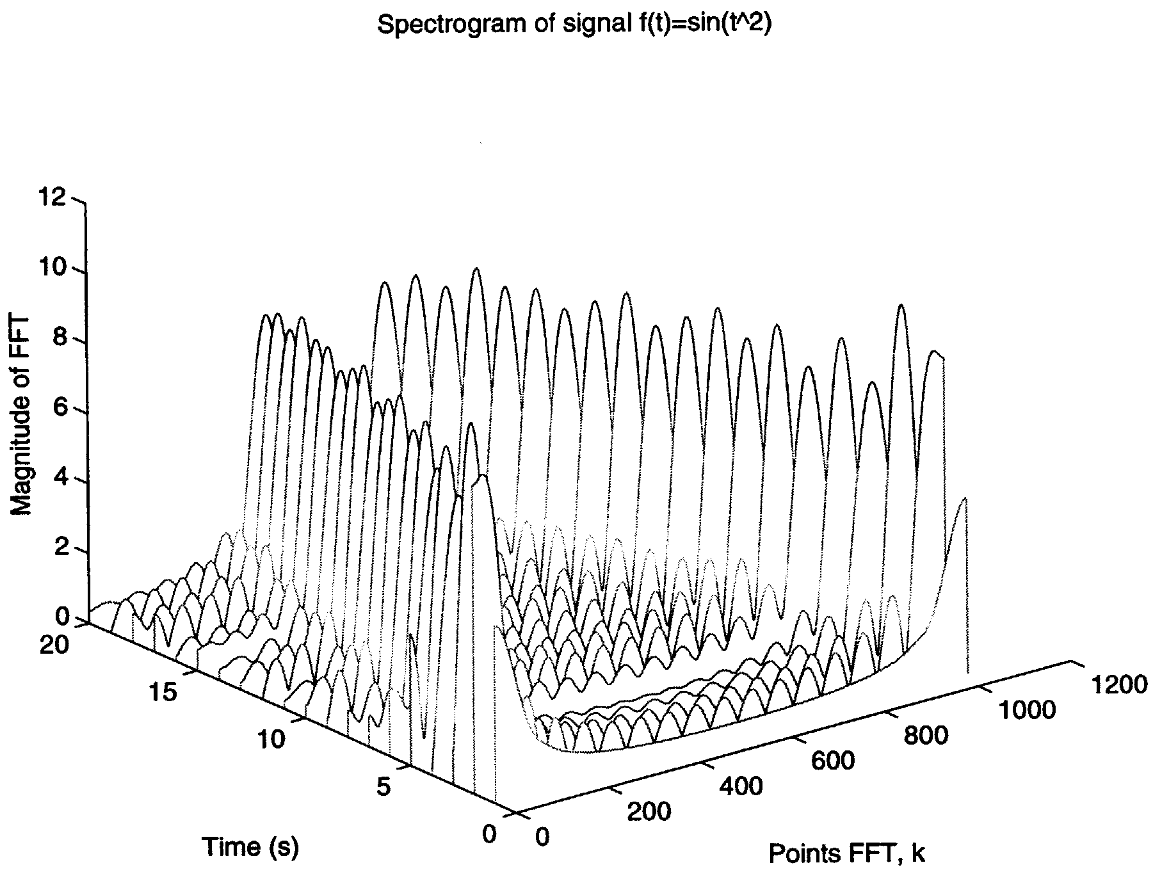

Figure 2-2: The Spectrogram of the signal $f(t) = sin(t^2)$

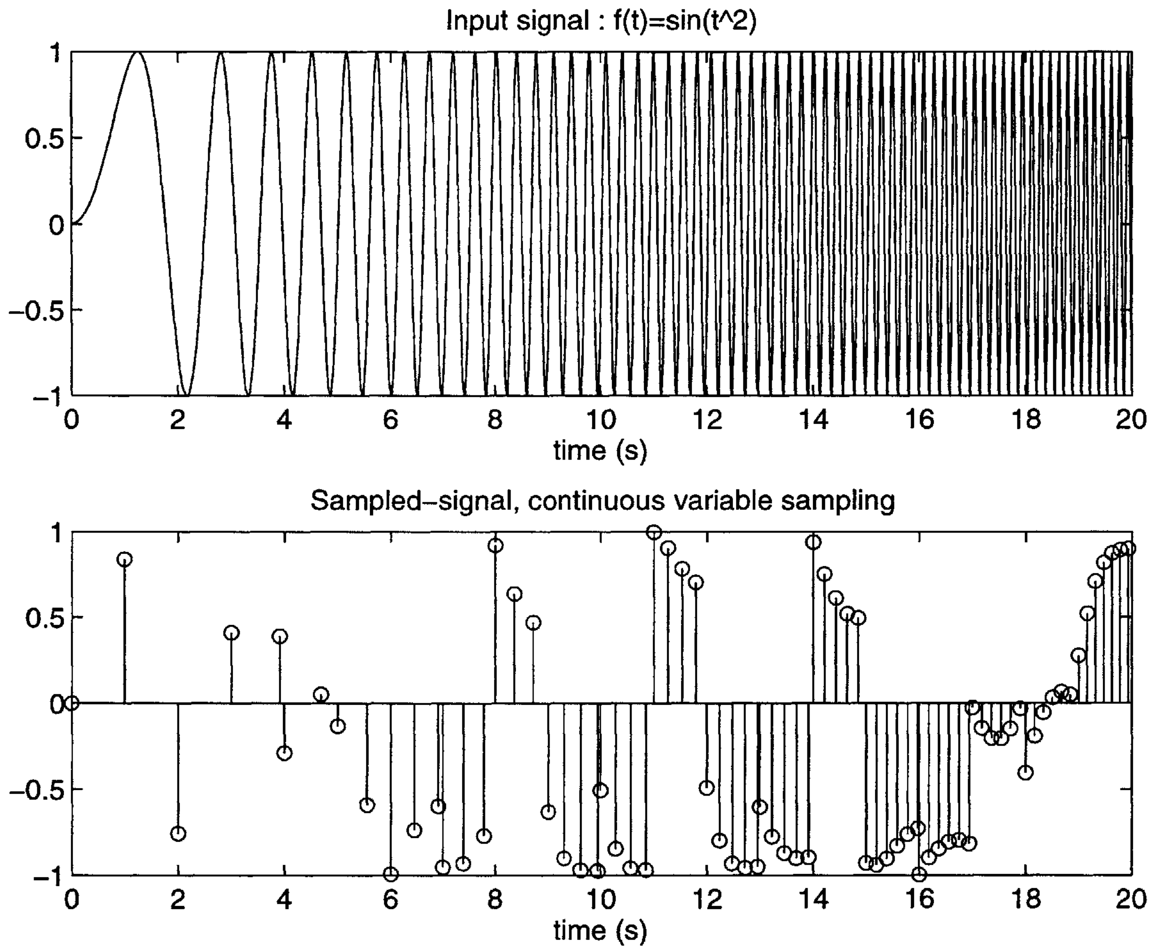

Figure 2-3: Continuously-changing sampling rate for an unknown signal

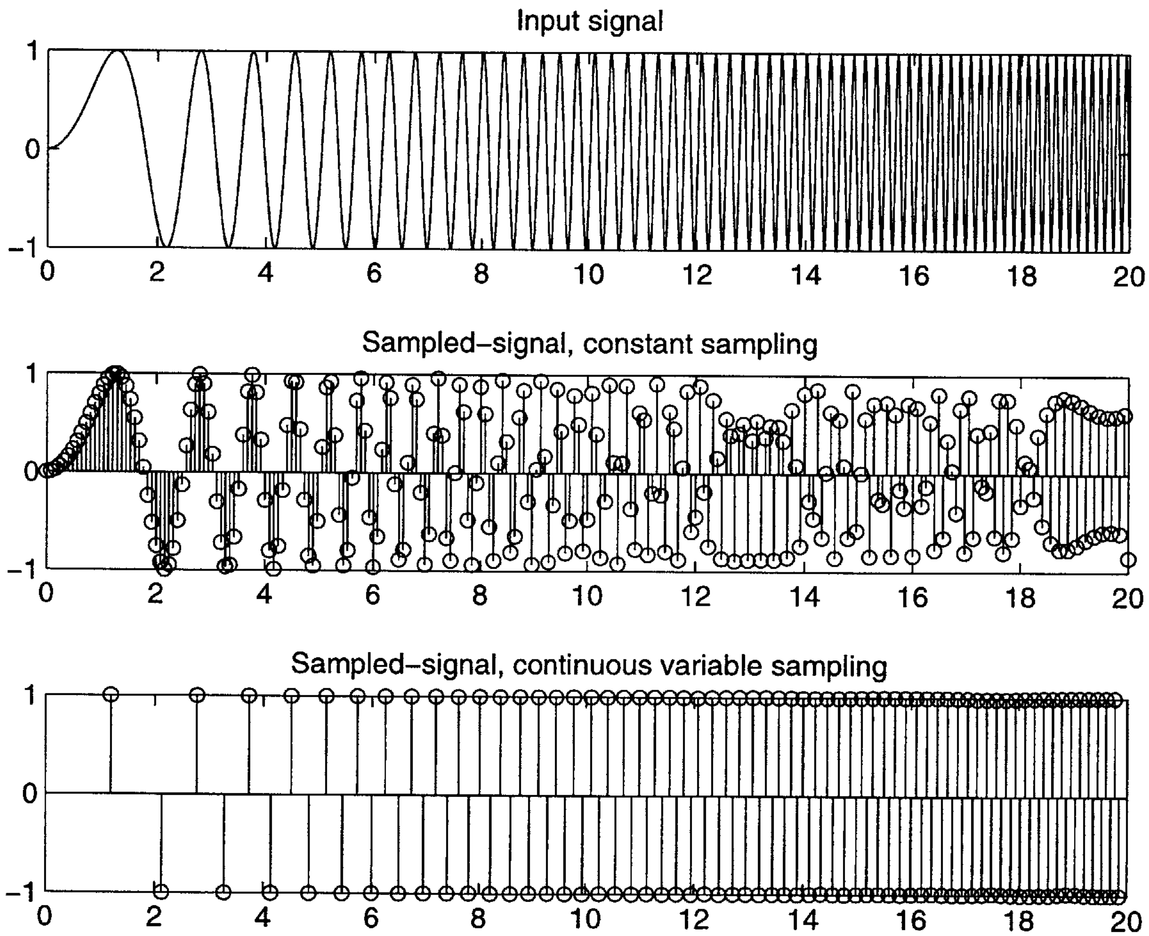

Figure 2-4: Comparison of constant, discrete and continuous sampling

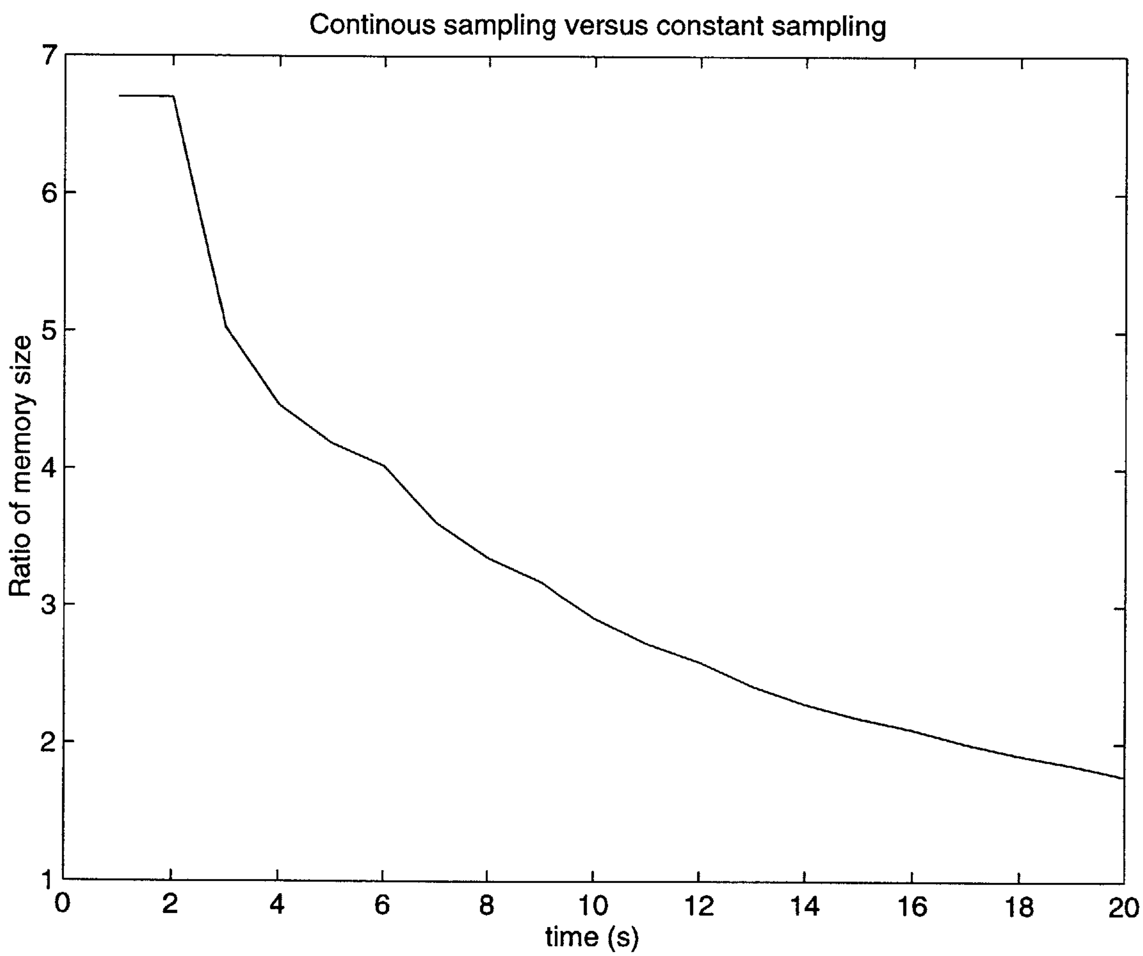

Figure 2-5: Potential memory saving

2.2.3 Signal Reconstruction

Signal reconstruction is an integral part of sampling. A sampling scheme is acceptable if the reconstruction of the original signal from its samples is guaranteed to have minimal errors. The errors in sampling include:

- *Truncation error* which results when only a finite number of samples are used instead of the infinite number of samples needed for sampling representation.
- *Aliasing error* which is caused by violating the assumption that the signal is band-limited.
- *Jitter error* which is caused by sampling at instants different from the sampling points.
- *Round-off error* which comes from the digital recording of the sample values.
- *Amplitude error* which is the result of the uncertainty in measurements of the amplitude of the sample values.

A rigorous treatment of some of these errors with their bounds were given by Papoulis [18] and Thomas and Liu[19].

To develop a reconstruction technique for a time-varying signal from its samples, we start with an equation for reconstructing the time-invariant signal given by Eqn. 2.4. In this case, we can say that any function limited to the bandwidth W and the time interval T can be specified by providing $2TW$ values. Based on our continuously-changing rate sampling as outlined in Section 2.2.1, our algorithm for the signal reconstruction is an extension of Eqn. 2.4 which can be summarized as follows:

1. For each cycle identify the sampling points. For Nyquist rate sampling, we have 2 samples and approximately $8 - 10$ samples for practical sampling.
2. Reconstruct the signal in the subinterval Δt_i corresponding to the above cycle by using Equation 2.4. From each of the cycles we will have one sinusoidal reconstruction, s_i.
3. Perform the reconstruction process for all intervals.

The results of the reconstruction process for the case presented in Section 2.2.1 are shown in Figure 2-6. One other way to reconstruct a signal is by using a polynomial of a certain order. A least squares criterion can be used to find the reconstructed signal that best fits the original signal. The result of this technique for reconstructing the original signal is shown in Figure 2-7. The resulting errors in both reconstruction techniques are shown in Figure 2-8. Further, Figure 2-9 gives a spectral version of the comparison between the original, sampled, and the reconstructed signals.

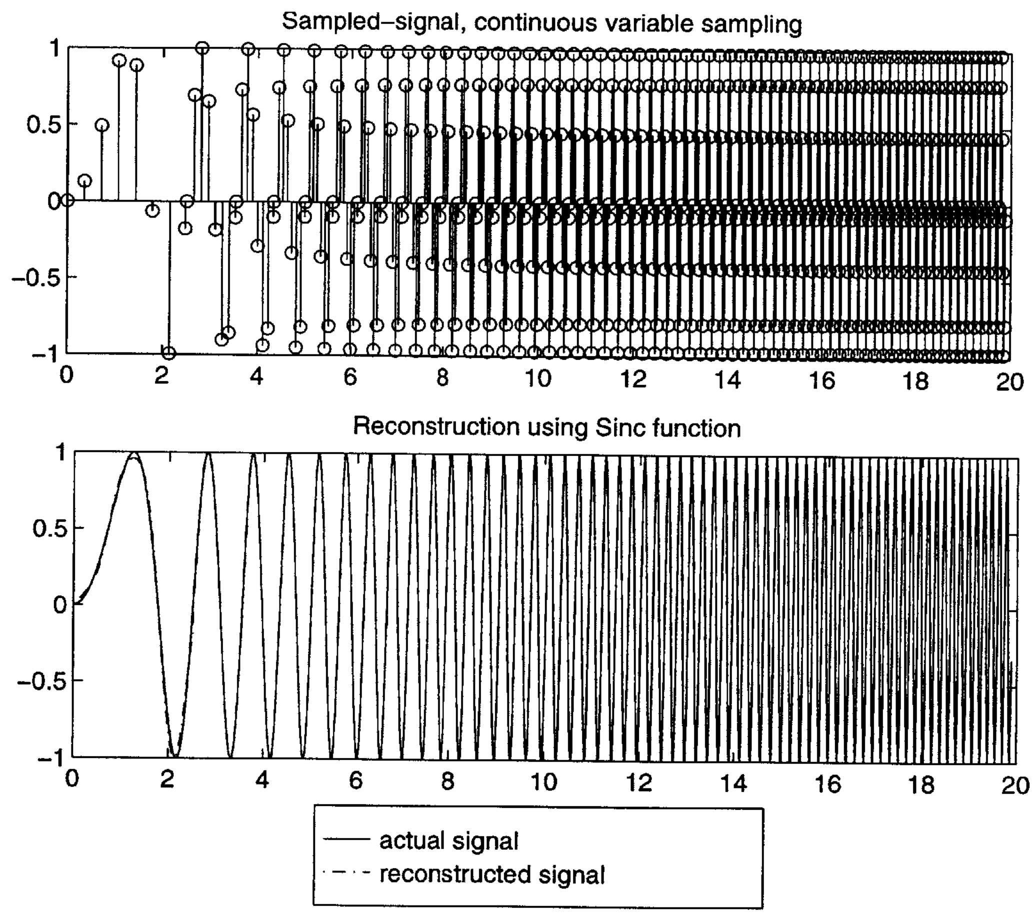

Figure 2-6: Signal Reconstruction Using Sinc Function

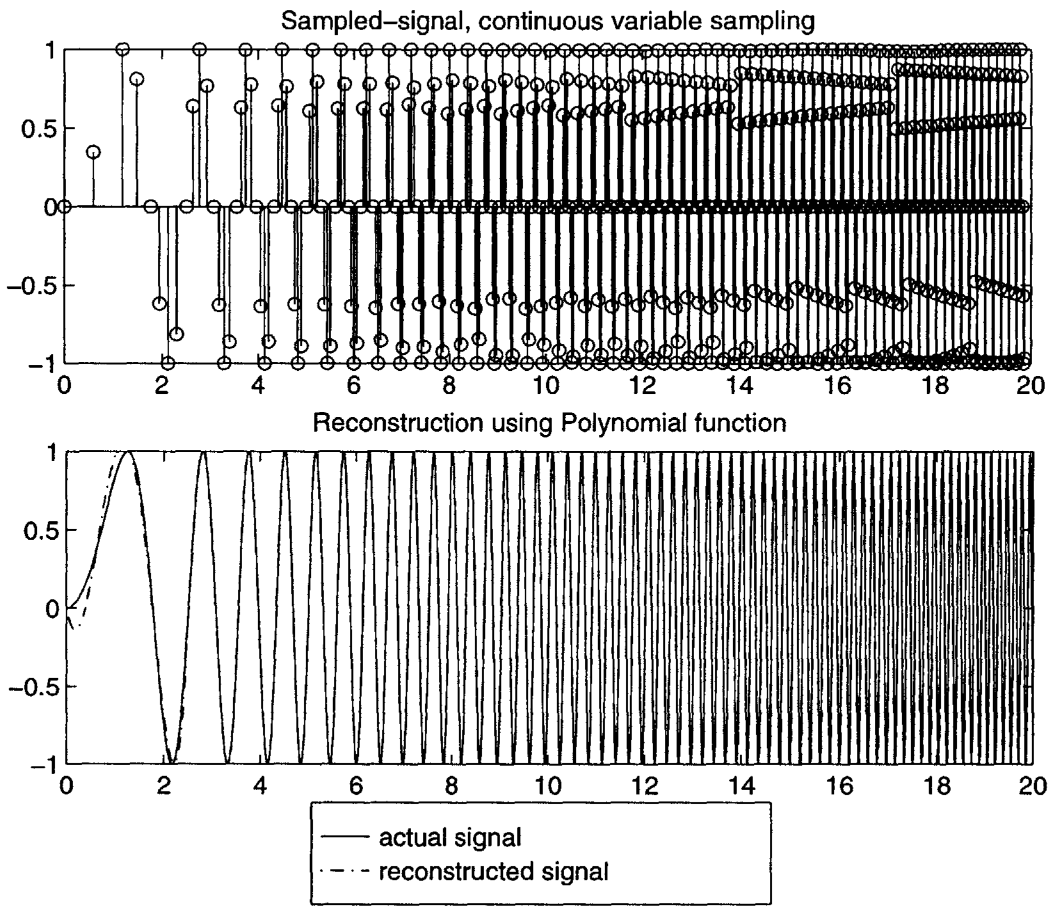

Figure 2-7: Signal Reconstruction Using Polynomial

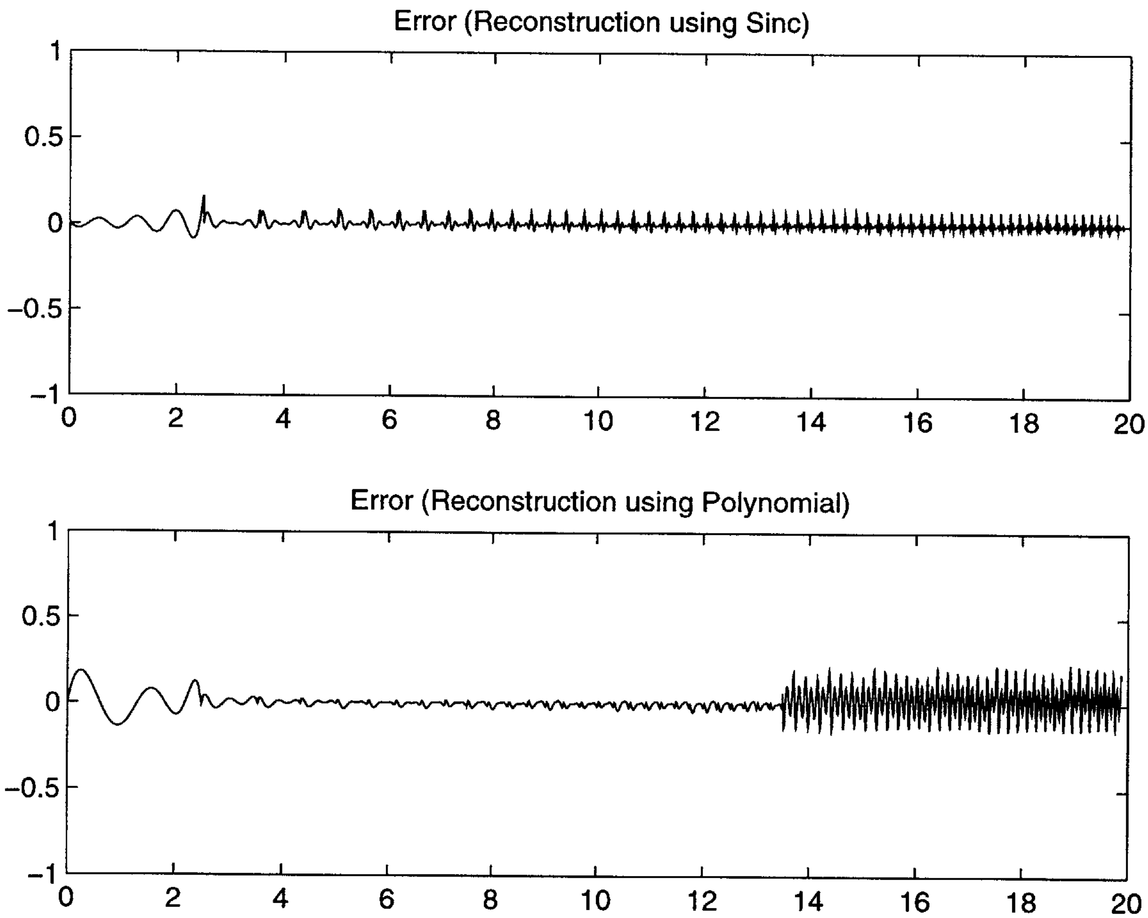

Figure 2-8: Errors of Signal Reconstruction

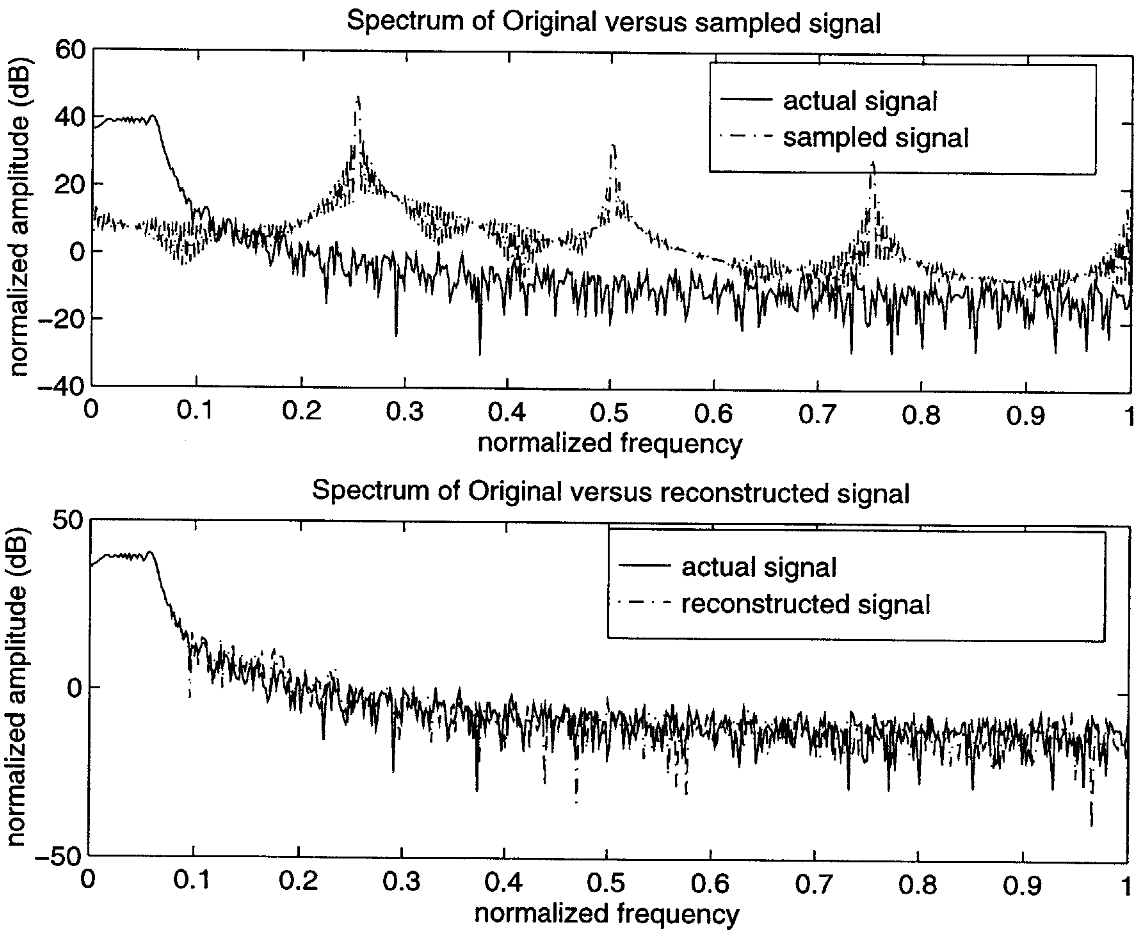

Figure 2-9: Spectrum of Original, Sampled and the Reconstructed Signal

Chapter 3

Applications to Aerospace Systems

3.1 On-line Variable Sampling Scheme

3.1.1 Application to GHAME vehicle dynamics

Dynamics Analysis

The dynamics of the angle-of-attack α of the vehicle is usually described by a linear time varying differential equation. Following reference [5], the equation for perturbation of the angle-of-attack α after linearizing the aerodynamics coefficients is given by:

$$\alpha'' + \omega_1(\xi)\,\alpha' + \omega_0(\xi)\,\alpha = f(\xi) \tag{3.1}$$

when $\omega_1(\xi)$ and $\omega_0(\xi)$ are slowly varying, the dominant approximation to the general solution of Equation 3.1 is given by [2]-[5]:

$$\hat{\alpha}(\bar{\xi}) = C_1\hat{\alpha}_1(\bar{\xi}) + C_2\hat{\alpha}_2(\bar{\xi}) \tag{3.2}$$

where

$$\hat{\alpha}_1(\bar{\xi}) = \exp\left(\int_{\xi_0}^{\bar{\xi}} k_r(\tau_0)d\tau_0\right) sin\left(\int_{\xi_0}^{\bar{\xi}} k_i(\tau_0)d\tau_0\right) \tag{3.3}$$

$$\hat{\alpha}_2(\bar{\xi}) = \exp\left(\int_{\xi_0}^{\bar{\xi}} k_r(\tau_0)d\tau_0\right) cos\left(\int_{\xi_0}^{\bar{\xi}} k_i(\tau_0)d\tau_0\right) \tag{3.4}$$

C_1, C_2 are arbitrary constants. Since these are fast scale solutions, they primarily describe variations in frequency and phase of the solution. In light of the sampling theorem, they give us the first approximation to the sampling of the solution of Equation 3.1. The fast solutions which represent rapid motion are sinusoidal functions with variable frequency. Once we obtain these solutions from the differential equation, we can use the algorithm developed in Section 2.2.1 to perform sampling. Thus, we will have developed a sampling scheme which is asymptotically optimal.

On-line Implementation

The method of frequency estimation outlined in Section 2.2.2, can handle the case where the dynamic behavior of the vehicle is unknown. In this study, we assume that we have an on-board sensing system to measure the angle-of-attack. This can be done, for instance, by using wind vanes or pressure tappings. The GHAME model shown in Figure 3-1 that has been developed in the previous work [6] is used in this study. We would like to sample the measured data on-line using continuosly changing sampling rate and compare the results to the one obtained using constant sampling rate. The block diagram for the on-line sampling scheme is presented in Figure 3-2. The analysis is summarized in Figures 3-3 - 3-5. Using the FFT analysis we can calculate the changes in frequency of the measured signal. The result is shown in Figure 3-3. In the on-line implementation, an on-line FFT is performed to make an estimate of the signal frequency in the interval of interest. The variable sampling is carried out based on the estimated frequency variations. Figure 3-4 depicts the comparison between the continuously changing sampling and constant sampling. The potential savings in memory allocation using the first method are shown in Figure 3-5. The results also indicate the potential benefit in computational time if we use a smaller vector size to represent the signal.

As mentioned previously, these potential savings can be very useful for many applications such as display, time history analysis or control. For the GHAME vehicle,this method is attractive particularly if a fully autonomous controller is to be designed. In this case, the success of such a system in practice would rely on how accurately and quickly the necessary information (for gain calculation) can be acquired using the on-board sensing system and, more importantly, data processed by an on-board computer.

3.1.2 Application to VTOL Aircraft

A similar scheme is implemented in the VTOL aircraft dynamics. A model that has been developed in the previous study [6] is shown in Figure 3-6. The model represents an on-line simulation of the pitch attitude angle θ dynamics during the hover-forward flight transition. The measured data is sampled on-line with the scheme shown in

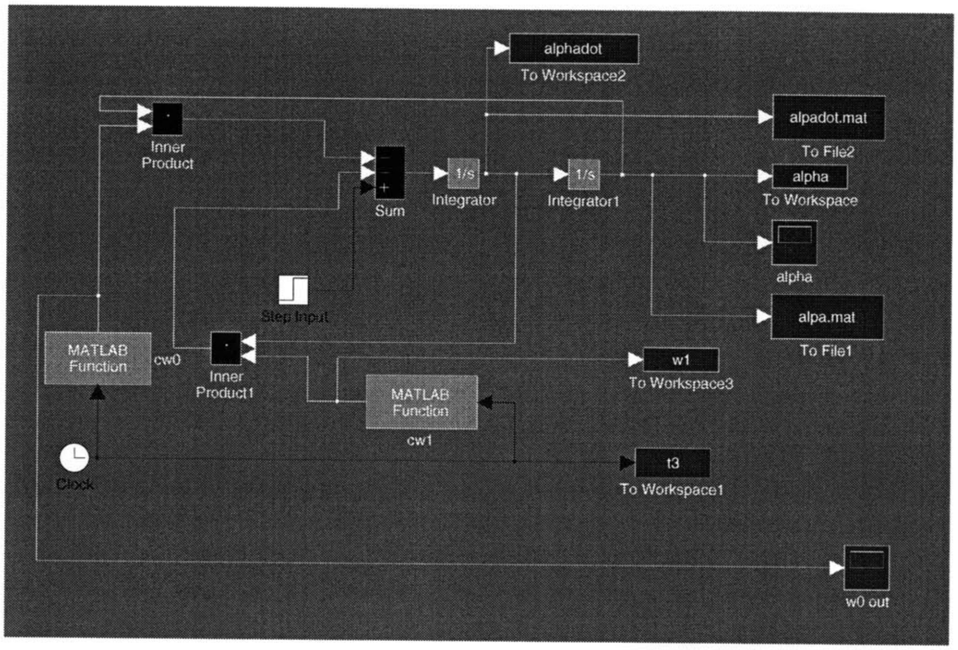

Figure 3-1: Block Diagram for GHAME vehicle dynamics

Figure 3-7. Again, an on-line FFT technique is performed during the sampling process. The result of the on-line sampling is presented in Figure 3-8. The benefit of the continuous variable sampling is not as obvious as that of in the case GHAME vehicle. As we can observe from the figure, there is not much difference between the highest and the lowest frequencies.

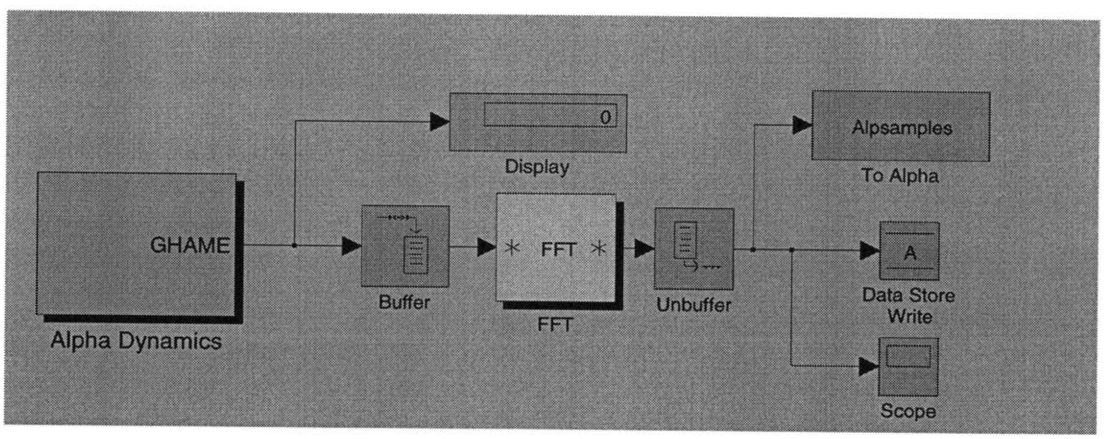

Figure 3-2: Block Diagram for GHAME On-line Sampling Scheme

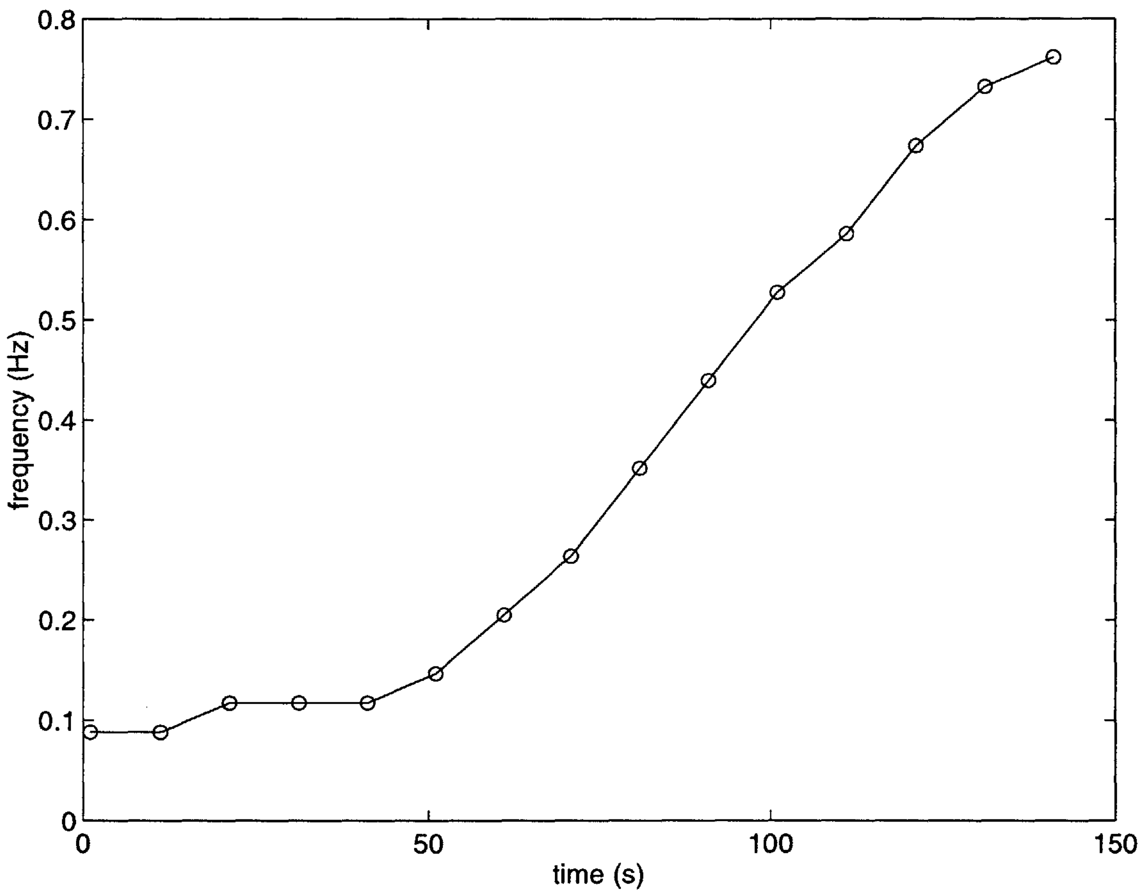

Figure 3-3: Frequency behavior of measured angle of attack α

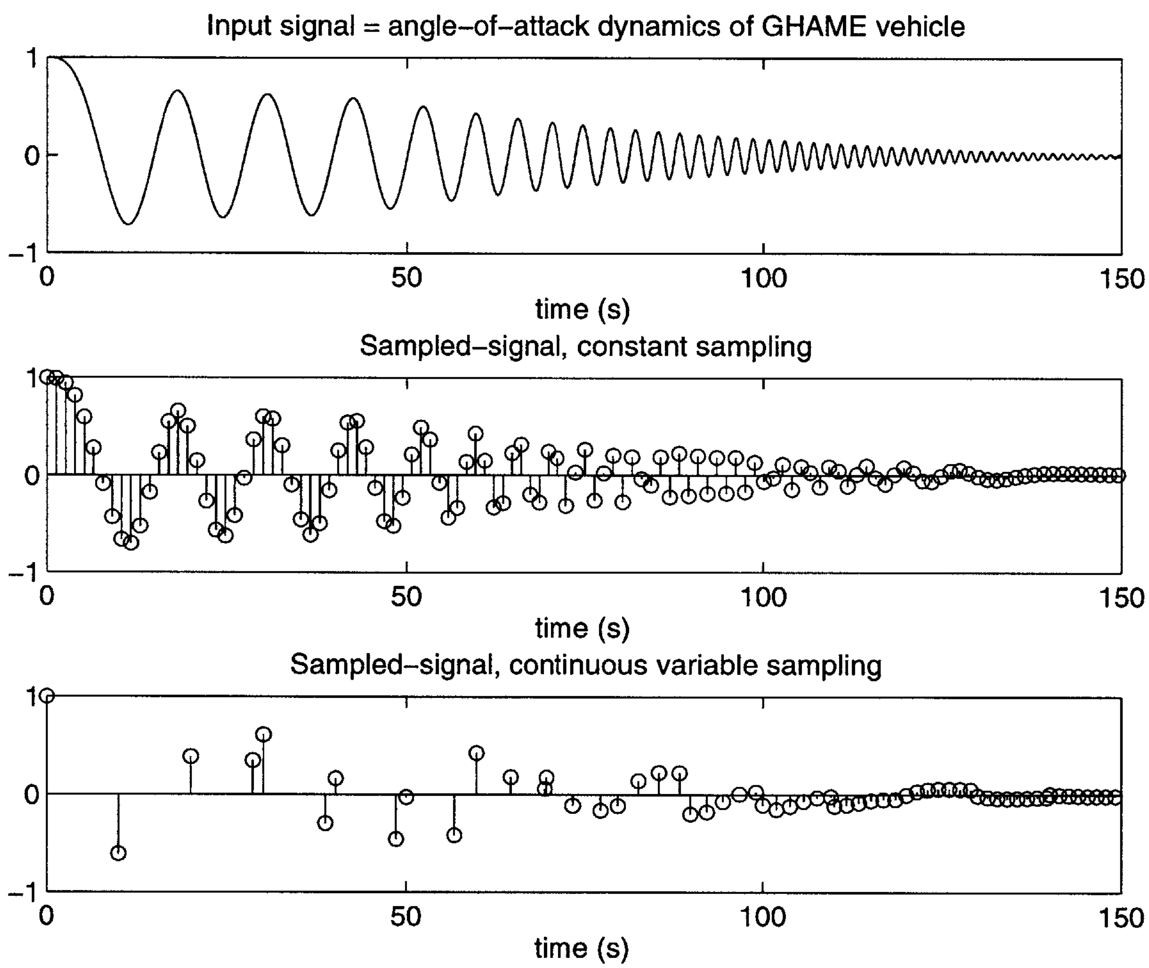

Figure 3-4: Comparison of constant and continuous sampling

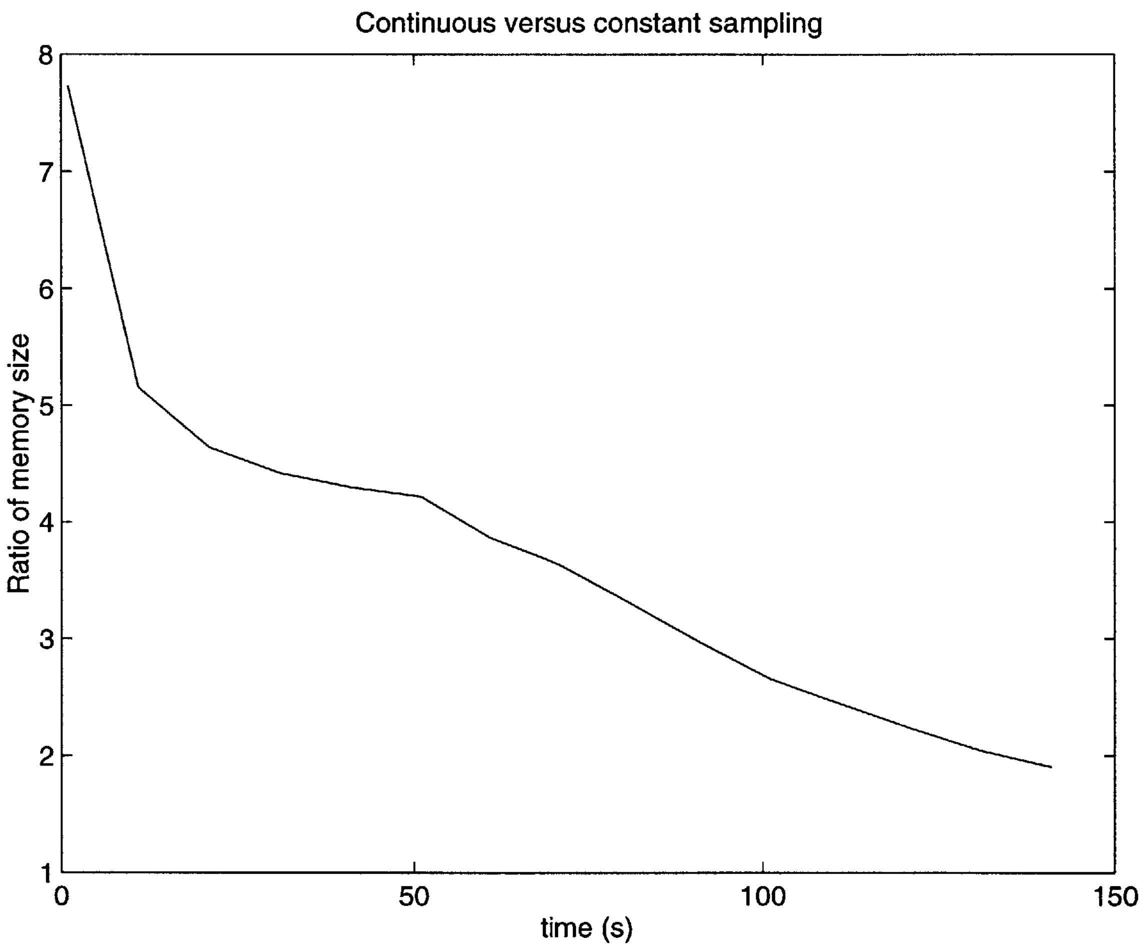

Figure 3-5: Memory potential saving

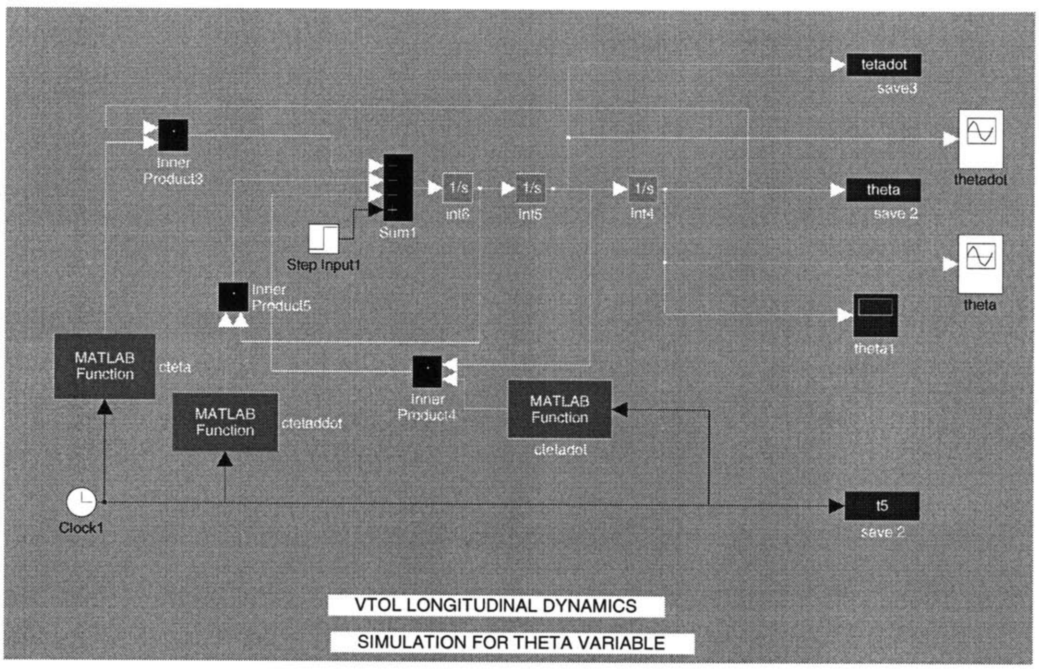

Figure 3-6: Block Diagram for VTOL θ Dynamics

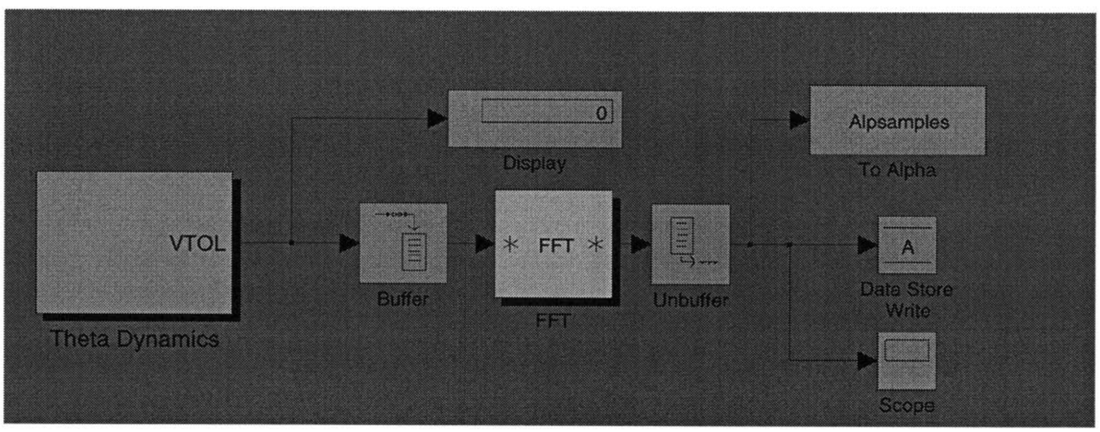

Figure 3-7: Block Diagram for VTOL On-line Sampling Scheme

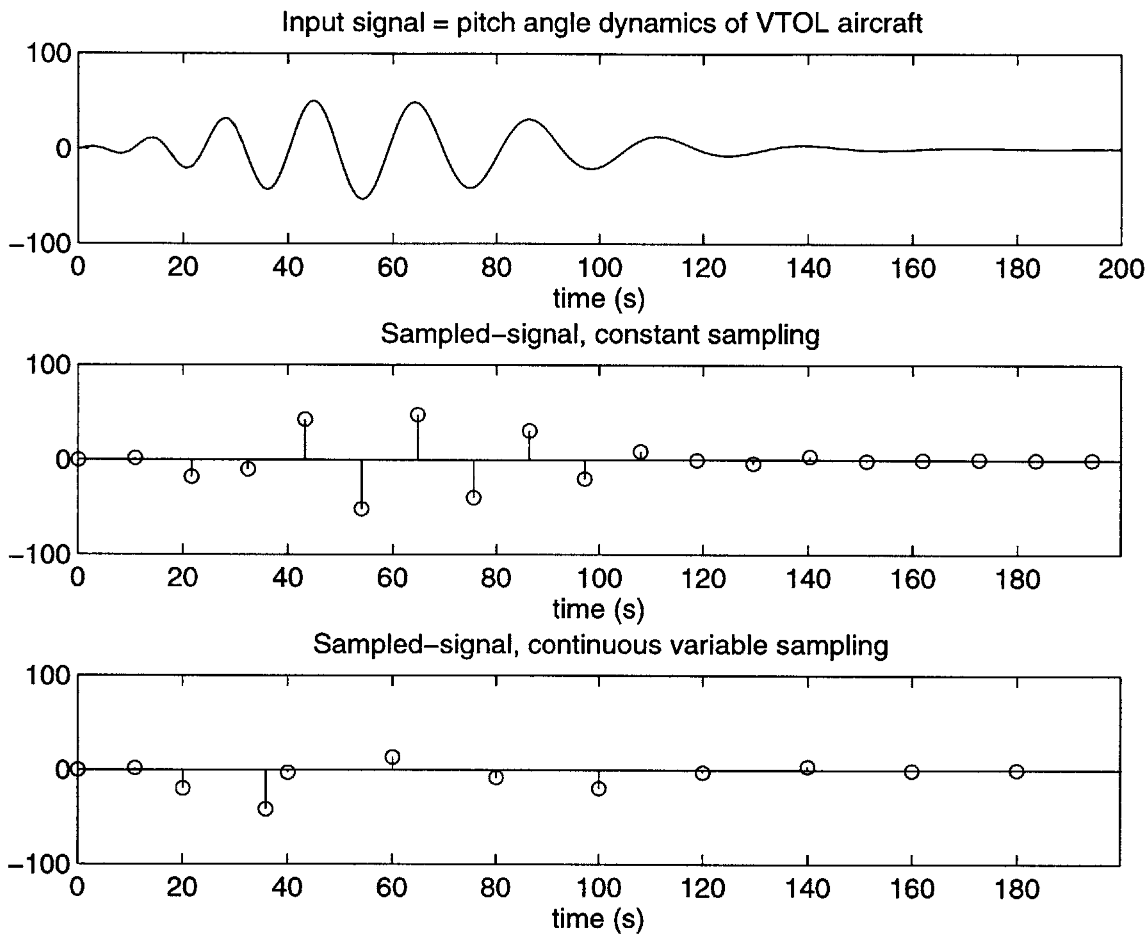

Figure 3-8: Online Sampling Results for a VTOL Aircraft

3.2 Signal Multiplexing Approach for an Aircraft Avionics System

3.2.1 Background

Avionics technology plays an increasingly important role in a modern aircraft's operation. The term avionics (aviation-electronics) embraces many aspect of electronic subsystems. In general it consists of *navigation, communications , flight control, engine control, flight management, subsystem monitoring and control, collision avoidance, weather detection, and emergency aid system.*

A study by AGARD [9] in 1985 predicted and identified key areas in the research and development of future avionics systems that are commensurate with the higher demand on cost and mission effectiveness of aircraft. The identified technology needs include: *system design methodology, high data rate technology, system management function, performance standards, and software (e.g. parallel processing).*

The current research is motivated by a need to handle the complex data/information flow within the modern aircraft systems characterized by different sources and receiver modules which process the data at different rates. As a case study, operation of a commercial aircraft will be considered. The roles of the avionics system will be described by various tasks performed during the mission.

To achieve the mission, aircraft must have reliable avionics systems that provide accurate information for navigation purpose and operation of the aircraft in general. The functions required of the avionic systems during a typical mission are as follows:

1. **Navigation** The aircraft carries inertial navigation systems (INS) that operate together with alternative navaid subsystems. The multisensor navigation system can include the following elements for updating the INS with information on position and velocity.

   (a) Position data

   - Radar. Including multimode radar (MMR)
   - Radio navigation aids. TACAN, Loran, Omega, GPS, VOR, DME, VOR/TACAN, and JTIDS RelNav
   - Position fixes. Flyover, FLIR, TERCOM, and star sightings

   (b) Velocity data

   - Doppler radar
   - Indicated Airspeed (IAS)
   - Global Positioning System (GPS)
   - EM-Log or Speedlog

(c) Attitude/heading
- Star tracker
- Multi-antenna GPS

2. **Mission planning**. On-board software replans routes (defined during pre-flight mission planning) through terrain based en-route observations. The routes are updated using information from digital maps of terrain and real-time detection of other aircraft.

3. **Special function**. For military aircraft, the role of avionics systems manifests itself in special subsystems such as weapon management and electronic countermeasures.

In some situations, all ground or satellite navigation systems could be disabled and the aircraft is required to fly over hostile terrain. Under these conditions, aircraft operation, sensor management, and information handling will increase the workload of an operator, so that a fully automatic operation will become necessary. To achieve the needed mission accuracy, therefore, future inertial navigation systems will most likely include more navigation sensors and subsystem modules. Consequently, a better technique is needed to handle complex information flows in the navigation subsystem.

Navigation data acquired by different sensors are usually sent to the on-board subsystems, i.e. flight control, flight management, engine control, communication protocol and crew display. In such a multisensor navigation system, the sensors are integrated by means of a mission computer. The rates at which the data is sampled can, in general, differ by an order of magnitude (See Tables).

It was observed by Ramnath that the presence of data rates of different orders of magnitudes suggests the use of multiple time scale methods [1]-[4] to track the different signals efficiently and effectively by a multiplexing approach.

Source-Destination	ϵ
Nav. Compt. to Antenna Servo vs. Gimbal pickoffs to Antenna servo	0.25
Nav. Compt to FCC vs. Gimbal pickoffs to clutter tracker	0.05
Clutter tracker to Kalman Filter vs. Radar to Kalman Filter	0.02
Nav. Compt to FCC vs. Accell. to SAR motion computer	0.01
Ext. to Kalman Filter vs. Accell. to SAR motion computer	0.001
Ext. to Kalman Filter vs. Accell to Nav. Compt.	0.00033

Table 3.1: Ratio of Rates between Different Signals

3.2.2 Signal Multiplexing Algorithm

For simplicity, the multiplexing scheme is described by a scenario in which there are only two signals to be managed by a central processing unit. The two signals modeled are two sinusoidal signals with different frequency behavior. The two signals are $f(t) = sin(t^2)$ and $f(t) = sin(t^{1.5})$ respectively. In conjunction with the information received by on-board sensing system of an aircraft, these signals represent two data measurements which are updated at two different rates e.g. the first one is a velocity data received by Doppler radar and the second one is an attitude data received by multi-antenna GPS. The behavior of these two signals is shown in Figure 3-11. The constant sampling and the continuously-changing rate sampling are presented in Figure 3-12 and 3-13, respectively. We can observe from these figures that there are idle times between sampling points where the processing unit does not operate. The idea of the multiplexing is filling in these idle times with the sampling of other signals. Figure 3-14 shows the schematic diagram of the multiplexing technique. Figure3-15 depicts the diagram for the sychronization scheme. The algorithm for the multiplexing of two incoming signals is summarized as follows

1. Identify input signals, $S_1(t)...S_2(t)$.
2. Measure the rise time (t_r) or the peak time of each signal to estimate the initial frequency f_{init} and to define the time to start the sampling, t_s.
3. Compare and synchronize the time to start sampling $S_1(t)$,t_{s1}, and the time to start sampling $S_2(t)$,t_{s2}

DESTINATION	SOURCE						
		IMU Pulses		Gimbal	Navigation Computer		
	Ext.	Gyro	Accell.	pickoffs	Posit.	Vel.	Torq. Comm.
Gyro Torquers	-	-	-	-	-	-	50
Paltform gimbal servos	-	Analog	-	Analog	-	-	-
Navigation computer	-	-	3000	-	-	-	-
Kalman Filter	<1	-	-	-	50	50	-
Antenna servos	-	-	-	200	50	-	-
Flight Control Computer	-	-	-	10	10	10	-
Clutter Tracker	-	-	-	200	-	25	-
Ballistics computer	-	-	-	50	25	25	-
SAR motion compensation	-	-	1000	1000	50	50	-
SAR cursor	-	-	-	-	50	50	-
Navaid aiding signals	-	-	-	-	-	50	-

Figure 3-9: Typical Data Rates from Source to Destination (Hz) [8]

DESTINATION	SOURCE					
	Local Inert.	Air Data	Nav-Aids	Kalman Filter	Radar	Clutter Tracker
Gyro Torquers	-	-	-	-	-	-
Paltform gimbal servos	-	-	-	-	-	-
Navigation computer	-	-	-	<1	-	-
Kalman Filter	-	-	<1	-	50	<1
Antenna servos	-	-	-	50	-	-
Flight Control Computer	30	10	-	-	-	-
Clutter Tracker	-	-	-	-	-	-
Ballistics computer	-	25	-	-	-	-
SAR motion compensation	-	-	-	-	-	-
SAR cursor	-	-	-	-	-	-
Navaid aiding signals	-	-	-	-	-	-

Figure 3-10: Typical Data Rates from Source to Destination (Hz) [8]

4. Define a window interval Δt_1 and Δt_2. For a deterministic case, a rectangular window can be used. A rectangular window is defined as:

$$w[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

For a more general case different windows (e.g. Kaiser or Hanning window) might be necessary to guarantee the robustness of peak detection algorithm. Kaiser and Hanning windows can be viewed as a tapered form of a rectangular window. The use of these windows can reduce or minimize the ringing effects that occur when a rectangular window is used. Kaiser and Hanning window are defined respectively as follows,
Kaiser:

$$w[n] = \begin{cases} \frac{I_0\left[\beta(1-[(n-\alpha)/\alpha]^2)^{0.5}\right]}{I_0(\beta)}, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

Hanning:

$$w[n] = \begin{cases} 0.5 - 0.5cos(2\pi n/M), & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

5. Compare f_{init1} and f_{init2}. If $f_{init1} > f_{init2}$ (case 1), take sample(s) of S_1 within $t = 0$ to $t = t_{s2}$. Otherwise (case 2), take sample(s) of S_2 within $t = 0$ to $t = t_{s1}$.

6. If case 1 is true, take the 1st sample of S_2. Take the next samples of S_1 within $t = t_{s2} + \Delta t_1$. Otherwise, take the 1st sample of S_1 and the next samples of S_2 within $t = t_{s1} + \Delta t_2$.

7. To sample the signals in the subsequence windows, perform the frequency prediction by taking the FFT of the windowed signal. To make a prediction we need to do the differential FFT (DFFT) and this can be done by buffering (storing) the sequence of the previous window.

8. If the sample of S_1 overlaps with the sample of S_2, shift it by a small amount of time ϵ as shown in Figure 3-15.

9. Combine the shifted and unshifted signals to form an appropriate sample of a signal.

10. Repeat the procedure fot the next windows

The result of the signal multiplexing is shown in Figure 3-16. In this example, we are dealt with two time-varying signal. Figure 3-12 and 3-13 show the benefit of using variable rate sampling scheme which is summarized in the following table.

Signal	Constant sampling	Varible-rate sampling
Signal 1 $f(t) = sin(t^2)$	501	126
Signal 1 $f(t) = sin(t^{1.5})$	84	28

Table 3.2: Comparison of memory allocation in terms of number of samples

We can extend the result to n signals and thus observe the potential savings that can be harnessed with this technique. Figure 3-17 shows different signals with different frequencies received by a typical avionics system. Using the previously described technique we can implement the signal multiplexing into the avionics system. The block diagram ot the algorithm is shown in Figure 3-18. In general, the data processed by an on- board sensing system comes from an online measurement. Thus we need to have an estimate of the signal frequencies in a real-time. In this case, we can implement the frequency prediction using online FFT outlined in 2.2.2. The real-time implementation of the multiplexing is beyond the scope of the thesis.

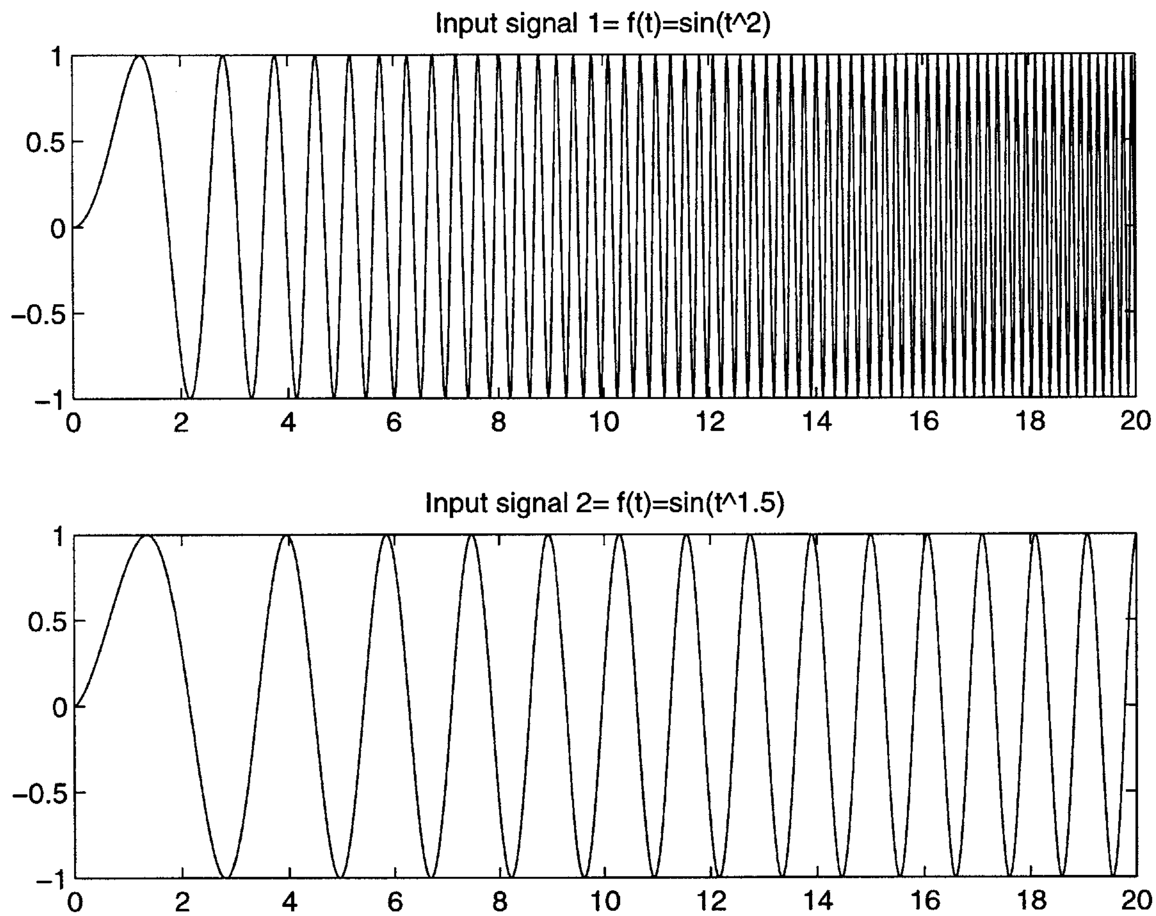

Figure 3-11: Two Different Signals Processed by an Onboard Computer

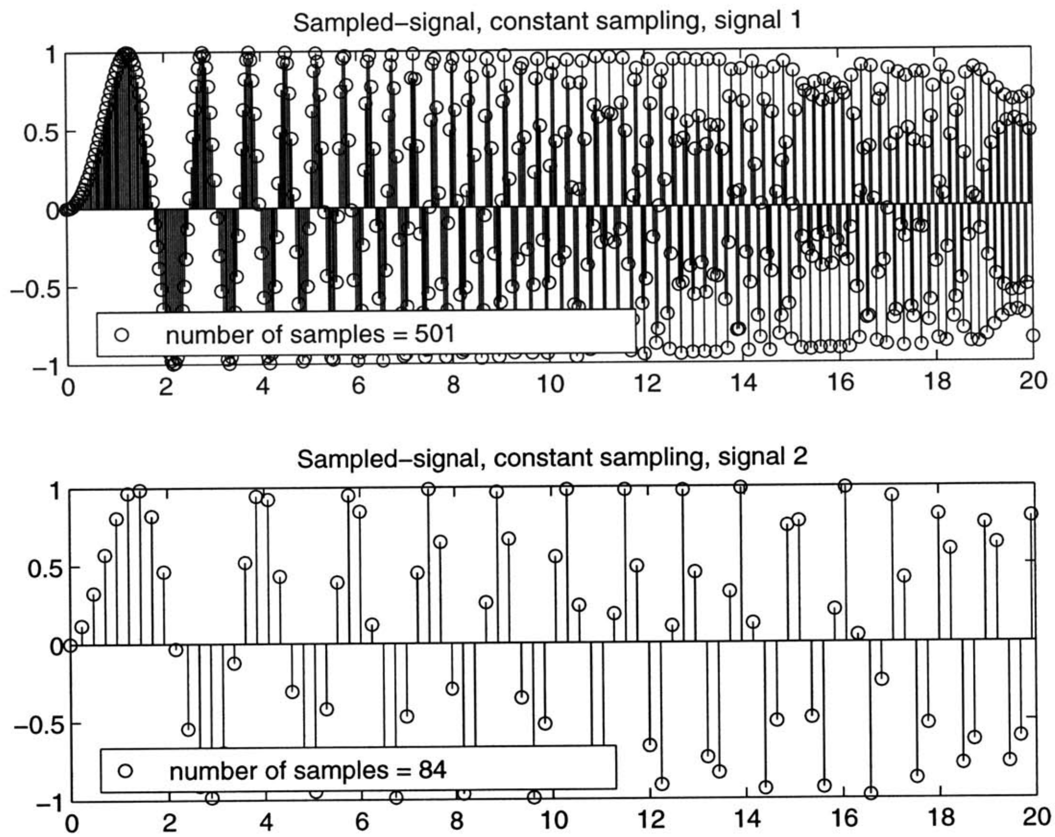

Figure 3-12: Constant Sampling

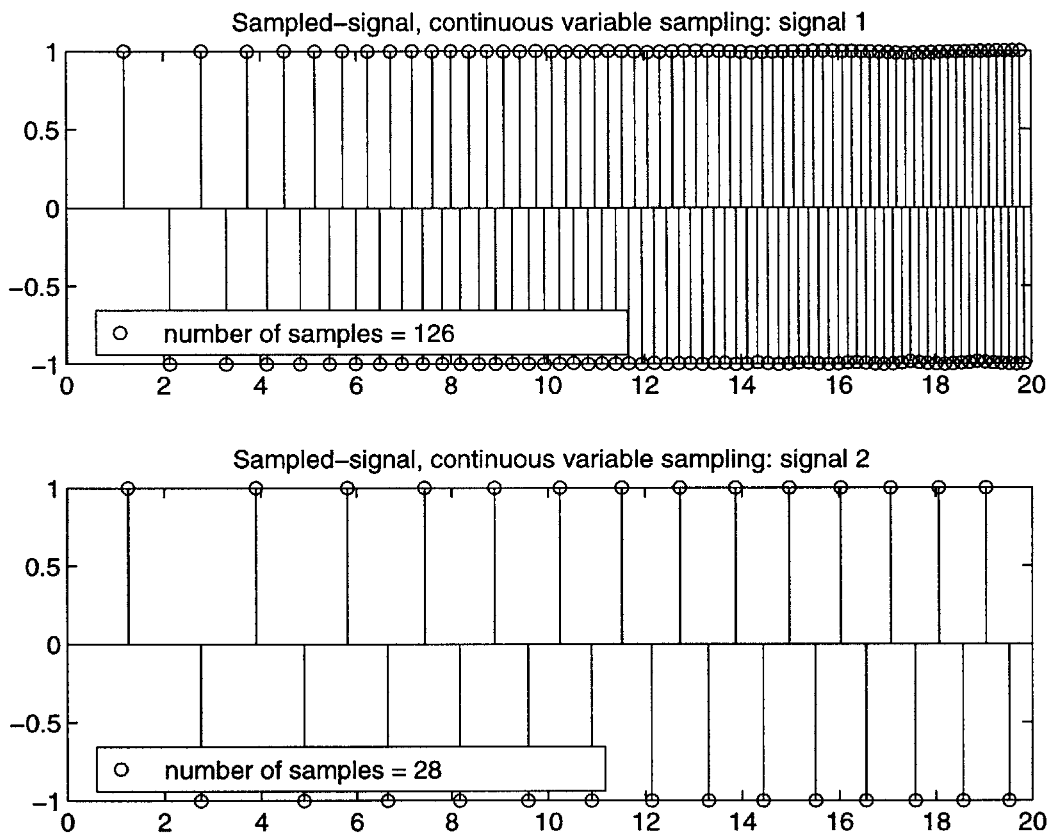

Figure 3-13: Continuously-changing Rate Sampling

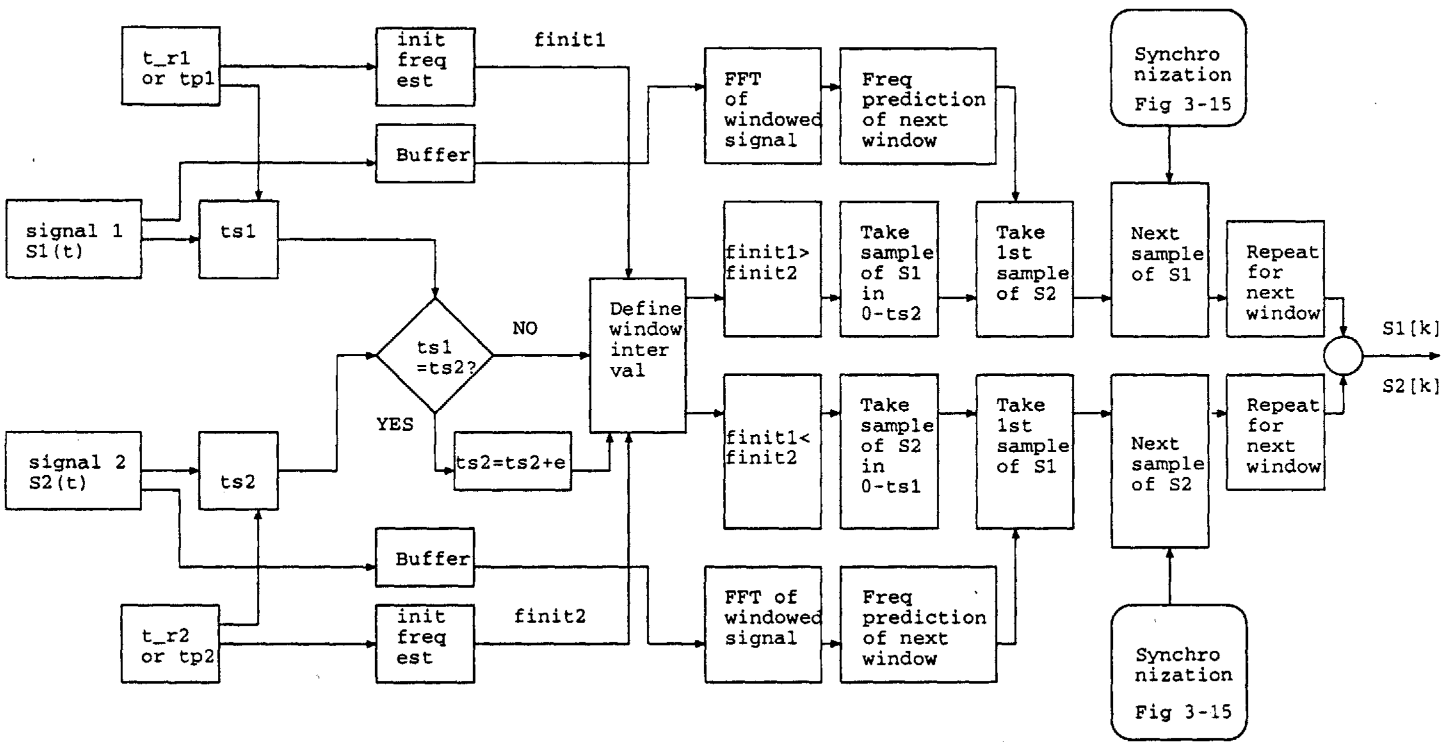

Figure 3-14: Multiplex Diagram

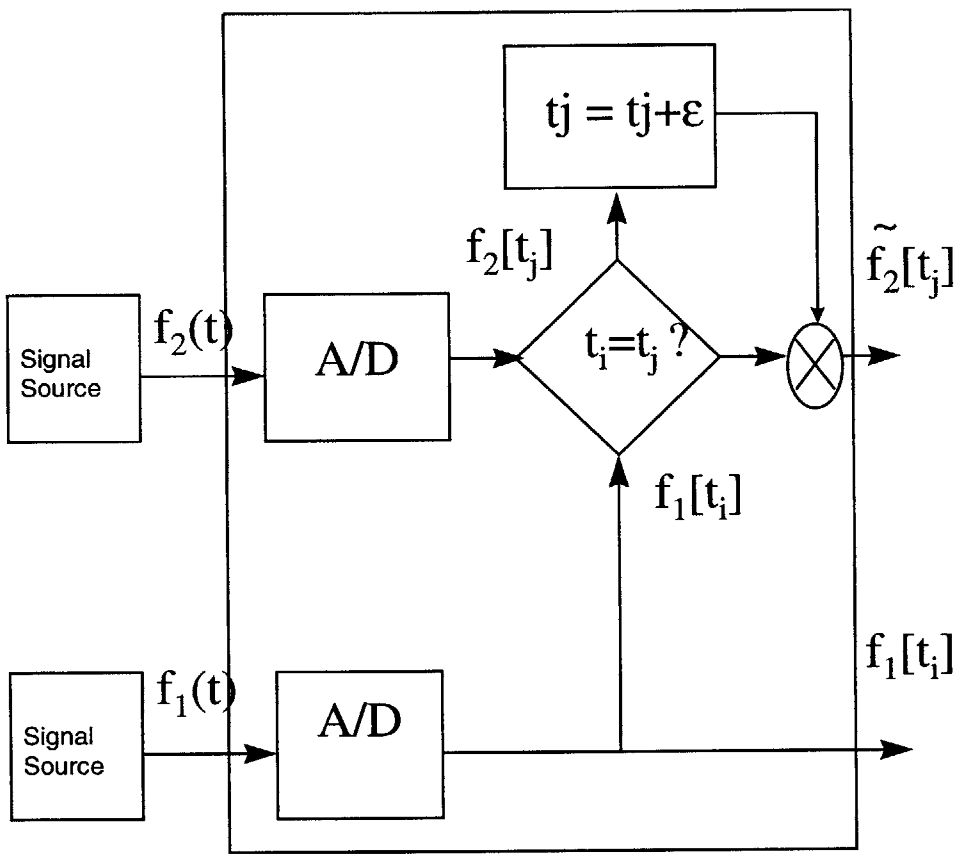

Figure 3-15: Synchronization Diagram

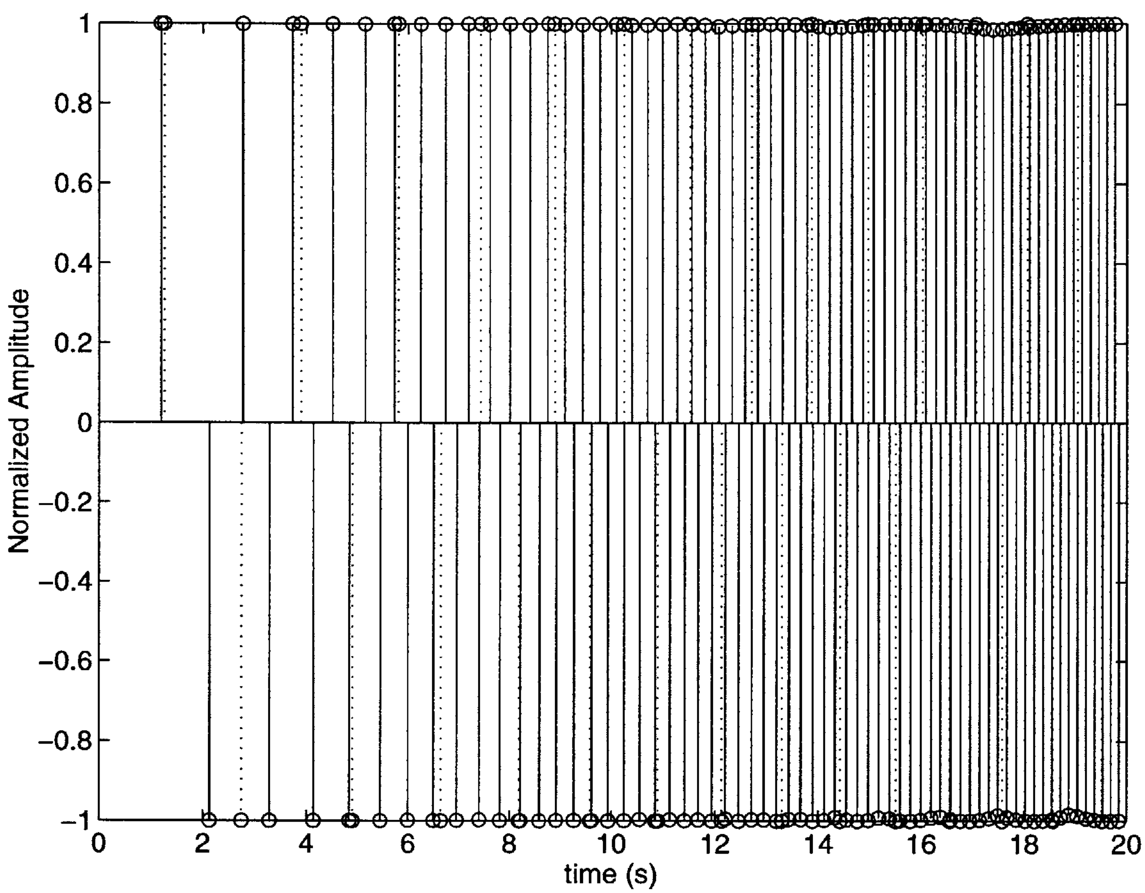

Figure 3-16: Multiplexing of the Two Input Signals

..... $\equiv$ $f(t) = sin(t^{1.5})$

—— $\equiv$ $f(t) = sin(t^2)$

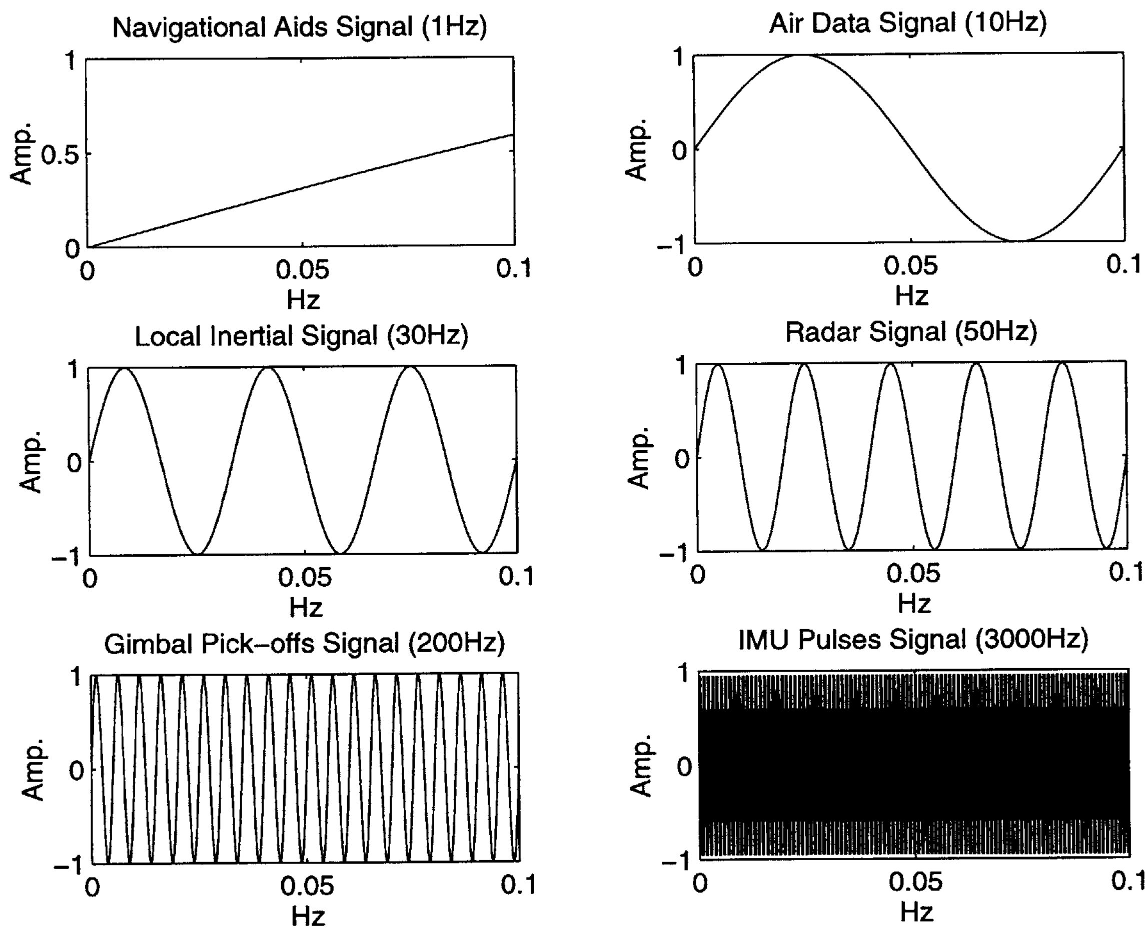

Figure 3-17: Different Signals Processed by an On-board Computer

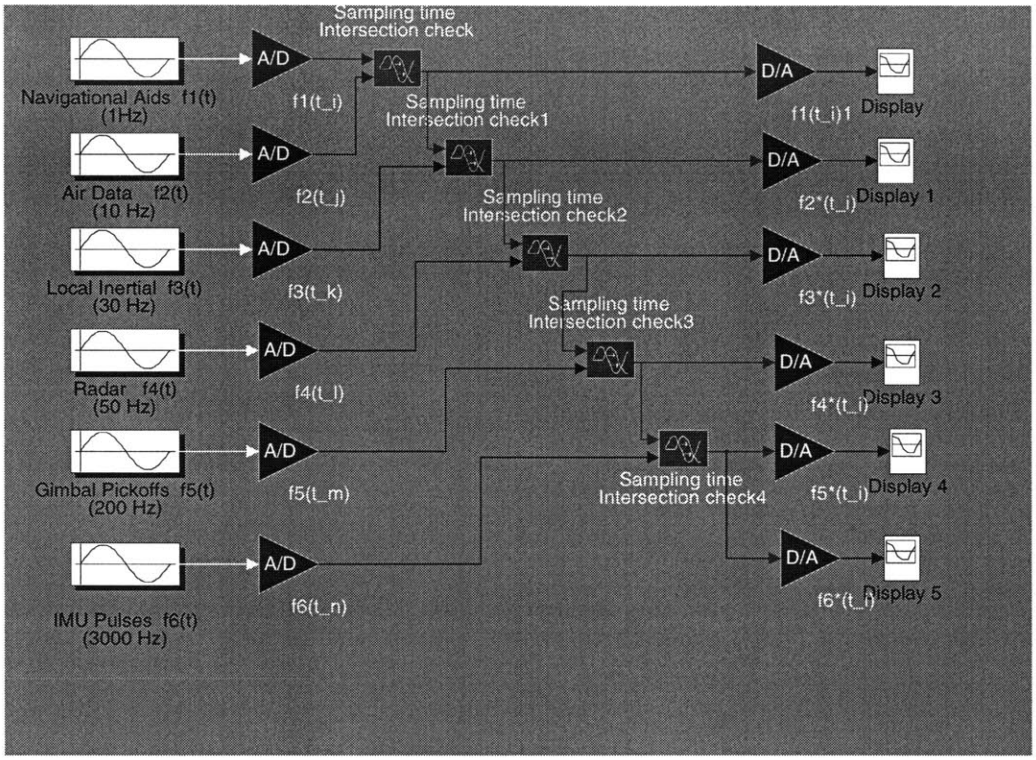

Figure 3-18: Multiplexing Diagram for an Avionic System

Chapter 4

Discussion and Conclusions

4.1 Concluding Remarks

The rapid advances in digital technology create many new windows of opportunity. Digital signals offer various advantages over the analog equivalents. In the technology of radio broadcast, for instance, trasmitting a digital rather than an analog signal offers clearer sound, interference-free reception and space for dozens of stations in the bandwidth that carries just two or three analog signals. The focus of the digital technology is the sampling theorem enabling representation of a continuous signal with equivalent discrete sequences. The traditional purpose of sampling has been to retain the information content within a certain signal in the most economical manner, primarily to reduce computational load or reduce memory requirements. In the realm of time-invariant signals, theoretically, the Shannon's sampling theorem guarantees a lossless reconstruction from samples taken at the Nyquist's rate. The extension of the theorem to include time- varying cases has not been fully addressed.

The contribution of of this work is centered on the top level demonstration of Ramnath's variable sampling approach incorporating multiple time scales theory. The approach is illustrated by means of analytical examples as well as in the context of a class of flight vehicle dynamics through variable flight conditions. The results of simulations show the promise of benefits to be gained by performing a continuous variable sampling with regard to computational and memory requirements. For the aerospace vehicle design, this benefit can pave the way for the design of a fully autonomous vehicle. This scheme can be extended to various engineering applications such as active vision for robotics, real-time control systems design, and speech processing.

The benefit of the signal multiplexing technique was also presented through a simulation of an aircraft avionics system. In general, there are two ways in which signals can be multiplexed: frequency division multiplexing (FDM) and time division multiplexing (TDM). In this work, the latter form of multiplexing is considered. In a digital mode, a channel can be shared by interleaving the pulses of different signals so that the channel can be shared on the time basis rather than on a frequency basis. In the simulation, it was shown that since the information is discrete in time, the transmission scheme can provide quiet periods between transmissions during which time other signals can be sent. Combined with the variable sampling scheme, this creates an efficient way to process and transmit multiple signals.

4.2 Recommendations for Further Work

Further work is required for the benefits to become useful for practical applications. The following is the list of suggestions:

1. Assess the real-time application. The real advantage can only be measured if the scheme is implemented on a real-time basis i.e. the computation efficiency of the variable sampling algorithm must outweigh the simplicity of a constant rate sampling. This can be accomplished by implementing the algorithm using C languange.

2. Investigate the benefits of a dedicated chip design as opposed to a multi-purpose computer for implementing the algorithm.

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